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THEORY OF SAWTOOTH OSCILLATIONS

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Sawtooth oscillations were first discovered on the ST tokamak and were reported in 1974 by von Goeler, Stodiek and Sauthoff. These oscillations are now seen regularly on all tokamaks and can be observed on many diagnostics including soft X-ray, temperature and density measurements.

The basic pattern consists of a slow increase in central temperature and density brought to an end by a sudden collapse, the whole process repeating in a periodic manner. Outside the central region an inverted sawtooth is seen, a slow decay being followed by a fast rise.

The theory of these oscillations has had a fascinating but rather contorted development. It would be satisfying if it were possible to say that this development had led us to an understanding of the subject but unfortunately the present state is one of mystery. However it could be argued that this makes a review of the subject more useful.

This talk will be in three parts. The first will sketch the historical development of the subject. The second will describe the more recent work and particularly that relating to JET. The third will review the problems which the experimental results pose for proposed theoretical explanations.

1. Historical Sketch

1970 Shafranov applied the mhd stability theory of cylindrical plasmas in the tokamak limit $nq=krB_z/B_\theta \sim 1$ with $kr \ll 1$. For the monotonically rising q profile shown in figure 1, the now well-known rigid displacement, ξ, inside q=1 was found to be unstable to the m=1 mode under normal conditions, the potential energy of this displacement being

$$\delta W = 2\pi^2 \frac{\xi^2}{R} \int_0^{r_1} \left(rp' - \frac{B_\theta^{\ 2}}{2\mu_0} (1-q)(1+3q) \right) rdr.$$

For $p' < 0$ the integrand is negative for $q < 1$.

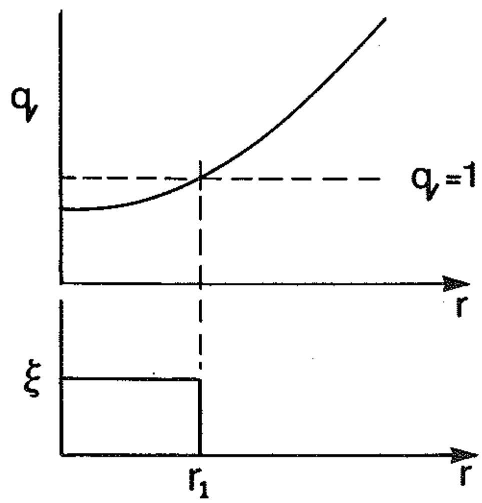

Fig.1 Monotonic q profile with q(0) < 1 and associated rigid displacement ξ inside the radius at which q=1.

1973 Rosenbluth, Dagazian and Rutherford analysed the non-linear behaviour of the m=1 mode described by Shafranov. The development was taken to be ideal and the saturated displacement was calculated. The non-linear displacement consists of a rigid shift of the core inside q=1, with the energy of the instability being taken up by the magnetic field in a narrow layer around the q=1 surface. For the simple case illustrated in figure 1 the saturated displacement is approximately

$$\frac{\xi}{r_1} = \frac{13}{4}\left(\frac{r_1}{R}\right)^2 \frac{\beta_p + \frac{1}{3}(1-q_o)}{(1-q_o)^2}$$

where q_o is the axial value of q and

$$\beta_p = \frac{2\mu_o R^2}{r_1{}^4 B_\phi{}^2} \int_o^{r_1} \left(-\frac{dp}{dr}\right) r^2 dr \qquad (1)$$

measures the destabilising effect of the pressure, B_ϕ being the axial magnetic field.

For expected values of the parameters, ξ/r turned out to be small. For example, for β_p=0.1, q_o=0.7, r_1/a=0.3 and R/a=3 the saturated displacement $\xi/a=2\times10^{-2}$.

1974 The sawtooth oscillations were observed on ST. The estimated displacements were larger than predicted by the non-linear ideal mhd theory and it seemed that resistive diffusion might be responsible. However, in round figures, the collapse time of the sawtooth was ~100μs whereas the resistive diffusion time $\tau_R=\mu_o\sigma r_1{}^2$ was ~10ms. Thus the theoretical question to be resolved was - how could this factor of 100 in timescales be reconciled?

1975 To answer this question Kadomtsev proposed that during the collapse there is a rapid reconnection of the helical magnetic flux inside the q=1 surface to that outside. The fast timescale is achieved by allowing the resistive diffusion to take place in a narrow layer at the q=1 surface. The timescale is then $\sim \mu_o\sigma r_1\delta$ where

δ is the thickness of the layer. This thickness is determined by the requirement of mass conservation as the plasma flows from the q<1 core into the m=1 magnetic island formed by the reconnection.

1976 Numerical simulations of the Kadomtsev model were reported. Danilov et al. followed the plasma through the sawtooth collapse and Sykes and Wesson reproduced several cycles of the full relaxation oscillation. Figure 2 shows the type of behaviour found, this behaviour being fully consistent with Kadomtsev's model.

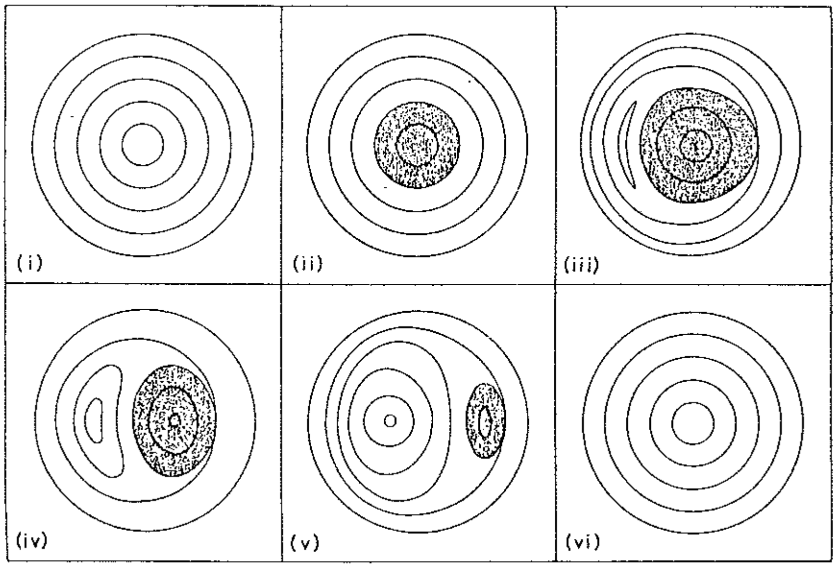

Fig.2 Development of magnetic field structure during the sawtooth collapse according to Kadomtsev model. The m=1 instability displaces the $q < 1$ region (shown shaded) and restores q to a value > 1. (From Sykes and Wesson (1976)).

The timescale resulting from Kadomtsev's model is the geometric mean of the resistive diffusion time τ_R (~10ms) and a timescale τ_A related to the mhd velocity characteristic of the helical magnetic field $B^*=B_\theta(1-q)$ and given by $\tau_A \sim r_1/(B^*/\sqrt{\mu_0\rho})$. Since $\tau_A \sim 1\mu s$ the timescale given by Kadomtsev's model was $\tau_K \sim (\tau_R\tau_A)^{1/2} \sim (10ms.1\mu s)^{1/2} \sim 100\mu s$ and this was in agreement with the experimentally observed values.

