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N. Lopez Cardozo, B. Tubbing, F Tibone and A. Taroni

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N. Lopez Cardozo[1], A. Taroni, F Tibone and B. Tubbing[1]

JET-Joint Undertaking, Culham Science Centre, OX14 3DB, Abingdon, UK

[1]*on attachment from FOM Rijnhuizen*

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ABSTRACT.

The problem of deducing χ_e from heat pulse propagation measurements is addressed. An extended diffusive model is described, which takes.into account perturbed source and sink terms. χ_e is expressed as a function of two observables, the heat pulse velocity ν_{HP} and the radial damping rate α: χ_e 4.3 a ν_{HP}/α. This expression is checked against full transport e simulations and found accurate. Finally it is shown that the thus inferred χ_e is a local value, not influenced by the χ_e-profile outside the measuring region.

INTRODUCTION

Energy confinement, and its deterioration at high heating power, is perhaps the most urgent issue in tokamak physics. One of the key parameters to be studied is the electron heat diffusivity (χ_e) in the confinement region. Experimentally χ_e can be assessed by analysing the spatial diffusion of a perturbation of the equilibrium profile of the electron temperature, T_e. To that end the local perturbation $\tilde{T}_e(r)$ is measured as a function of time at various positions in the plasma simultaneously. Usually the sudden change of the central part of the T_e-profile as a result of the sawtooth instability is used as initial perturbation. The original discussion of this method is by Callen and Jahns [1]. Since then the method has been applied to various machines [2-8].

Until recently noise on the measured signal severely limited the obtainable accuracy. It is only with the present ECE diagnostic systems and the long time scales involved on the new generation large tokamaks that an experimental error of as small as 20% may be realised. These developments on the experimental side now lead us to reconsider the technique of deriving a local value for χ_e from measurements of $\tilde{T}_e(r,t)$. The standard technique is based on the assumption that the relaxation of the perturbed T_e-profile is a purely diffusive process, involving only the electrons. As was shown by Goedheer [9] this approximation is not always adequate. It leads to an overestimate of χ_e of typically a factor of 1 to 2.

In this paper we will present an extended diffusive model which reduces systematic errors to order 10%. In Section 2 we briefly recall the standard diffusive model. The extended diffusive model is introduced, and checked with numerical transport simulations. In Section 3 we analyse the influence of the shape of the χ_e-profile on the determination of χ_e. We address the question whether heat pulse propagation analysis yields a local measurements of χ_e, or rather a global average. Section 4 gives a discussion of the merits of the new model.

2. DIFFUSIVE MODELS

A description of the relaxation of the perturbed T_e-profile in principle requires solving the complete set of transport equations for electrons and ions. However, the problem may be simplified by neglecting effects of $\tilde{n}_e$, changes in the electron-ion energy exchange etc. The simplest equation then achievable is the diffusion equation $\dot{\tilde{T}} = {}^2/_3 \chi_e \nabla^2 \tilde{T}$ which for suitable initial perturbation profiles can be solved analytically. This approach was pursued first by Soler and Callen [3], who found that χ_e can be expressed as

$$\chi_{HP}^{DIF} = \frac{r^2 - r_{mix}^2}{8t_p} \qquad (1)$$

where r_{mix} is the mixing or reconnection radius and $t_p(r)$ is the time at which $\tilde{T}_e(r)$ reaches a maximum. The superscript DIF refers to the fact that the relaxation is assumed to be a purely diffusive process.

Two questions concerning this approach have been raised. First, are the simplifications made to arrive at the simple diffusion equation justified? Second, how good are the analytically obtained results?

The latter question is addressed in a recent paper by Fredrickson et al [6]. It gives some improvements on the original analytical work and checks the results with a code that numerically solves the diffusion equation for more general conditions. They find that the original result (1) holds to good approximation.

The first question can be studied by comparing the results from solving the purely diffusive equation (either analytically or numerically) to those obtained with a full numerical transport simulation. This was done by Goedheer [9]. He finds that, due to the perturbation of the sink terms (electron-ion energy exchange, mainly) which in the diffusive model are neglected, the temperature perturbation spreads faster in the transport simulation, typically by a factor between 1 and 2. His conclusion is that in order to reliably derive χ_e from measurements of $\tilde{T}_e(r,t)$, a full transport simulation for the specific plasma conditions should be run. This, however, is too time-consuming, in terms of both manpower and cpu-time, to be a viable option. The extended diffusive model we present here takes into account variations of sinks and sources. Yet it leads to a simple expression for χ_e in terms of two measured quantities, the heat pulse velocity and the rate at which the amplitude of the perturbation decreases towards the wall. Both quantities are experimentally well accessible with ECE diagnostic systems [8,10,11]. The heat pulse velocity has been used routinely in heat pulse analysis studies [4,8,9,10,11]. The use of the radial damping rate as additional input is introduced here.

