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ABSTRACT.
A consistent theoretical description of the global plasma modes that are excited or suppressed by the presence of a high energy particle population is given. The process stabilising resistive modes that are considered to be responsible for the so-called “sawtooth” oscillations of the central region of the plasma column is identified and described analytically.

Recent experiments on well-confined plasmas in regimes of thermonuclear interest have convincingly demonstrated the influence of high energy particle populations on the stability of global modes. During neutral beam injection experiments, fishbone-like bursts in the temporal evolution of the soft X-ray emission and of the poloidal magnetic field were observed by the PDX machine experiments [1]. These bursts were correlated with the scattering of beam ions. It was then pointed out [2], and subsequently reported [3], that the excitation of a mode with toroidal mode number $n = 1$ and dominant poloidal mode number $m = 1$, and with a frequency of oscillation ω, in the plasma rest frame, about equal to the thermal plasma ion diamagnetic frequency $\omega_{di} = -(c/eBrn)(dp_i/dr)$, evaluated at the radius r_o of the surface where the magnetic helical parameter $q(r) = 1$, was a likely explanation for the onset of these oscillations. In an alternative interpretation [4], the limit $\omega_{di}/\bar{\omega}_{Dh} = 0$ was considered and a mode with $\omega = \bar{\omega}_{Dh}$ was proposed as the relevant candidate. Here $\bar{\omega}_{Dh}$ is a characteristic value (see below) of $\omega_{Dh}^{(o)}(E,\zeta,r)$, the bounce-averaged magnetic drift frequency of energetic particles with poloidally trapped orbits, with E their energy, $\zeta \equiv (v_\perp/v)^2 (B_o/B)$ the pitch angle in velocity angle, and B_o the value of the magnetic field B on the magnetic axis. In both models, the instability relies on a resonant interaction between the mode and energetic trapped ions satisfying the condition $\omega = \omega_{Dh}^{(o)}(E,\zeta,r)$. This viscosity-like process allows the excitation energy associated with the plasma pressure gradients to be tapped.

The relationship between these two models has been clarified recently [5-9]. In Ref.[5] it was first reported that, in the limit $\hat{\omega}_{di} \equiv \omega_{di}/\bar{\omega}_{Dh} << 1$, two distinct "fishbone" regimes exist for different values of the energetic ion poloidal beta, β_{ph}, separated by a stable window. The two regimes coalesce, and the stable window disappears when $\hat{\omega}_{di} \sim 1$, which is the ordering appropriate to the early experiments of the PDX machine. An example of the stable domain that can separate these two regimes in the plane defined by the parameters $\hat{\omega}_{di}$ and $\hat{\beta}_{ph} \equiv (\omega_A/\bar{\omega}_{Dh})(\epsilon_o/s_o)\beta_{ph}$ is shown in Fig.1 for $\hat{\gamma}_{MHD} \equiv \lambda_H\omega_A/\bar{\omega}_{Dh} = 0.1$. Here, λ_H is the ideal MHD stability parameter [10,11] related to the negative of the well-known functional δW, $\omega_A \equiv v_A/\sqrt{3}\, R_o$, $v_A = (B^2/4\pi m_i n_i)^{1/2}$, $s_o \equiv r_o q'(r_o)$ the local magnetic shear parameter, $\epsilon_o \equiv r_o/R_o$, and R_o is the torus major radius. The appropriate definition of β_{ph} in the anisotropic limit $p_{\|h} \sim \epsilon_o p_{\perp h}$, with $p_{\|h}$ and $p_{\perp h}$ the components of the energetic ion pressure tensor, is $\beta_{ph} = -[8\pi/B_p^2(r_o)] \int_0^1 d\hat{r}\, \hat{r}^{3/2}\, d(\hat{r}^{1/2} p_{\perp h})/d\hat{r}$, where $\hat{r} \equiv r/r_o$ and B_p is the poloidal magnetic field.