1974-75 (going back): In parallel with the experimental developments and the attempts at interpretation as described above there had been further work on the stability of the ideal m=1 mode. Sykes and Wesson had discovered from numerical calculations that the m=1 mode could be stabilised by toroidal effects. The problem was investigated analytically by Bussac et al. They found that, allowing for the toroidal coupling to the m=2 component of the mode, the mode is stable for β_p (as defined above) below a critical value. This critical value depends on the configuration but typically was 0.2 to 0.3. Although the value of this β_p was not known in the early experimental work it seems likely that it was in the stable regime.

In Kadomtsev's paper the nature of the linear instability is not defined. Given the historical context however, the model seems to be describing the non-linear resistive development of a linearly ideal mode. However once it appeared that the ideal mode was probably stable, the Kadomtsev model could also be applied to a resistive m=1 instability. There was therefore a need to understand the stability of the resistive mode.

1976 Coppi, Galvao, Pellat, Rosenbluth and Rutherford developed the theory of the resistive m=1 mode. The mode was found to be unstable if there is a q=1 surface in the plasma. For the case where the ideal mode is weakly stable the growth rate is given by

$$\gamma = \frac{(rq'/q)_{r_1}^{2/3}}{S^{1/3}\,\tau_H}$$

where $S=\tau_R/\tau_H$ and $\tau_H=r_1/(B_\theta/(\mu_o\rho)^{1/2})$. This mode was therefore a satisfactory candidate for the sawtooth instability and combined with Kadomtsev's reconnection model seemed to provide an explanation of the experimental results.

1980 onwards: The TFR Group expressed uncertainties about the validity of the Kadomtsev model for the TFR sawteeth. Their analysis of the experiment indicated that although an island was formed its growth was limited and it did not undergo a complete reconnection. In the model proposed by Dubois and Samain the final collapse is due to the onset of fine-scale turbulence while the island is still small. This turbulence is initiated by a current layer which appears on the separatrix and the turbulence then propagates over a large region as shown in figure 3.

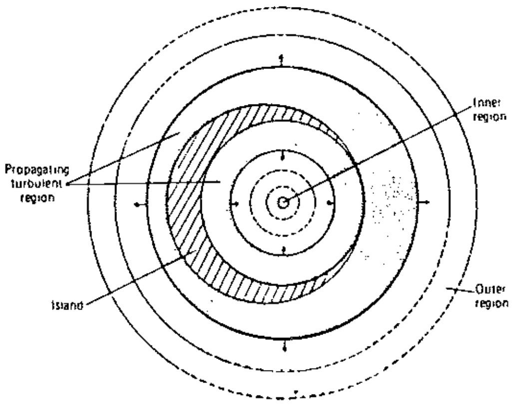

Fig.3 Geometry of the turbulent region in the Dubois-Samain model.

1984 Bussac, Pellat, Soule and Tagger also developed a model based on the idea that the magnetic island is only partially grown at the time of the sawtooth collapse. The sawtooth collapse is then due to an ideal mhd instability of this magnetic island.

1985 Denton, Drake, Kleva and Boyd carried out numerical simulations to elucidate the physical processes involved in sawtooth relaxations. They found that behaviour of their sawteeth was very dependent on the cross-field thermal conduction. Taking $\kappa_\perp$ small near the magnetic axis skin currents are formed leading to periodic sawteeth with a number of the experimentally observed features.

1985 We now come to the early observations on JET (Campbell et al), which led to the proposal of a new theoretical model of the sawtooth. This was followed by a number of experimental and theoretical papers from JET and an account of the theoretical analysis is given below.

2. Theoretical Analysis of JET Sawteeth

The experimental results on JET inaugurated a new phase in the development of the theory of sawteeth. The starting point was the simple observation that the sawtooth collapse was usually precursorless and the collapse was very rapid. The collapse time was ~100μsec. This is not significantly slower than observed in much smaller tokamaks and so, since the time predicted by the Kadomtsev model is proportional to $(\text{radius})^{3/2}$, this result was not consistent with the model.

A new model to explain these results was proposed at the Budapest Conference in 1985. This section will summarise this model and the subsequent developments reported at the Kyoto Conference in 1986.

The problem is seen most clearly if we recognise that there are three basic questions concerning sawtooth oscillations:

i) what instability drives the oscillations?

ii) what is the mechanism of the collapse?

iii) why does the oscillation have the form of a relaxation?

These questions will be considered in turn.

i) The Instability

The rapidity of the collapse suggests that an ideal mode is involved. The theory of Bussac et al predicts that for simple q profiles which are parabolic around the magnetic axis, the m=1 ideal mode is only unstable if β_p (as defined in equation 1) is above a critical value which is in the range 0.2 to 0.3 for $r_1/a < 0.3$. When the experimental values of β_p were calculated for JET it was found that they were much smaller, a typical value for ohmic discharges immediately prior to the collapse being 0.05. Thus if an ideal mode was involved there was a discrepancy to be explained.

This difficulty was resolved by reconsidering the form of the q-profile.

Simple current profiles which are parabolic around the magnetic axis have axial values of q such that $1-q_o \sim (r_1/a)^2$ and this typically gives $1-q_o \sim 0.1$. However if the current profile is flat and q_o is close to unity immediately after the sawtooth collapse, the resistive diffusion of the current during the ramp phase is so slow that q will remain flat up to the next collapse. The amount of the change in q was estimated on the assumption that after the collapse the temperature and the resistivity are flat inside the q=1 radius, and that the current increase is brought about by the increase in central temperature during the ramp phase. This process is limited by electromagnetic induction and the resulting change in q is given by

$$\Delta q \approx \frac{\Delta T}{T} \cdot \frac{\tau_s}{\tau_\eta} \qquad (2)$$

where $\Delta T/T$ is the fractional change in temperature during the sawtooth, τ_s is the sawtooth period and τ_η $(= \mu_o \sigma r_1{}^2/4)$ is the characteristic resistive diffusion time. Since, for the ohmic JET sawteeth, $\Delta T/T \sim 0.1$ and $\tau_s/\tau_\eta \sim 0.1$ we have $\Delta q \sim 10^{-2}$. This means that the q profile used in the theory makes it inapplicable to the actual experimental case. The theoretical model was therefore modified and the stability limits recalculated. The model used is illustrated in figure 4, for which

$$j = j_c \left(1 - \frac{r^2}{a^2}\right)^\nu \qquad r > r_1$$

$$j = \frac{2B_\phi}{Rq_o} \left(1 - 2(1-q_o)\frac{r^2}{r_1{}^2}\right) \qquad r < r_1$$

