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Abstract

Experimental observations of sawtooth periods in tokamaks are consistent with a model in which the magnetic field after sawtooth collapse is given by $\nabla\times\underline{B} = \mu\underline{B}$ with μ a constant.

Introduction

One picture of sawtooth activity is of a magnetic field configuration evolving towards q=1 as current penetrates into the plasma core. The plasma becomes unstable to an m=1 mode causing a change in magnetic configuration in the core, raising q above unity. The instability causes a reduction in the central current density, hence field energy is converted to plasma energy increasing the entropy of the core.

The two main factors which determine the sawtooth period in such a model are the magnitude of the change in magnetic configuration during the collapse and the size of the sawtooth region. The first of these is considered here.

Kadomtsev's model [1] of sawtooth activity assumed a rigid displacement of the plasma core leading to field line reconnection. Wesson [2] proposed that the collapse should proceed through two convective cells as subsequently observed on JET. In that case Kadomtsev's prescription for the change in magnetic configuration during the collapse cannot apply. A new prescription is required.

The Model

The sawtooth collapse is a very rapid event compared to a transport timescale so it is assumed that the collapse is confined within a mixing radius, r_m. This implies three constraints which must apply during the sawtooth collapse.

1) The total current flowing within r_m must be conserved since $B_\theta(r_m)$ is unchanged. In general this may include a skin current induced at r_m.
2) The toroidal field flux within r_m is conserved since $A_\theta(r_m)$ is unchanged.
3) The helicity $K = \int \underline{A}\cdot\underline{B}\, dV$ of the core plasma is conserved on the rapid collapse timescale. This assumes the flux surface at r_m is unbroken during the change in magnetic topology necessary to achieve an axisymmetric state after sawtooth collapse. Here $\underline{A}$ is the vector potential and the integral is taken over the volume enclosed by the mixing radius.

The change in plasma and field configuration during the sawtooth collapse is derived assuming the associated change in field energy is as large as possible under the above constraints. As the sawtooth period is the time taken for the plasma and field configuration to evolve diffusively back to the pre-collapse state, maximising the change in field energy allows the longest possible sawtooth period to be derived. The model is independent of the instability responsible for the collapse and the derived final configuration will approximate the post-collapse configuration produced by the non-linear evolution of an ideal or resistive instability.

All calculations are carried out in cylindrical geometry. The word toroidal is used to mean the longitudinal direction in a cylindrical tokamak.

The Post-Collapse Configuration

It has been shown previously [3] that the helicity of a volume enclosed by a flux surface can only change on a resistive diffusion timescale, long compared to the sawtooth collapse time. Woltjer [4] and Taylor [5] have shown how minimisation of magnetic field energy

$$E_m = \frac{1}{2\mu_0} \int \left|\underline{B}\right|^2 dV \qquad (1)$$

subject to variations in the vector potential under the constraint of helicity conservation yields the Euler-Lagrange equation,

$$\nabla\times\underline{B} = \mu\underline{B} \tag{2}$$

with μ a constant. The current in such a configuration flows parallel to the magnetic field and hence

$$\underline{j}\times\underline{B} = \nabla p = 0. \tag{3}$$

The axisymmetric solution in cylindrical geometry to Eq.(2) is

$$B_z = B_1 J_0(\mu r) \tag{4}$$

$$B_\theta = B_1 J_1(\mu r) \tag{5}$$

where J_0 and J_1 are Bessel functions of the first kind. Expansion near the axis results in the safety factor profile

$$q = q(0)\left(1 - \frac{r^2}{2q(0)^2R^2}\right) \tag{6}$$

a low shear configuration with variation in q of order ε^2. In toroidal geometry this result is modified by $O(\varepsilon^2)$ corrections [6] but the

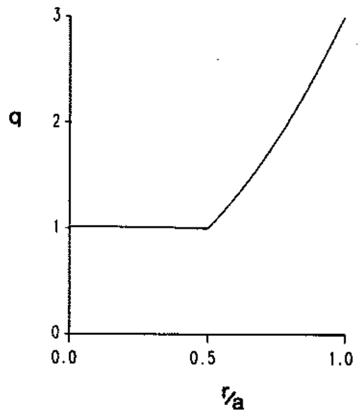

Fig.1: Schematic plot of q profile where the core magnetic field is given by Eqs.(4) and (5)

essential result of a configuration with low shear is still valid. Fig.1 shows a schematic q profile with the magnetic field in the core given by Eqs.(4) and (5).

The magnetic field configuration in the plasma core is assumed to be given by Eqs.(4) and (5) after a sawtooth collapse. If the further assumption is made that plasma energy and particles are redistributed during the collapse in such a way as to maximise the entropy of the core, one obtains

$$\nabla n = \nabla T = 0 \tag{7}$$

consistent with observation.

To calculate the post-collapse configuration requires evaluation of B_1 and μ in Eqs.(4) and (5). These are obtained by conserving the toroidal field flux and the helicity of the mixing region. Since these constraints involve $\nabla\times\underline{A}$ it is also necessary to include the boundary condition that $A_z(r_m)$ is held constant during the collapse.