The existence of a "stability window" is important for the understanding of plasma regimes recently attained by the JET machine, where the $m = 1$, $n = 1$ mode activity has been suppressed for periods, in some cases, exceeding 3s during ion cyclotron resonant frequency (ICRF) heating (where highly anisotropic ions with $p_{\perp h} \gg p_{\|h}$ and energies in the MeV range are produced) and/or during neutral beam injection [12]. However, while the absence of fishbone oscillations in these plasma discharges could be so understood [5-7], the problem of explaining the concurrent suppression of the larger and more important soft X-ray relaxation oscillations (the so-called "sawtooth oscillations") remained partly open [6]. In fact, the sawtooth relaxation is generally believed to be initiated by a low-frequency $m = 1$ mode with $\omega_{*e}/\omega_{di} < \mathrm{Re}\,(\omega/\omega_{di}) \leq ½$, which can be unstable for a wide range of values of the relevant plasma poloidal beta, $\beta_p = [8\pi/B_p^2(r_o)][\langle p\rangle - p(r_o)]$, in the presence of finite electrical resistivity (angle brackets denote volume average in the region where $q \leq 1$). Here ω_{*e} ($\omega_{*e}/\omega_{di} < 0$) is the frequency of the electron drift mode. This instability, usually referred to as the resistive internal $m = 1$ mode [10], is associated with a branch of the $m = 1$ dispersion relation that is different from that related to the fishbone instability. It is therefore important to show, as we do in this Letter, that full stabilization of resistive modes can be achieved for relatively small values of β_{ph} and realistic values of β_p.

When the plasma resistivity is neglected, the dispersion relation for $m = 1$ internal modes was shown in [5-8] to be

$$[\omega(\omega - \omega_{di})]^{1/2} = i\omega_A[\lambda_H + \lambda_K(\omega)], \quad (1)$$

valid for $\lambda_H + \mathrm{Re}\,\lambda_K > 0$, where $\lambda_K \sim \epsilon_o\beta_{ph}/s_o$ is a complex function of ω that is contributed by the energetic particle population. The real part of $\lambda_K(\omega)$ represents the work needed in order to displace the energetic particles. This work is done by the perturbed electric field, against the perturbed current density of the energetic particles in the plasma volume inside the $q = 1$ surface. For monotonically decreasing pressure profiles of the energetic particles it is positive (stabilizing) when $\omega/\bar{\omega}_{Dh} < 1$ and negative (destabilizing) when $\omega/\bar{\omega}_{Dh} > 1$. The imaginary part of $\lambda_K(\omega)$ is contributed by the energetic particles that resonate with the mode and can be viewed as a form of energy dissipation. These combined contributions can be rewritten as $\lambda_K \propto \int d^3\vec{x}\vec{\xi}^*\cdot(\nabla\cdot\tilde{\overleftrightarrow{P}}_h)$ with $\vec{\xi}$ the plasma displacement and $\tilde{\overleftrightarrow{P}}_h$ the perturbed pressure tensor of the energetic particles. The full definition of λ_K and details of its derivation can be found in Refs.[3,6-8]. In Fig.2 we show the real and imaginary parts of $\Lambda_K(\omega) \equiv (s_o/\epsilon_o\beta_{ph})\lambda_K(\omega)$. We have used the model distribution function given in Ref.[7], describing energetic ions, with mostly trapped orbits, accelerated by ICRF waves [12]. Note that $\mathrm{Re}\,\Lambda_K = 0$

for $\omega = \bar{\omega}_{Dh}$. For this distribution, and neglecting plasma shaping contributions, $\bar{\omega}_{Dh} = 1.2\, cE_h(1 + 2s_o)/(Z_h eBR_o r_o)$, with E_h the average value of E and $Z_h e$ the fast ion charge.