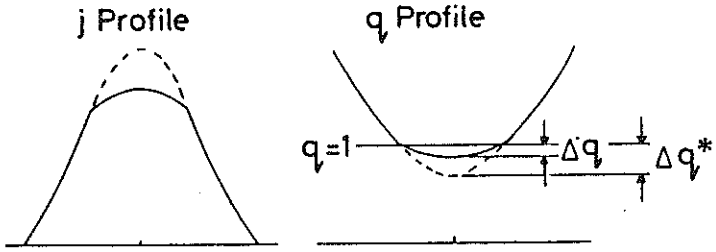

Fig.4 Form assumed for j(r) and q(r) in the calculation of the stability of flattened q profiles.

where $j_c = (2B_\phi/R)(2-1/q_o)(1-(r_1{}^2/a^2))^\nu$. The change in the coupling to the m=2 component leads to the new criterion for instability

$$\beta_p > \left(\frac{13}{144}\,\frac{3\Delta q}{\Delta q+2\Delta q^*}\right)^{1/2}, \quad q_o < 1$$

$$\beta_p > 0 \qquad\qquad , \quad q_o > 1$$

where $\Delta q=1-q_o$ and $\Delta q^*(=\nu r_1{}^2/2a^2)$ is the value of $1-q_o$ which would have occurred without the flattening. The resulting stability boundary (for $\nu=1$) is shown in figure 5 together with the result of a more accurate numerical calculation.

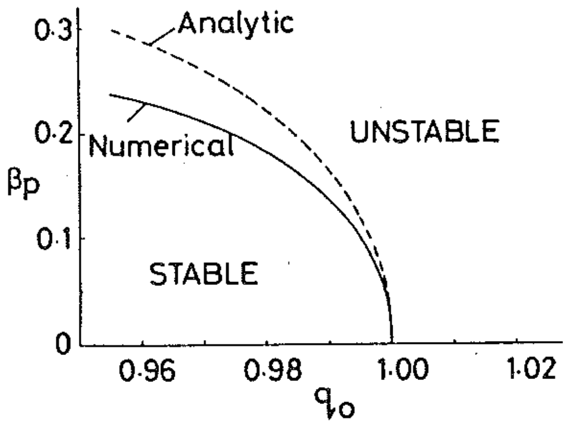

Fig.5 Stability diagram for flattened q profiles (for $\nu=1$ and $r_1/a = 0.3$) giving the critical β_p against q_o.

The strong influence of flattening the profiles is clear. For a profile with completely flat q inside the q=1 radius the ideal m=1 mode is unstable for all β_p. It is also interesting to note that, despite the magnetic shear introduced, cases with $q_o>1$ (but with an off-axis minimum value of q=1) are also unstable for all β_p.

This result restored the possibility of the sawtooth instability being an ideal m=1 mode. However although this provides a fast linear growth rate it reintroduced the problem of the small non-linear displacement expected for the ideal mode at the experimentally observed low β_p. This then was the next problem to address.

ii) The Mechanism

The introduction of the concept of a flattened q-profile leads to a completely different picture of the behaviour of the plasma during the sawtooth collapse. It is different from Kadomtsev's model of course because no significant magnetic reconnection takes place during the fast collapse. But equally fundamentally, the low shear arising from the flattened q-profile leads to a different non-linear behaviour for the ideal mode.

Consider the idealised case where q=1 everywhere inside the central region. The absence of shear would then allow an interchange of flux tubes without line bending, leading to a non-linear rearrangement of the temperature and density profiles on an inertial timescale. A calculation of this timescale using δW to estimate the potential energy gave a sawtooth collapse time τ_c~100μs, in agreement with the observed value. In reality there will be some shear and we would expect a quasi-interchange. In this case the energy required to provide the small energy of the line bending will be taken from the energy of the instability.

This model leads to a quite different picture of the plasma motion. To see this it is necessary to return to the theory of the instability. The potential energy may be written

$$\delta W = \delta W_2 + \delta W_4$$
$$= \frac{\pi^2 B_\phi^{\ 2}}{2R} \int \left(1 - \frac{1}{q}\right)^2 \left(\frac{d\xi}{dr}\right)^2 r^3 dr + O(\varepsilon^4)$$

where the expansion is in the inverse aspect ratio $\varepsilon = a/R$. For the usual m=1 instability it is necessary that the large δW_2 term, which is positive definite, be reduced to zero. This is achieved by taking the leading order displacement of the core to have the well-known "rigid-shift" form shown in figure 1. Thus, where $q \neq 1$ we have $d\xi/dr = 0$. However if 1-q is sufficiently small (that is $1-q = O(\varepsilon)$) the ordering fails and there is no longer a necessity for ξ to be constant inside the q=1 radius. This was illustrated originally by calculations for the cylinder, where the same argument holds. The resulting eigenfunction for cases with $q_0 = 0.6$ and $q_0 = 1$ are shown in figure 6. This change of the eigenfunction has since been substantiated for the toroidal case.

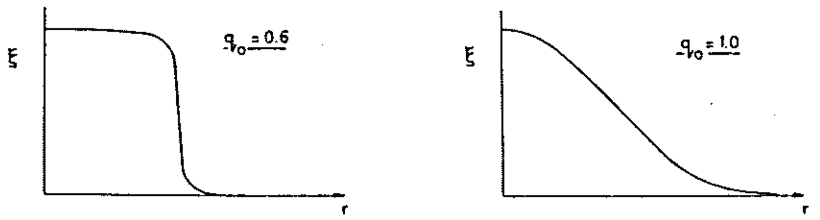

Fig.6 Eigenfunctions for the radial displacement ξ, for $q_0 = 0.6$ (internal kink) and $q_0 = 1.0$ (quasi-interchange).

This model of the plasma flow made a clear prediction as to the expected behaviour of the plasma temperature, density and soft X-ray emission. Figure 7 shows the original drawings of the quasi-interchange flow pattern and the rearrangement of the magnetic field and plasma. The flow pattern has two broad convective cells and results in a squeezing out of the hot plasma to form a crescent and of the influx of colder plasma to form a bubble.

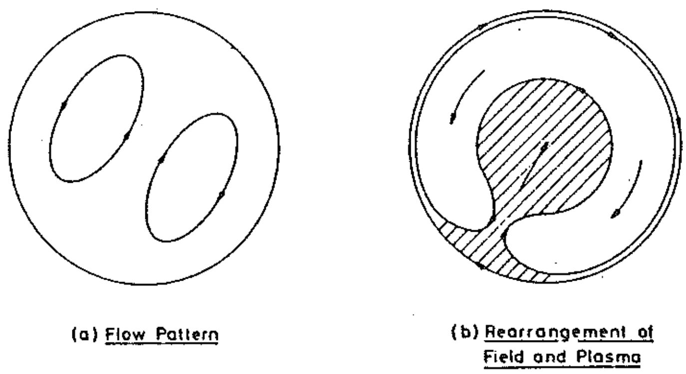

Fig.7 Drawings of the quasi-interchange flow pattern and the associated rearrangement of the field and plasma (Budapest Conf. 1985).

Thus in this model the fast fall in the temperature in the centre of the plasma is not the result of a reconnection of the magnetic field. On the timescale of the initial motion the plasma behaves as a perfect conductor and the magnetic topology is unchanged. The fast temperature collapse at the axis is simply due to the rapid sideways motion of the plasma. Any reconnection takes place in a later stage. This is consistent with the observation of "post-cursor" oscillations following the collapse since a longer time is required for the subsequent resistive reconnection.