Comparison with Kadomtsev's Model

A comparison with Kadomtsev's model can be made by initially assuming the toroidal field constant as Kadomtsev did. The conservation of helicity can then be replaced by the conservation of helical flux, $\int\psi dV$, where

$$\frac{d\psi}{dr} = B_\theta(1-q). \tag{8}$$

Minimising the poloidal field energy subject to this constraint yields

$$B_\theta = \frac{\mu_o j_o r}{2} \tag{9}$$

with j_o a constant.

Kadomtsev gave a simple example of the change in ψ during collapse with

$$\psi_o = \frac{\alpha r^2}{2}\left(\frac{r_m^{\,2}}{2} - \frac{r^2}{2}\right) \text{ initially}$$

and

$$\psi_\infty = \frac{\alpha r_m^{\,4}}{16} - \frac{\alpha r^4}{4} \text{ finally.}$$

Calculating the associated change in field energy shows it to be 94% of that obtained when the final configuration is given by Eq.(9). Kadomtsev's model serves to lower the poloidal magnetic field energy almost to the minimum possible under the applied constraints.

The assumption of uniform and constant toroidal field constrains the system and replacing this assumption with the conservation of toroidal field flux allows a larger decrease in field energy during the collapse. Taking the simple example above with

$$\alpha = \frac{0.1\ B_z}{R r_m^{\,2}}\ , \quad \frac{r_m}{a} = 0.5 \quad \text{and} \quad \frac{R}{a} = 3$$

the change in field energy for the three prescriptions for post-collapse configuration can be calculated:

$\frac{\Delta E_m}{E_{m\theta}} = -\ 0.054\%$ with Kadomtsev's model;

$\frac{\Delta E_m}{E_{m\theta}} = -\ 0.058\%$ when the poloidal field energy is minimised,

and

$\frac{\Delta E_m}{E_{m\theta}} = -\ 0.50\%$ when the post-collapse configuration is that given by Eqs.(4) and (5).

$E_{m\theta}$ is the initial poloidal field energy within the mixing radius. There is a factor of 10 larger decrease in field energy in the last case. This is not due to a decrease in toroidal field energy since the lowest energy distribution of toroidal field flux is with B_z uniform as it was initially. The extra field energy released comes from the poloidal field and is available because the system is less constrained.

The Sawtooth Period

The new prescription for the change in plasma and magnetic field configuration during sawtooth collapse has been included in a 1-D simulation. The simulation follows the resistive diffusion of the magnetic fields and the evolution of the temperature perturbation following a sawtooth collapse in a 1m minor radius plasma with parameters typical of a JET ohmic discharge. It is assumed that a collapse is triggered when q falls to unity somewhere. The mixing radius is half the minor radius.

Fig.2 shows the evolution of the simulated electron temperature over 400ms whilst Fig.3 shows the period of sawtooth produced by the simulation for 40 consecutive sawteeth.

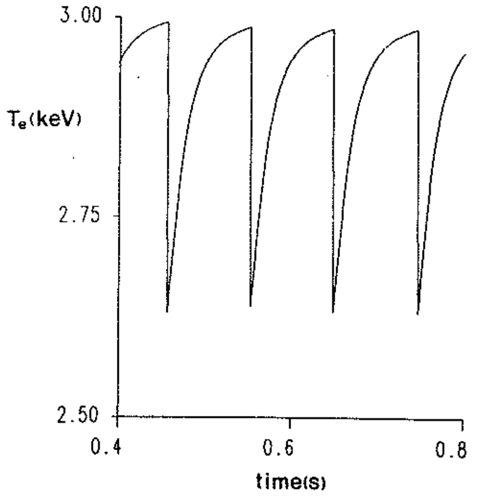

Fig.2: Simulated time variation of central electron temperature

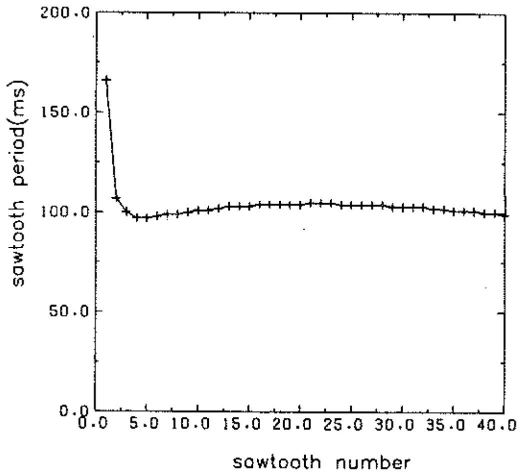

Fig.3: Plot of the simulation sawtooth period for 40 sawteeth

The initial configuration evolves to one with regular periodic sawteeth with a period of approximately 100ms and demonstrates that regular sawtooth oscillations result from this model. Fig.4 shows typical electron temperature sawteeth on JET under different plasma conditions.