Equation (1) has a "high frequency" branch with $\omega_{di}/2 \le \omega \lesssim \bar{\omega}_{Dh}$, corresponding to the mode responsible for the excitation of fishbone oscillations, and a low frequency branch with $0 \le \omega \le \omega_{di}/2$, corresponding to the mode which becomes unstable in the presence of finite electrical resistivity. If $0 < \lambda_H + \mathrm{Re}\, \lambda_K < \omega_{di}/(2\omega_A)$, the resonant interaction tends to damp this mode [6]. However, as $\lambda_H + \mathrm{Re}\, \lambda_K \to 0$, the mode frequency tends to zero and the resonant contribution disappears. Equation (2) cannot be used for non-positive values of $\lambda_H + \mathrm{Re}\, \lambda_K$, where the stability of the lower frequency branch is determined by the zero frequency limit of the energetic ion response and by dissipative processes mainly involving the bulk plasma species in a layer around the $q = 1$ surface. Referring to the extensively developed theory of $m = 1$ resistive modes, the role of λ_H is now taken by $\lambda_H + \lambda_K(0)$ which depends on the energetic particles and becomes increasingly negative with increasing β_{ph}.

The expression of $\lambda_K(0)$ in the limit where most particles are trapped is [6,13]

$$\lambda_K(0) \simeq \frac{4\pi^3\epsilon_o}{B_p^2 s_o} \left(\frac{2}{m_h}\right)^{3/2} \int_0^1 d\hat{r}\, \frac{\hat{r}^{1/2}}{q} \int_{h_{min}}^{h_{max}} \sigma_v \frac{I_q^2 - qI_cI_d}{I_d} \zeta d\zeta \int_o^\infty dE\, E^{3/2} \frac{\partial F_{oh}}{\partial r}. \quad (2)$$

where $h = B_o/B$, $h_{max,min} = 1 \pm r/R_o$, $\sigma_v = \int_{-\theta_o}^{+\theta_o} (d\theta/2\pi)(1 - \zeta/h)^{-1/2}$, θ_o is the magnetic turning point of the trapped particles, $I_c = (\cos\theta)^{(0)}$, $I_q = (\cos q\theta)^{(0)}$ and $I_d = [(\cos\theta + s\theta \sin\theta)]^{(0)}$. The combination $I_q^2 - q\, I_c I_d$

vanishes identically for $q \equiv 1$. For the model distribution function of Ref.[7], $\Lambda_K(0)$ depends on r_o/a, on $\langle r_p \rangle/a$, where $r_p = -(d\ell n\, p_{\perp h}/dr)^{-1}$, on the ratio $p_{\parallel h}(0)/p_{\perp h}(0)$, and scales approximately as $\langle 1 - q \rangle$. Examples of these dependences are shown in Fig.3. Note that, because of a residual contribution from circulating particles, $\Lambda_K(0)$ is not exactly zero for $q_o \to 1$. The next order term in the Taylor expansion of $\lambda_K(\omega)$ is independent of $\langle 1 - q \rangle$. Its real part is of the form

$$\delta\lambda_K(\omega) = - C\ (\epsilon_o \beta_{ph}/s_o)(\omega/\bar{\omega}_{Dh}) \tag{3}$$

with $1 \lesssim C \lesssim 2$ for $0.5 \le q_o \le 1$ and $0.3 \le r_o/a \le 0.5$ (see Ref.[8] for full expression). In Ref.[6], where the $\omega \gtrsim \omega_{di}$ branch was considered, $\langle 1 - q \rangle$ was assumed to be small compared with $\omega_{di}/\bar{\omega}_{Dh}$, and $\lambda_K(0)$ was neglected. However, neglecting $\lambda_K(0)$ is not appropriate for the lower frequency branch whose oscillation frequency goes to zero at marginal stability.

The relevant dispersion relation which extends Eq.(1) to negative values of $\lambda_H + \lambda_K(0)$, considering resistivity as the only dissipative process present in the thermal plasma, and adopting a two-fluid collisional model, is [3,14]

$$[\omega(\omega - \omega_{di})]^{1/2} = i\omega_A[\lambda_H + \lambda_K(0)](Q^{3/2}/8)\Gamma[(Q - 1)/4]/\Gamma[(Q + 5)/4] \tag{4}$$

where $Q^2 \equiv i\omega_A^{-3}\ \epsilon_\eta^{-1}\ \omega(\omega - \omega_{di})(\omega - \omega_{*e}^*)$, $\epsilon_\eta \equiv \eta_\parallel s_o^2 c^2/(4\pi r_o^2 \omega_A)$, $\eta_\parallel$ is the parallel resistivity, $\omega_{*e}^* \equiv \omega_{*e} + (1.71\ eT_e/eBr_o)(dT_e/dr)_o$ and $\omega_{*e} = (cT_e/eBrn_e)_o(dn_e/dr)_o$. Equation (1) is recovered in the limit $|Q| \gg 1$.