A calculation of the type of flow pattern which might be expected from linear theory was carried out in the cylindrical approximation and the result is shown in figure 8 where a comparison with the rigid-shift flow pattern is made.

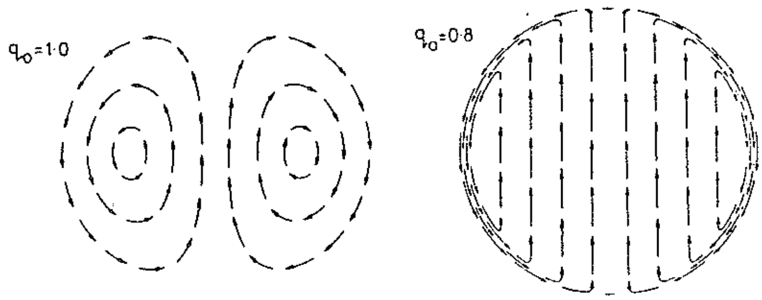

Fig.8 Flow pattern for a quasi-interchange (q_0 = 1.0) and "rigid shift" (q_0 = 0.8).

These predictions had a remarkable confirmation when results were obtained from tomographic reconstruction of the soft X-ray emission during a JET sawtooth (Edwards et al). An example of such reconstructions is shown in figure 9. The convective motion and the formation of the cold bubble are clear.

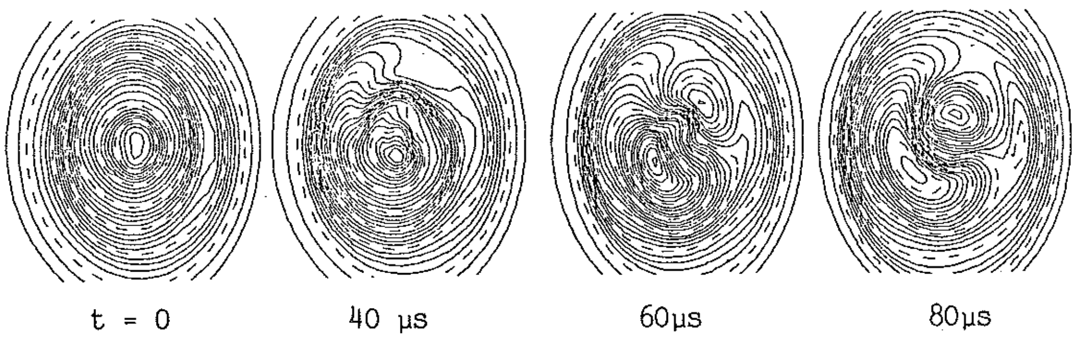

Fig.9 Sequence of tomographic reconstructions of the soft X-ray emission during a JET sawtooth collapse (Soft X-ray Group, P. Smeulders).

This behaviour can be compared with non-linear simulations of the quasi-interchange instability carried out in cylindrical geometry. Such a comparison is made in figure 10. While no precise agreement is to be expected the general similarity is apparent.

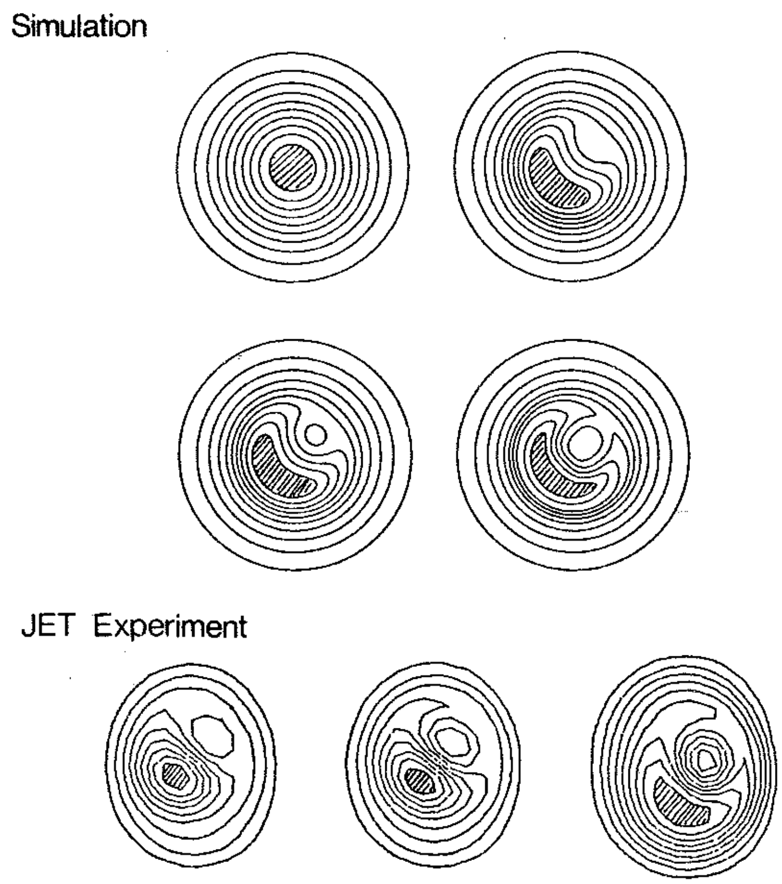

Fig.10 Comparison of type of plasma deformation predicted for the quasi-interchange (simulation by P. Kirby) and the observed deformation as reconstructed from the soft X-ray emission on JET (R. Granetz).

It is important to clarify the basic differences between the form of the ideal quasi-interchange instability and that of the tearing or Kadomtsev model in which an island is formed. Figure 11 gives schematic diagrams of the plasma re-arrangement in the two cases. It is seen that in the tearing mode case the colder plasma in the island forms a crescent around the displaced hot core whereas in the quasi-interchange the displaced hot plasma forms a crescent around the cold bubble. These cases are readily distinguishable and in JET the behaviour corresponds to that of the quasi-interchange.

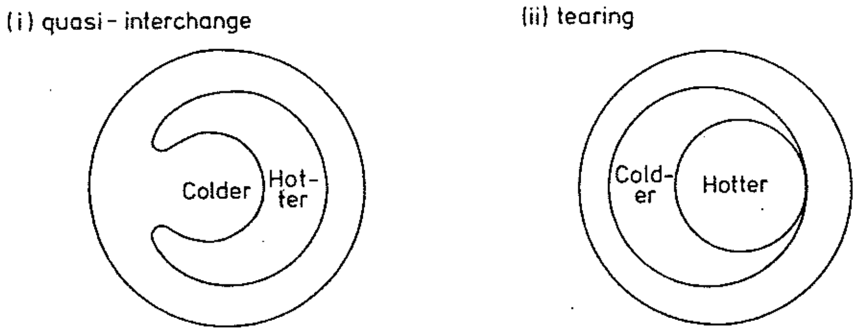

Fig.11 Schematic diagrams showing the contrasting non-linear topological development for the quasi-interchange and the tearing mode. This distinction makes experimental identification straightforward.