The basic equation governing the perturbed T_e-profiles is

$$\frac{3}{2} n_e \dot{\tilde{T}}_e = - \nabla \cdot \tilde{Q} + \tilde{P} \tag{2}$$

where $\tilde{T}_e$ is the perturbation of the electron temperature, $\tilde{Q}$ the perturbation of the heatflow per unit area and $\tilde{P}$ the perturbation of the source power density, i.e. electron-ion energy exchange, radiation loss and ohmic dissipation. If we assume that χ_e is independent of T_e and spatial derivatives thereof (see section 4) and linearise $\tilde{P}$ in $\tilde{T}$, equation (2) takes the form

$$\frac{3}{2} n_e \dot{\tilde{T}}_e = \nabla \cdot (n_e \chi_e \nabla \tilde{T}_e) - \frac{1}{\tau} n_e \tilde{T}_e \tag{3}$$

with $\tau = n_e \left\{ \frac{\partial}{\partial T_e} (P_\Omega - P_{ie} - P_{rad}) \right\}^{-1}$ where P_Ω, P_{ie} and P_{rad} denote power densities of ohmic dissipation, electron-ion energy exchange and radiation loss, respectively. The damping term $(\frac{1}{\tau} n_e \tilde{T}_e)$ is important if τ is comparable to the typical time scale of the diffusion process. In normal tokamak conditions the electron-ion exchange is the leading term in τ, so that $\tau \sim \tau_{ie}$. For JET a typical value is $\tau_{ie} = 50$ ms in the confinement region, compared to a typical time of 30 ms for the diffusive relaxation process. Hence we cannot neglect the term $\frac{1}{\tau} n_e \tilde{T}_e$. Naturally, τ has a radial profile, about which we lack precise knowledge. However, we can improve on the standard zero-order approximation $\tau = \infty$ discussed above, by making the first order approximation τ = constant. Results of comparison with full transport simulations will be given below.

As is clear from Eq. (3), the effect of the damping term is to add a factor $\exp(-t/\tau)$ to the solution of the initial value problem without damping. Consequently, with damping (see Fig. 1):

- the time $t_p(r)$ at which $\tilde{T}(r)$ is maximal comes earlier;
- the actual maximum $\tilde{T}(r,t_p)$ is reduced by a factor $\exp(-t_p/\tau)$.

From ECE measurements $\tilde{T}(t)$ can be deduced at different radii, from which we derive two quantities:

- the heat pulse velocity v_{HP}, defined (locally) by $v_{HP} = (\frac{d}{dr} t_p)^{-1}$
- the radial damping rate α, defined by $\alpha = 10\ a \frac{d}{dr} \log \tilde{T}(r,t_p)$ expressed in dB.

Both are in principle local values which can be determined as a function of radius. To reduce noise, however, it is generally necessary to average over regions of at least 10% of the minor radius. An absolute lower limit is set by the spatial resolution of the ECE system, naturally. Below we will show that the two measured quantities v_{HP} and α enable us to determine unambiguously the two unknowns in the extended diffusion equation, τ and χ_e. Hence by including the damping rate in the analysis we are able to account for the effect of perturbed sinks and sources.

We solved Eq (3) numerically in cylindrical geometry, symmetric in z and Θ, for the choice of profiles $\chi_e(r) = \chi_{e,o}$ and $n_e(r) = n_e(o)\ \{0.1 + 0.9\ [1-(r/a)^2]\}$ and the boundary condition $\tilde{T}_e(a) = 0$. Profile effects are investigated in Section 3. The initial dipole perturbation was of a parabolic shape, close to the experimentally observed perturbation profiles.

$v_{HP}(r)$ and $\alpha(r)$ were determined from the calculated $\tilde{T}(r,t)$ and averaged over a radial interval of 0.1a, ie. from r−0.05a to r+0.05a. In Fig. 2 results are shown for r_{mix}=0.5a, for r ranging from 0.6a to 0.8a. Plotted are the dimensionless quantities $v_{HP}a/\chi_{e,o}$ and α (both are independent of system dimensions) as a function of the dimensionless parameter $K=a^2/\tau\chi_{e,o}$. K determines the relative strength of the damping term. We see that both α and $av_{HP}/\chi_{e,o}$ can to a very good approximation be represented by linear functions of K:

$$av_{HP}/\chi_{e,o} = p_v + q_v K \tag{4a}$$

$$\alpha = p_\alpha + q_\alpha K \tag{4b}$$

Hence $\chi_{e,o}$ can be expressed as a function of v_{HP} and α, yielding χ_{HP}^{EXT}:

$$\chi_{HP}^{EXT} = c_1 \frac{av_{HP}}{\alpha + c_2} \tag{5}$$

with $c_1=q_\alpha/q_v$ and $c_2=c_1p_v-p_\alpha$. The constants p_v, p_α, q_v and q_α are nontrivial functions of r and r_{mix}. However, the resultant constants c_1 and c_2 are well behaved in the radial range of interest. In Fig. 3 c_1 and c_2 are plotted as a function of $r-r_{mix}$, for r_{mix} ranging from 0.3a to 0.6a.