As shown in Refs.[15,16], a stable domain against both resistive internal modes and $m = 1$ drift-tearing modes exists for $\Omega_{di} \equiv |\omega_{di}|/(\omega_A \epsilon_\eta^{1/3}) > 1$ and for $\Lambda_{HK} \equiv [\lambda_H + \lambda_K(0)]/\epsilon_\eta^{1/3}$ negative and such that $\Omega_{di}^{-1/2} < |\Lambda_{HK}| < \Omega_{di}$.

In the drift-tearing domain, characterised by the inequalities $\left|\Lambda_{HK}\right| > \Omega_{di} > |\Lambda_{HK}|^{4/5}$ and $\Lambda_{HK} < 0$, modes are very weakly unstable or completely stable in the presence of electron thermal conductivity [17,18].

For the high temperature regimes of interest, the parameter $\Omega_{di} (\propto T^{3/2}) \gtrsim 1$. Thus the relevant threshold is $\Lambda_{HK} < - \Omega_{di}^{-1/2}$. Expressing this in terms of β_{ph} we conclude that the resistive m = 1 branch is fully stabilised by the energetic ions when

$$|\Lambda_K(0)| \frac{\epsilon_o}{s_o} \beta_{ph} \gtrsim \lambda_H + |\epsilon_\eta \omega_A/\omega_{di}|^{1/2}. \qquad (5)$$

The stable domain is obtained by combining the threshold condition in Eq.(6) at small β_{ph} with that against the upper frequency branch at larger β_{ph}. Stabilisation can occur for lower values of β_{ph} than indicated by Eq.(6) when, as shown in Ref.[19], transverse viscosity arising from ion-ion collisions is considered and $|\Lambda_{HK}| < 1$ while Ω_{di} is large. Indeed, the threshold given by Eq.(5) is intended to be indicative since, as noted in Ref.[6], the two-fluid model is not entirely appropriate in the low collisionality regimes attained in recent experiments.

A consistent theoretical framework for interpreting the sawtooth-free regimes in the JET experiments is thus brought together. In these experiments, typical values of the relevant parameters are $\omega_{di}/\bar{\omega}_{Dh} \simeq 10^{-1}$, $\Omega_{di} \simeq 1 \rightarrow 4$, $R_o/a \simeq 3$, while β_p varies in a wide range ($\beta_p \sim 0.05 \rightarrow 0.5$). For values of β_p below the ideal MHD threshold ($\lambda_H < 0$), stability against the resistive branch is achieved by the combined effect of the zero frequency response of the energetic ions and of finite ω_{di} and ω_{*e} effects within the transition layer. This applies, e.g., to high current, sawtooth-free regimes in JET and more generally to the onset phase of the sawtooth-free regimes when β_p is increasing but still comparatively low. For larger values of β_p, and in particular for values significantly above the ideal MHD threshold, the higher frequency branch of Eq.(1) is to be considered (see Fig.1). Stability against this branch can be achieved by the sole effect of the energetic ions up to values of β_p as large as three times the ideal MHD threshold. This applies to the sawtooth-free regimes in discharges with moderate currents ($I_p \lesssim$ 3MA).

In Fig.4 we show the combined stability domain in the $(\Delta\beta_p^2, \beta_{ph})$ plane as obtained by adopting the approximate model $\lambda_H \simeq (3\pi/2)\epsilon_o^2(\Delta\beta_p^2)$, where $\Delta\beta_p^2 = \beta_p^2 - \beta_{p,mhd}^2$, and the model distribution function of Ref.[7] for mostly trapped energetic ions produced by ICRF heating. No stabilization by energetic trapped particles is possible above a maximum value of $\Delta\beta_p^2$. The experimental findings are consistent with the fact that centrally peaked heating profiles are advantageous for stability as they correspond to finite positive values of β_{ph} and are compatible with the extent of the stability window. In addition, large currents causing $q \le 1$ on a large portion of the plasma cross section reduce significantly the stable domain, since the maximum stable values of β_p and β_{ph} scale approximately as $\epsilon_o^{-3/2}$ and ϵ_o^{-2}, respectively.