The distinction between the quasi-interchange and the tearing mode is brought out most clearly by considering the azimuthal displacement eigenfunction $\xi_\theta(r)$. Using $\underline{\nabla}\cdot\underline{\xi}=0$ and $r_1/R<<1$, ξ_θ is related to ξ_r by $\xi_\theta = -\frac{d}{dr}(r\xi_r)$. For illustration the calculated forms of a typical quasi-interchange and tearing mode are shown in figures 12(a) and (b).

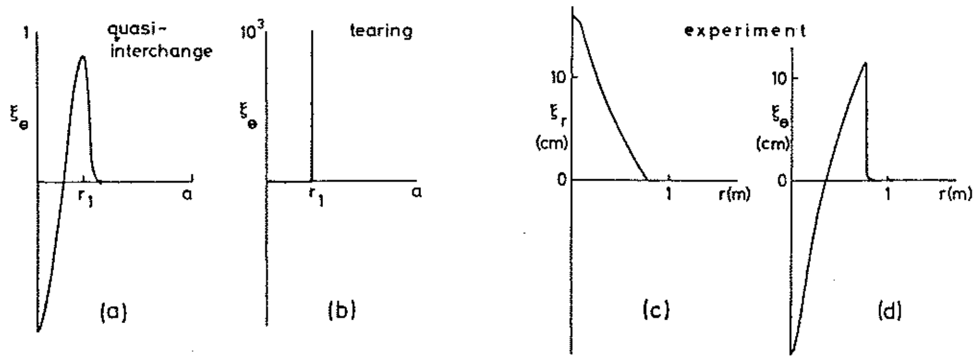

Fig.12 Eigenfunction $\xi_\theta(r)$ for (a) quasi-interchange and (b) tearing mode together with experimental forms for (c) $\xi_r(r)$ and (d) $\xi_\theta(r)$ from the JET experiment.

A quantitative assessment of the experimental behaviour can be made by analysing the displacement of the plasma in the early stage of the sawtooth collapse. Figure 13 shows the initial soft X-ray emission across the median plane together with the emission 50μs later. If we assume that on this timescale the magnetic field carried the plasma properties, including soft X-ray emission, with it, the radial displacement ξ_r of the plasma is given by the displacement of a given value of the emission. The derived ξ_r and ξ_θ are shown in figures 12(c) and (d). Comparison with figures 12(a) and (b) gives clear evidence that the displacement has the form of a quasi-interchange.

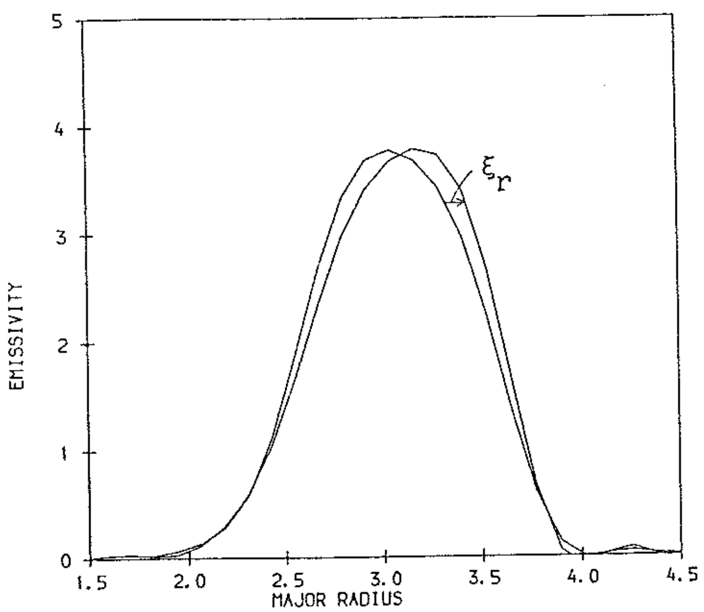

Fig.13 The initial soft X-ray emission across the median plane together with the emission 50μs later (JET), allowing a calculation of the displacement $\xi_r(r)$ shown in Fig.12.

iii) The Relaxation

The fact that the m=1 instability gives rise to a relaxation oscillation has never been explained. The simpler behaviour of a saturated mode would probably have been predicted in the absence of experimental evidence. A consequence of this is that it is not clear why the sawtooth collapse occurs when it does rather than at some other time.

A clue to this behaviour is suggested by the sawtooth behaviour in JET when additional heating is applied. Let us first consider the behaviour we would expect if the sawtooth collapse is caused by a critical temperature or pressure gradient. Figure 14 shows the change in sawtooth period which would result from switching on the additional heating. Because the rate of temperature rise is increased it would be expected that the critical condition would be reached earlier and the sawtooth period shortened, roughly in the ratio of the heating rates.

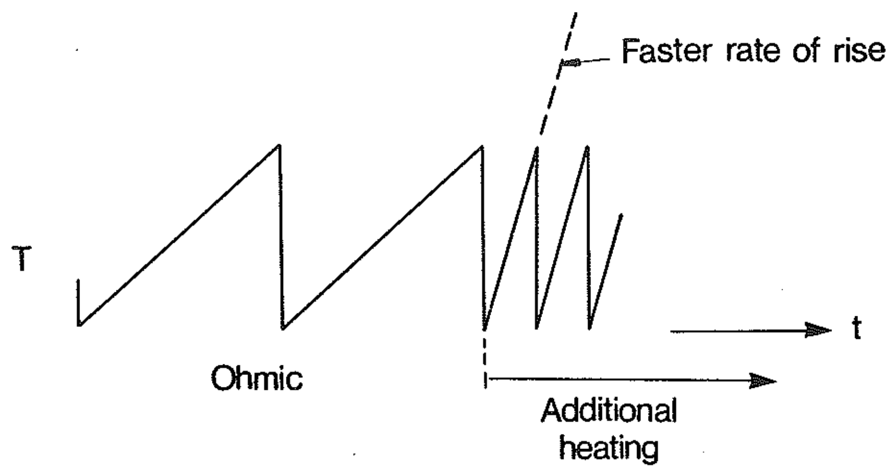

Fig.14 Showing how the sawtooth period would be shortened on the application of additional heating if the sawtooth collapse were triggered by the temperature or pressure gradient.

Figure 15 shows an early example of the behaviour in JET. The heating rate is increased by a factor of about 5 when the ICRH is switched on but the period is unchanged. As a consequence the temperature fluctuation is about 5 times that of the ohmic sawteeth. This clearly rules out the temperature or pressure gradient as the cause of the collapse. In subsequent experiments it was found that heating actually increases the period, making the argument even stronger.

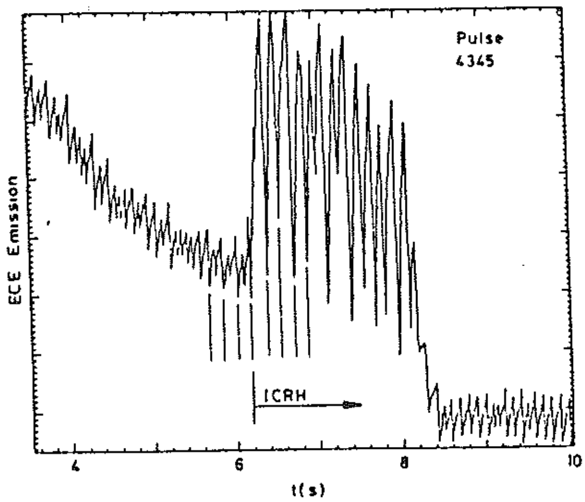

Fig.15 Shows that the application of additional heating in JET does not decrease the sawtooth period.