The range of $r-r_{mix}$ in which heat pulse measurements can be made sensibly is restricted by several constraints:

- in the direct vicinity of r_{mix}, ie. $r-r_{mix}$ < 0.05a, the solution of the diffusion equation is very sensitive to the shape of the initial T_e perturbation;
- with α typically ≥ 40 dB, the amplitude of $\tilde{T}_e(r)$ falls by a factor of 10 over Δr=0.25a, which makes reliable measurements at r > r_{mix}+0.25a very difficult.
- for r≥0.85a the effects of the wall and the impurity radiation dominate the transport processes, and no reliable χ_e measurement can be expected anyway.

Therefore the radial range of useful $\tilde{T}(r)$ measurements is effectively r_{mix}+0.05a < r < r_{mix}+0.25a. Hence, due to the minimal averaging interval of 0.1a necessary to determine v_{HP} and α, the useful range of r on which χ_{HP} can be determined is just $(r_{mix}-r)$ = 0.1a to 0.2a. On this interval χ_{HP}^{EXT} can for all practical purposes be expressed as: (around $r=r_{mix}+0.15a$, taking $\langle\alpha\rangle$=40dB, $\langle c_2\rangle=-5$)

$$\chi_{HP}^{EXT} = 4.3\ a\ \frac{v_{HP}}{\alpha} \qquad (6)$$

In Fig. 4 this relation is plotted together with the points $(\alpha,\ av_{HP}/\chi_{e,o})$ calculated for K=0 to 40. Clearly expression (6) gives a good fit to the data, independent of r_{mix}.

Note that the purely diffusive model is found as a special case of the extended model. For K=0 with $r_{mix}=0.4a$ and $r-r_{mix}=0.15a$, we have $\alpha=32$ dB, hence

$$\chi_{HP}^{EXT}\ (K=0) = 0.14\ av_{HP}$$

This corresponds to expression (1) for the diffusive case: here one should insert $r=r_{mix}+2 \times 0.15a$ (because in this formula r and r_{mix} are defined as the boundaries of the averaging interval) to find:

$$\chi_{HP}^{DIF} = \frac{1}{8}\ v_{HP}\ (r_{mix} + r) = 0.14\ a\ v_{HP}$$

The same correspondence is found for other values of r_{mix}, demonstrating that indeed the diffusive model is reproduced as a special case of the extended model.

The extended model was tested by applying it to a number of computer simulations of JET plasmas. The simulations were done with a full, 1.5 D transport code, JETTO [12,13]. It produced T_e traces at various radii so that heat pulse analysis could be performed. In table 1 the χ-values derived with both the diffusive and the extended model are compared to the actual values used in the code. All values are taken at $r_{mix}+0.15a$. We see that the extended model produces χ_e-values that are slightly low, whereas the purely diffusive model yields values that are a factor 1.6 too high. (This factor depends on K, evidently, and therefore depends on machine size and plasma conditions).

We conclude that, to 10% accuracy expression (6) for χ_{HP}^{EXT} can be used to derive χ_e from measurements of v_{HP} and α. The extended model correctly accounts for the effects of the perturbation of the sinks and sources. As will be shown in Section 3, the χ_e values with the extended model should correspond to χ_e averaged over the radial interval on which v_{HP} and α are determined, with a strong emphasis on the small radius side of this interval. If we account for this effect the extended model reproduces the modelled χ_e values to within few percent.