We observe that the model adopted in Ref.[20] to analyse the effects of finite electrical resistivity assumed $\lambda_K(0) \equiv 0$. On this basis, stability for the resistive mode could be achieved only when the resonant interaction dominates over resistivity [6], i.e. for values of β_p well in excess of the ideal MHD stability threshold ($\beta_{p,mhd} \sim 0.1 \div 0.3$) of internal modes. In fact, this condition does not appear to be met in many sawtooth-free discharges in JET. Furthermore, for the higher frequency root, taking $\lambda_K(0) = 0$ leads to the incorrect result that the width of the stable window shrinks to zero in the limit $\omega_{di}/\bar{\omega}_{Dh} \rightarrow 0$ (see also Ref.[9]).

The α-particle population that is produced in a D,T burning plasma with an energy at birth of 3.5MeV and with an isotropic distribution in velocity space, has been shown [7,8] to enhance significantly the stability against internal m = 1 modes. The main differences between an isotropic and an anisotropic distribution of energetic ions is that $\lambda_K^{iso} \sim \epsilon_o^{3/2}\beta_{ph}$, in contrast to $\lambda_K^{an} \sim \epsilon_o\beta_{ph}$, and that $\lambda_K^{iso}(0)$ does not vanish with ⟨1 − q⟩ [8]. Thus, the stabilizing influence of isotropic energetic ions on resistive modes persists even when ⟨1 − q⟩ is relatively small. With these differences taken into account, the JET experiments with ICRF heating can be seen to provide first hand experimental information on the enhancement of the stability of global modes that can be expected to occur in ignited plasmas.

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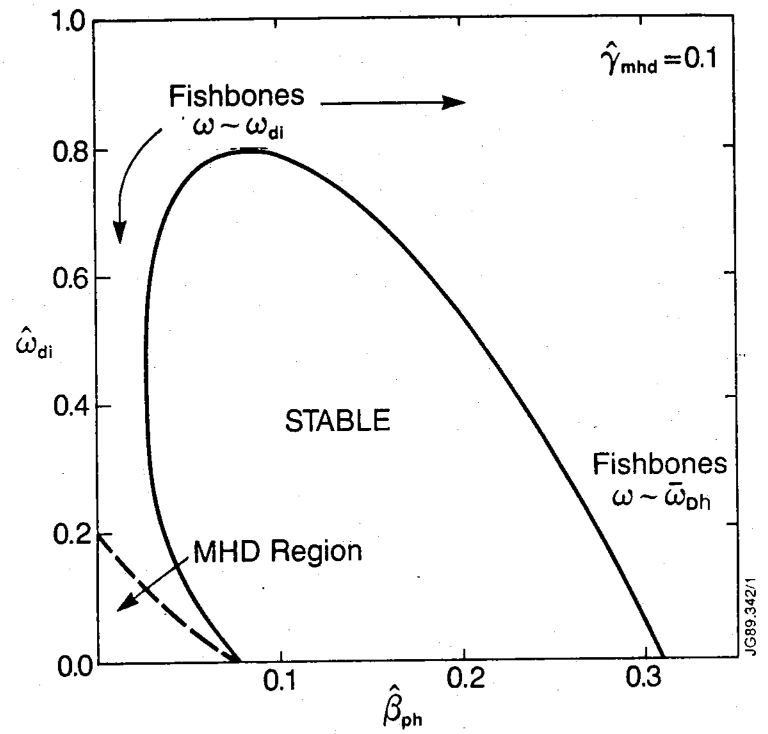

Fig. 1 Representative stability diagram in the ($\hat{\omega}_{di}$, $\hat{\beta}_{ph}$) plane at constant $\hat{\gamma}_{mhd}$.