As a result of this analysis it was proposed that the energy built up during the ramp phase was released by a magnetic trigger. In the next section we shall see that there is a difficulty with this idea but let us nevertheless look briefly at the original suggestion. The requirement is that a small change in q causes a gross instability. Figure 16 shows the proposed way in which this would arise. The q profile falls during the ramp until its off-axis minimum touches unity. At this point a gross instability covering the whole central region becomes possible. An idealised form of the plasma displacement is shown in the figure.

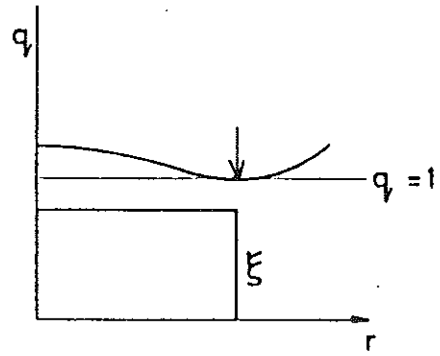

Fig.16 Showing how a falling profile with an off-axis minimum can trigger a large displacement when q_{min} reaches one (Budapest Conf. 1985).

Now of course the switch will not occur exactly when q=1. Figure 17 shows a typical form of the growth rate as a function of q_{min}. This illustrates how a sudden instability might be brought about as q_{min} falls to the critical value.

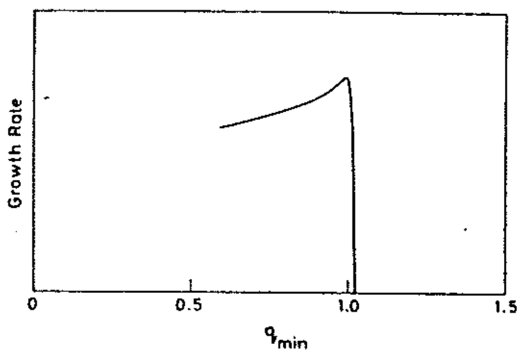

Fig.17 Showing the rapid (but it turns out insufficiently rapid) rise of the growth rate as a q profile of the type shown in Fig.16 falls to one.

The above account gives a description of the theory proposed to explain JET sawteeth. The principal features are summarised as follows.

i) An ideal m=1 mode is predicted when flattening of the q profile is allowed for.

ii) In a q flattened core an ideal instability can produce the sawtooth collapse on an inertial timescale. Reconnection does not occur during the sawtooth collapse but on a longer timescale during the subsequent ramp phase.

iii) A destabilising potential energy builds up during the ramp phase to be released by a magnetic trigger. Such a trigger could be provided by an off-axis minimum in q.

3. The New Problems

Although there may be some truth in the theoretical sawtooth model described in the last section, subsequent experimental results have introduced quite fundamental difficulties.

These difficulties fall into three categories

i) Although at first sight plausible, the magnetic trigger proposal does not withstand a careful quantitative analysis.

ii) Experimental measurements of q have been made on several tokamaks. Some of these give values of q_o much below unity (down to q_o~0.6) in the presence of sawteeth. Such values of q_o would rule out the quasi-interchange. The situation is made even more difficult by apparent differences in q_o between machines. There are indications from JET of quasi-interchange sawteeth with values of q_o closer to unity.

Furthermore, the plasmas with measured low q_o profiles would be expected to be unstable to an m=1 tearing mode <u>before</u> the sawtooth collapse.

iii) Recent experiments on JET have produced two sawtooth phenomena which are not understood. These are the so-called monster sawteeth and neutron sawteeth.

We shall now look at these three problems in turn. However it is regretted that, at best, this offers only clarification and certainly doesn't lead to solutions.

i) The Trigger Problem

Careful examination of this problem reveals that not only is the explanation proposed above inadequate but there is a fundamental difficulty for all models of "precursorless" sawteeth. This difficulty is associated not with the speed of the collapse but with the rapidity of its onset.

If the collapse time is τ_c, it is necessary for a growth rate $\gamma \sim \tau_c^{-1}$ to develop on a timescale τ_c. Let the change in central q necessary to produce this γ be δq. If this change in q results from resistive diffusion, then

$$\delta q \sim \frac{\tau_c}{\tau_s} \Delta q \qquad (3)$$

where Δq is the change in q during the ramp phase, the duration of which is approximately the sawtooth period τ_s. The magnitude of this Δq can be estimated from the resistive diffusion of the current and is given by equation (2), thus

$$\Delta q \sim \frac{\Delta T}{T} \frac{\tau_s}{\tau_\eta} \qquad (4)$$

where $\Delta T/T$ is the fractional change in the temperature during the sawtooth, and the resistive diffusion time $\tau_\eta = \mu_0 r_1^2/4\eta$. Relations (3) and (4) determine the expected changes δq during the collapse time. It is then necessary that the required instability growth rate can be switched on by this δq.

Typical JET values are $\Delta T/T \sim 10^{-1}$ and $\tau_s/\tau_\eta \sim 10^{-1}$ giving $\Delta q \sim 10^{-2}$. Then since $\tau_s \sim 100$ms and $\tau_c \sim 100\mu$s, relation (3) gives the required sharpness of the switch-on of the instability, $\delta q \sim 10^{-3} \cdot 10^{-2} \sim 10^{-5}$.

Even allowing for the uncertainty in the estimation, this δq seems to be improbably small. We can make an estimation of δq required to switch on the quasi-interchange mode as follows. Let the growth rate for zero shear be γ_0, then the destabilising energy density available for the instability is $\sim \frac{1}{2}\rho\gamma_0^2\xi^2$. Putting $d\xi/dr \sim \xi/r$ the stabilising energy density is $\sim (1-q)^2(B_\theta^2/2\mu_0)(\xi/r)^2$. Thus the dispersion relation can be characterised by

$$\gamma^2 = \gamma_0^2 - (1-q)^2\tau_A^{-2} \qquad \text{where} \qquad \tau_A = r_1/(B_\theta/(\mu_0\rho)^{\frac{1}{2}}). \qquad (5)$$

Although the initial switch-on, $d\gamma/dq$, at $\gamma=0$ is very rapid, it is necessary for γ to grow to give a value (in JET) of $\gamma^{-1} \sim 100\mu$s. Now for the ideal m=1 mode $\gamma_0 \sim \epsilon\beta_p/\tau_A$, where $\epsilon = r_1/R \sim 10^{-1}$, $\beta_p \sim 10^{-1}$ and $\tau_A \sim 1\mu$sec. Thus $\gamma_0^{-1} \sim 100\mu$s and this is of the order of the required growth time. Consequently the change in q necessary to produce the required growth rate is given by Eq.(5) to be

$$\delta q \sim \gamma_0\tau_A.$$

Hence the theoretical switch-on width in q is $\delta q \sim 10^{-2}$, and this is orders of magnitude greater than the required experimental value of 10^{-5}.