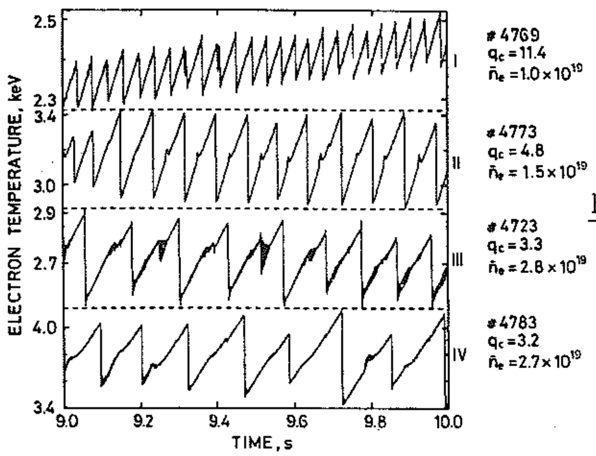

Fig.4: Sawteeth in electron temperature measured on JET under different plasma conditions (Taken from Campbell et al [7])

Sawteeth on JET have a mixing radius of ~ 0.5a in plasmas with $q_c(a) \approx 3.0$ and typically have a period of between 100 and 140ms. Considering the degree of approximation in the simulation where Spitzer resistivity and uniform thermal conductivity are assumed, the agreement between the simulation and the experiment is good.

Sawtooth Period Scaling

A scaling of sawtooth period with plasma parameters based on the present model can be derived by assuming the current density uniform across the mixing region. Because the current density is flattened by the sawtooth collapse and the resistive diffusion time is long compared to the sawtooth period this is a reasonable approximation. This gives

$$(\nabla\times\underline{E})_\theta = (\nabla\eta\times\underline{j})_\theta \tag{10}$$

which, assuming B_z uniform and constant, gives

$$\frac{\partial q(0)}{\partial t} = -\frac{2q(0)}{\mu_o r}\frac{\partial\eta}{\partial r}\,. \tag{11}$$

Since η is flattened during the sawtooth collapse and decreases as the temperature increases through ohmic heating during the ramp phase of the sawtooth, Eq.(11) gives an expression for the change in q(0) in a time, t, after collapse

$$\frac{\Delta q(0)}{q(0)} \alpha \frac{t^2}{\tau_H\tau_R} \tag{12}$$

with $\tau_H = \frac{nT}{\eta j^2}$, $\tau_R = \frac{\mu_o r_m^2}{\eta}$. Rearranging Eq.(12) gives

$$t_{st} \alpha \left(\frac{\Delta q(0)}{q(0)}\tau_H\tau_R\right)^{1/2} \tag{13}$$

with $\Delta q(0)$ the change during the collapse. Assuming $\frac{\Delta q(0)}{q(0)}$ to be the same for all devices gives

$$t_{st} \alpha \tau_H^{1/2}\tau_R^{1/2} \tag{14}$$

and with the additional assumption that q(0) = 1 this becomes

$$t_{st} \alpha \frac{r_m n^{1/2} T^{1/2} R}{V_\ell} \tag{15}$$

with V_ℓ the loop voltage, R the major radius. Fig.5 shows a plot of this scaling against data from 10 tokamaks tabulated previously [8] and also for JET.

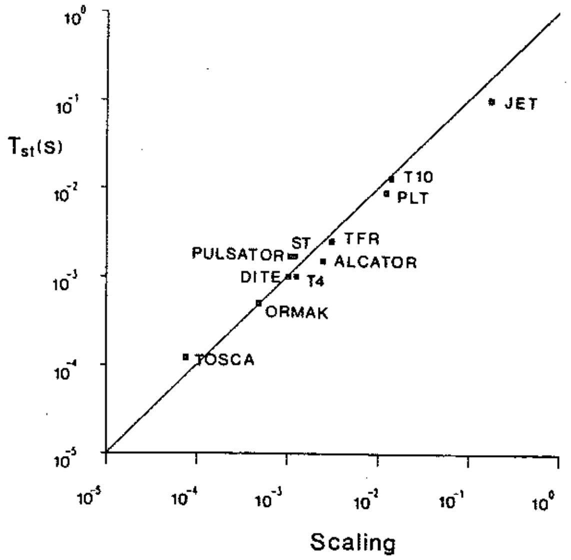

Fig.5: Comparison of experimental sawtooth period with the scaling given in Eq.(15)

Summary

Maximising the field energy change during a sawtooth collapse results in a new prescription for the post-collapse magnetic configuration which satisfies $\nabla\times\underline{B} = \mu\underline{B}$.

For a simple example this gives a factor of 10 increase in the field energy change over that predicted by Kadomtsev's model.

A 1-D simulation incorporating this prescription produces regular sawtooth oscillations with a period of 100ms for JET in approximate agreement with observation.

A scaling of sawtooth period with plasma parameters gives reasonable agreement with data from 11 tokamaks.

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