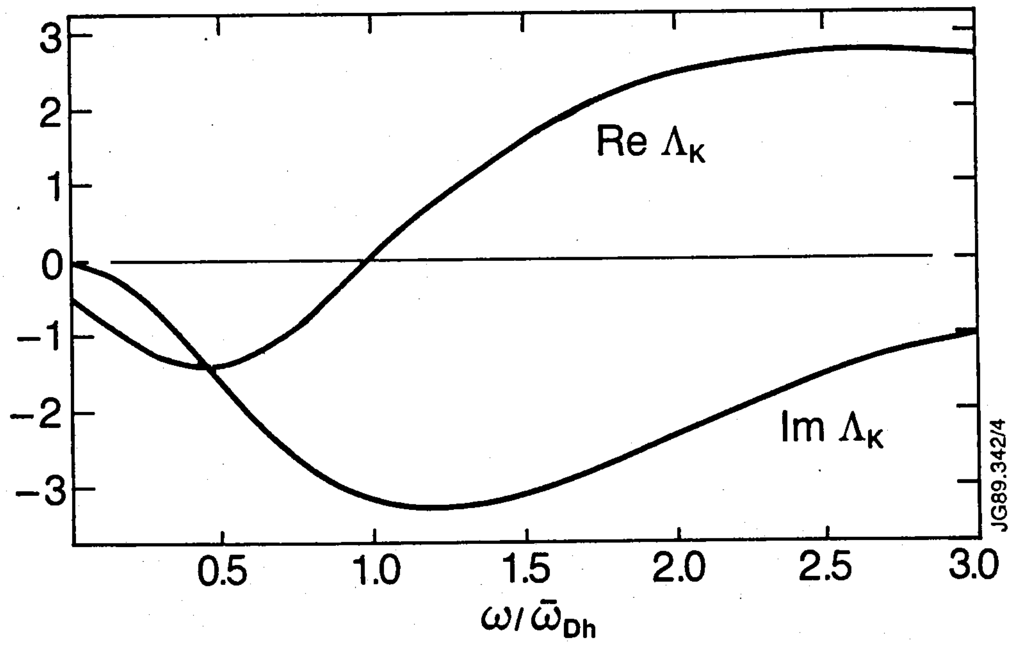

Fig. 2 Frequency dependence of the normalized reactive, Re Λ_K, and dissipative, Im Λ_K, parts of the energetic ion contribution to the m = 1 dispersion relation. The definition of $\Lambda_K(\omega)$ is given below Eq.(1).