The problem is even more severe for tearing modes where the theoretical value for the δq necessary to provide a growth rate γ is

$$\delta q \sim \gamma^{3/2} \tau_A \tau_\eta^{1/2}.$$

The typical JET values given above with $\tau_\eta \sim 1$sec give $\delta q \sim 1$ to be compared with a required switch-on $\delta q \sim 10^{-5}$.

It is clear therefore that a fundamental element is missing in our theoretical understanding of fast precursorless sawteeth. Thus although the experimental evidence on the flow behaviour seems to support the quasi-interchange model, the validity of this model cannot be accepted while we still lack an explanation of the rapidity of the onset of the collapse. An apparent amelioration of this situation is obtained by calculating $\exp \int \gamma dt$ which for the quasi-interchange leads to $\exp(t/\gamma_0^{-2/3}\tau_\eta^{1/3})^{3/2}$ and gives a characteristic time $\gamma_0^{-2/3}\tau_\eta^{1/3}$ of several ms. However the basic problem is unresolved.

ii) The q_0 Problem

It is difficult to measure the magnetic field with the precision required to elucidate sawtooth behaviour. However, recently such measurements have been made and theoretical models of sawtooth behaviour must take account of the values of q obtained.

McCormick has made measurements on ASDEX using a lithium beam. A spectral line excited in the atoms of this beam undergoes Zeeman splitting to produce a triplet. The polarisation of the undisplaced line lies along the direction of the magnetic field at the point of emission and therefore gives the angle of the magnetic field at this point. This then gives the required q-value. With these measurements it is found that when q_o is lowered to unity, q shows a "resistance" to going below unity. The measured values of q_o with sawteeth present are typically 1.05±0.05. When sawteeth are removed with lower-hybrid current drive q_o is observed to rise above unity. However a qualification is that these results are for a limited range of the edge value of q with $q_a \sim 4$.

Soltwisch used a different technique to determine q on TEXTOR. This measurement uses the Faraday rotation of a laser beam as it passes through the plasma. This measurement is not straightforward since the effect is locally proportional to the plasma density and the observed rotation is given by a line integral. However, careful unfolding of the measurements gives a reliable q-profile. The result is that values of q_o below unity are found and for $q_a \approx 2$ the value of q_o is 0.6±0.15. It is also found that q_o does not change by a large amount during the sawteeth. This result rules out quasi-interchange behaviour because the ideal m=1 mode is stable under these conditions. It also presents a problem in that the tearing mode would be expected to be unstable during the whole sawtooth period with a growth rate

$$\gamma \sim \frac{(1-q_o)^{2/3}}{\tau_A S^{1/3}}$$

In an attempt to resolve this problem it has been suggested that some flattening of the q-profile around the q=1 surface might provide the required stability during the ramps and such a stabilising effect has been found in numerical stability calculations.

West et al have used the lithium beam technique to measure q on TEXT. They find results similar to those of Soltwisch. For q_a=7.4, q_o=1.1±0.1 but for q_a≈2.3, q_o=0.7±0.05. Thus we now have low q_o measurements using two completely different diagnostic techniques.

An estimate of q_o on JET was made possible by the chance discovery of a phenomenon called the snake. When a pellet is injected into the plasma it sometimes leaves an almost permanent remnant effect at the q=1 surface (appearing as a snake-like trace on the soft X-ray measurement). The radial movement of the snake during the sawtooth period together with the estimated change in q from resistive diffusion gave a value of q_o≈0.97. This is consistent with the observed quasi-interchange behaviour and prompts the (unattractive) thought that there may be two types of sawtooth behaviour.

iii(a) <u>Monster Sawteeth</u>

It was found on JET that the application of ICRH heating can produce very large sawteeth as illustrated in figure 18. These sawteeth have been obtained with progressively longer durations, now exceeding two seconds. This behaviour might therefore be regarded as an "inadvertant" stabilisation of the sawtooth instability. However thought of, monster sawteeth call for an explanation.

The first monster sawteeth were obtained with ICRH together with neutral beams and it was natural to think that the current drive caused by the beams was altering the current profile and raising q_o above unity. However when the same effect was observed with only ICRH, which in JET is symmetric and is not expected to drive currents, this explanation was lost.

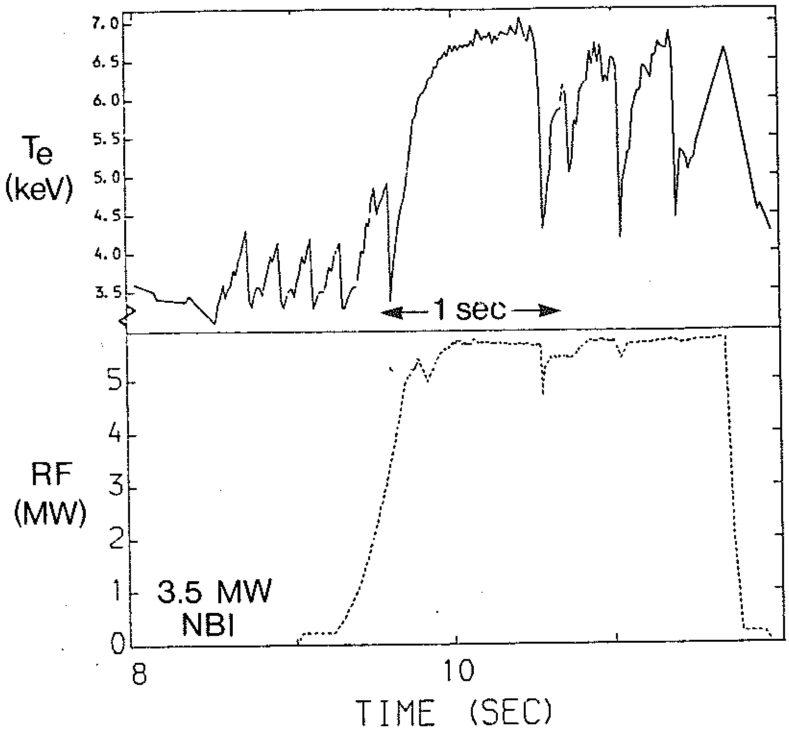

Fig.18 A "monster" sawtooth (JET). The application of additional heating leads to a long delay before the next collapse.

It was then thought that there might be impurity accumulation in the central region as a result of the heating. However the change in resistivity during the monster is given by

$$\frac{\Delta\eta}{\eta} \approx \frac{\Delta Z_{eff}}{Z_{eff}} - \frac{3}{2}\frac{\Delta T_e}{T_e}$$

and the second term is of order 1. It would therefore require a very large change in the impurity level to explain the result.

The third possibility is that the increased pressure drives a bootstrap current outside the q=1 surface, taking current from the central region and raising q_o above unity. This cannot be ruled out but the calculated bootstrap current, ~50kA, seems rather small.