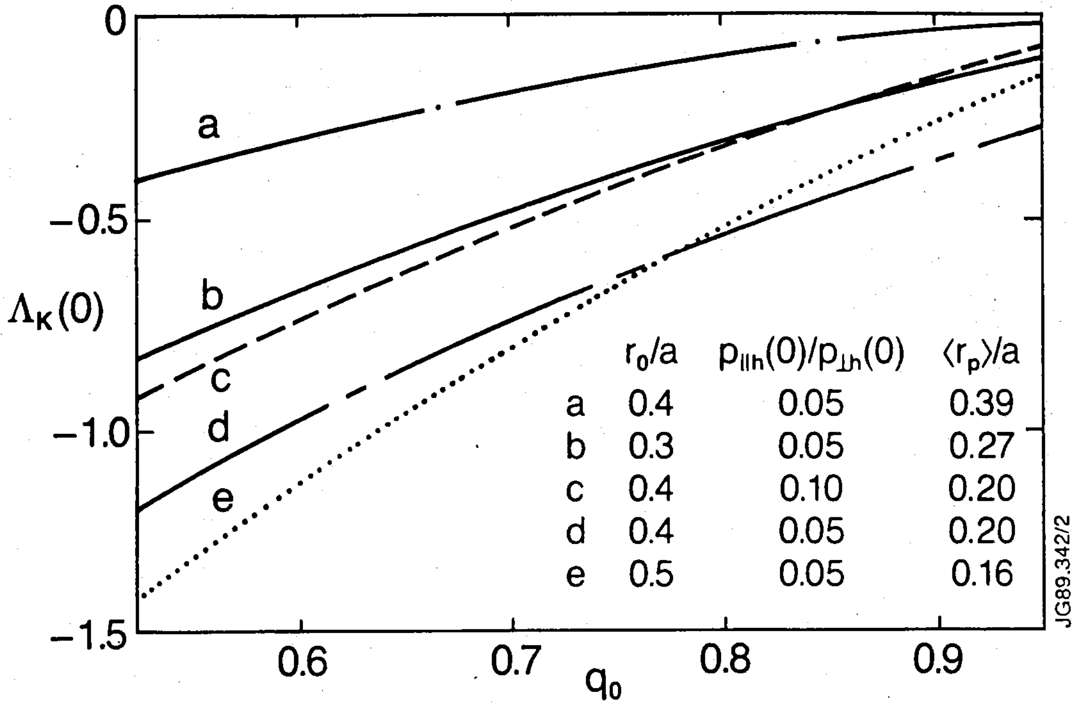

Fig. 3 Dependence of $\Lambda_K(0) \equiv (s_o/\epsilon_o \beta_{ph})\ \lambda_K(0)$ on q_o, with $q(r) = q_o + (1 - q_o)\,(r/r_o)^2$ for $r \lesssim r_o$.

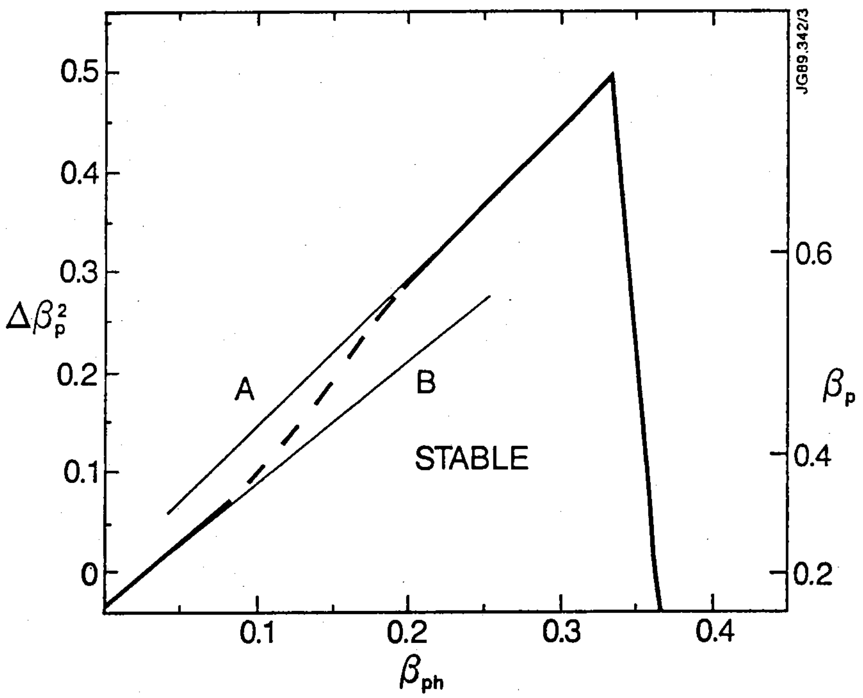

Fig. 4 Stable domain, including resistive effects, in the $(\Delta\beta_p^2, \beta_{ph})$ plane, where $\Delta\beta_p^2 = \beta_p^2 - \beta_{p,mhd}^2$. Values of β_p corresponding to $\beta_{p,mhd} = 0.2$ are indicated on the scale to the right. Curves (A) and (B) correspond to the stability thresholds of the high frequency branch of Eq.(1) and of the low frequency branch [Eq.(5)], respectively. We have chosen: $\bar{\omega}_{Dh}/\omega_A = 0.25$; $\omega_{di}/\omega_A = 0.01$; $\epsilon_o = 0.13$; $q(r) = 0.7 + 0.3\,(r/r_o)^2$; $\epsilon_\eta = 10^{-7}$. The two curves are connected by the dashed line when transverse viscosity is considered. Viscosity is important for $\beta_p \sim \beta_{p,mhd}$ and for finite values of β_{ph} such that, as a consequence of the heating, $\omega_{di}/(\omega_A \epsilon_\eta^{1/3})$ attains significant values.