A fourth possibility is that the switching on of additional heating has allowed a transition to the stable low-q_o regime. This is illustrated in figure 19.

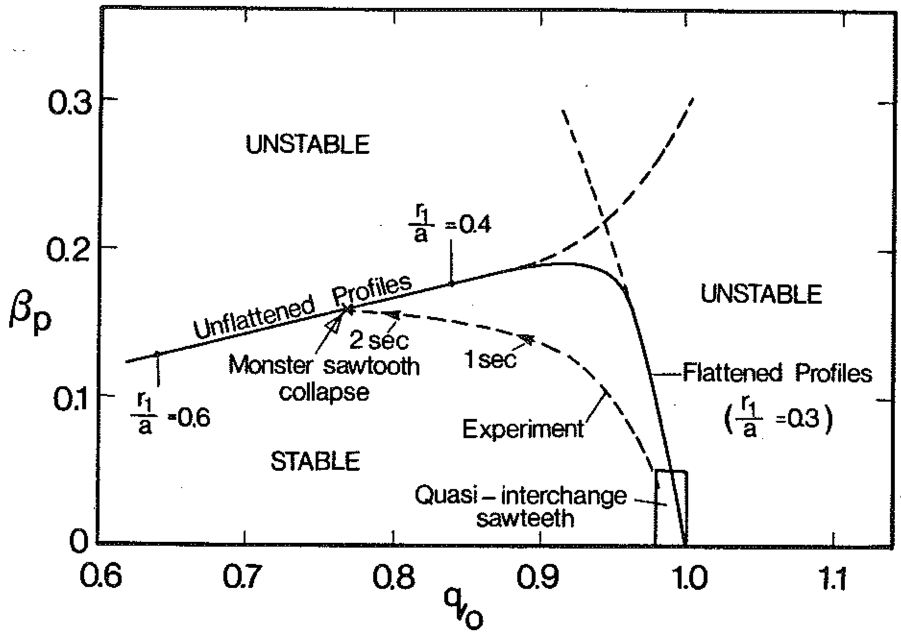

Fig.19 Conjectured behaviour of plasma during monster sawteeth. The q profile ceases to be flattened and the plasma enters the stable $q_o < 1$ configuration. The ultimate collapse then occurs when the change in β_p and q_o takes the plasma across the stability boundary.

In this picture the normal sawteeth have small β_p and $q_o \approx 1$. The additional heating rapidly raises $T_e(0)$ and pulls q_o below unity, taking it into the stable regime. The heating then increases β_p and causes q_o to fall. After one or two seconds the plasma reaches the stability boundary and the sawtooth collapse occurs. One might conjecture that the initial transition is brought about by a small but rapid upward shift of the $q_o = 1$ stability boundary as a result of the changed Larmor radii of the plasma ions or the production of fast ions, placing the plasma in the stable region. The absence of tearing modes under these conditions would be consistent with the observations of their stability at low q_o on other machines as described earlier.

The ideas described above are merely conjectures and it must be said that we do not have a convincing explanation of monster sawteeth.

iii(b) Neutron Sawteeth

Figure 20 shows a phenomenon which has been observed in some JET discharges with neutral beam heating. The soft X-ray emission and electron temperature traces show a monster sawtooth with the three preceding normal sawteeth. The neutron emission has sawteeth which are synchronous with the three preceeding sawteeth but the "normal" neutron sawteeth continue unchanged through the monster phase. As a challenge to sawtooth theory, figure 20 speaks for itself.

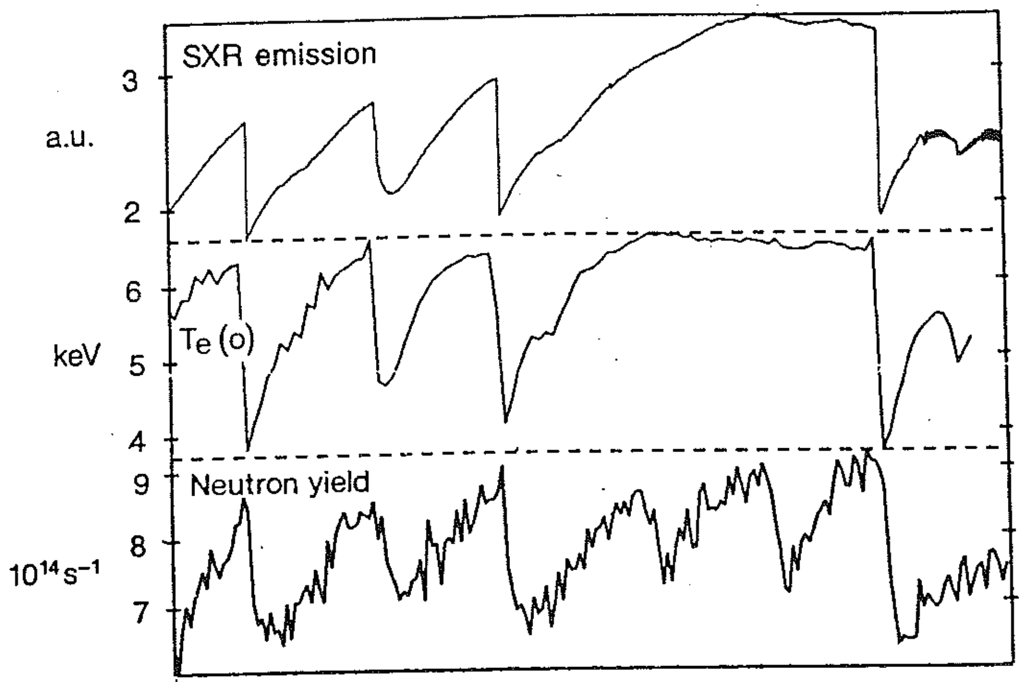

Fig.20 A case where neutron sawteeth, initially in phase with the temperature sawteeth, continue during the monster sawtooth (JET).

Summary

When sawtooth oscillations were first observed, theory had available an m=1 mhd mode to provide the required instability. Although the perceived form of this m=1 instability has changed over the years there has always been a plausible candidate available.

A more challenging problem has been that of the rapidity of the collapse. Kadomtsev's model provided an elegant solution to the problem but recent experimental results have been in conflict with this model and its validity is now seriously questioned. The quasi-interchange model was suggested as an alternative but the position regarding this proposal is at present perplexing. The model was apparently verified by observations on JET but measurements of q_0 on some other tokamaks give levels of magnetic shear which would not allow this mode. Are there different types of sawteeth in different machines? We certainly need an explanation of the low-q_0 sawteeth.

The other problem which has been persistent is that of the trigger. Although this problem is in a sense obvious it has only been formally recognised with the observation of precursorless sawteeth on JET. Stated simply, the problem is how is the rapid growth rate γ switched on in a time of the order γ^{-1}?

More problems arise. The observations of monster sawteeth and the associated neutron sawteeth perhaps provide clues, but at present seem only to add to the list of difficulties.

It seems clear from experience that good experimental work, such as we have seen recently, will be an essential element in resolving the sawtooth problem and that theory has a long way to go.

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