Table 1a. Runs of the full transport code JETTO used for comparison with heat pul models. B_T is the toroidal field, I_p is the plasma current, $\langle n_e \rangle$ is the line avera density; the χ_e-profile is inversely proportional to the n_e-profile, the absolu value is matched to produce the correct global energy confinement time.

nr	B_T [T]	I_p [MA]	$\langle n_e \rangle$ [$10^{19}m^{-3}$]	$\chi_e(r=\frac{2}{3}a)$ [m^2/s]	v_{HP} [m/s]	α [dB]
1	3.4	4.0	1.8	1.5	10	43
2	3.4	4.0	2.6	1.1	6.9	45
3	3.4	4.0	3.1	1.0	6.4	43
4	3.4	3.0	1.6	1.7	11	47
5	2.5	2.0	1.5	1.8	12	45

Table 1b. Comparison of the heat pulse models with the full transport code JETTO. χ_H is derived from v_{HP} and α given in Table 1a. The resulting χ_{HP}-values are list together with their factor of deviation from the local χ_e value in JETTO.

nr	JETTO $\chi_e(r=\frac{2}{3}a)$ [m^2/s]	DIFFUSIVE MODEL $\chi_{HP}^{DIF}(r=\frac{2}{3}a)$ [m^2/s]	DIFFUSIVE MODEL factor	EXTENDED MODEL $\chi_{HP}^{EXT}(r=\frac{2}{3}a)$ [m^2/s]	EXTENDED MODEL factor
1	1.5	2.5	1.7	1.5	1.0
2	1.1	1.7	1.5	1.0	0.9
3	1.0	1.6	1.6	1.0	1.0
4	1.7	2.8	1.6	1.5	0.9
5	1.8	3.0	1.7	1.7	0.9

3. The influence of the shape of the χ_e-profile on the determination of χ_{HP}

The diffusive and extended diffusive models yield a value for χ_{HP} expressed as a function of the measured quantities: delay time $t_p(r)$ or heat pulse velocity and radial damping rate. These quantities are measured locally, ie. on a radial interval Δr which is small compared to a; usually $\Delta r/a$ = 0.1 to 0.2. However, the diffusion equation is solved on the entire interval r/a = 0 to 1. Therefore the local values $t_p(r)$, v_{HP} and α are not only determined by the value of χ_e in the measuring interval Δr, but also by the χ_e-profile outside Δr. Consequently, χ_{HP} is not a measurement of the local χ_e, but a weighted radial average of χ_e.

Fredrickson et al. [6] investigated the influence of the shape of the χ_e-profile on χ_{HP} by numerically solving the diffusion equation for a number of different χ_e-profiles. They found that, indeed, χ_{HP} was clearly influenced by the shape of the χ_e-profile.

In this section we will determine more quantitatively how different parts of the χ_e-profile contribute to the measurement of χ_{HP}. The basic approach is to write χ_{HP} as a weighted line average of χ_e

$$\chi_{HP} = \int_0^a w(r)\chi_e(r)dr \tag{7}$$

and then to determine the weight function w(r) with the help of the cylindrical diffusion code. One may consider this weight function as the instrument function of the heat pulse propagation method. If it peaks strongly in the measuring region Δr this implies that χ_{HP} is a local measurement of χ_e. Conversely, if w(r) has significant contributions outside Δr, χ_{HP} is a broader line average of $\chi_e(r)$.

This method is not generally applicable. In particular, representation (7) breaks down for χ_e-profiles that approach zero (or go to infinity) at any radius, due to the nonlinearity of the problem. However, as we show in the

Appendix, eq. (7) is accurate for χ_e-profiles spanning a χ_e-range of a factor 10. This means that the ensuing analysis is applicable to all physically interesting profiles.

The weight function w(r) can be determined with standard techniques (see Appendix). In Fig.5 the result is shown for both the diffusive and the extended diffusive model. For the curves shown v_{HP} and α are determined and averaged over the interval r/a = 0.6 to 0.73, as indicated in the figure, with $r_{mix}/a = 0.5$. It is evident from Fig.5 that the weight function for the extended model is strongly peaked in the measuring interval. Clearly the extended model yields a very localized measurement of χ_e. The diffusive model, on the other hand, yields a rather broad line averaged value of χ_e. The reason for this difference in behaviour is that a variation in the χ_e-profile outside the measuring region changes α and v_{HP} in much the same way, leaving their ratio unaffected.

The above conclusion was tested by running the cylindrical diffusion code with a number of different χ_e-profiles, in the same way as Fredrickson et al [6] have done. We found that χ_{HP}^{EXT} is virtually insensitive to the shape of the χ_e-profile. For very different profile shapes χ_{HP}^{EXT} corresponded to the local value of χ_e within 5%, in corroboration with the above analysis. For the same profiles χ_{HP}^{DIF} could deviate up to 60% from the local χ_e value.

4. DISCUSSION

In the previous sections we have introduced the extended diffusive model, which expresses χ_e in terms of v_{HP} and α. The inclusion of the damping term proved: a) to account for the effect of the perturbed sinks and sources and b) to provide far more localized measurements than the original diffusive model. How can a single parameter account for these different effects?

First we must notice that the effect of any sink is to increase both v_{HP} and α. The most obvious sink is electron-ion energy exchange. However, if somewhere χ_e is larger than was assumed in the model this influences α and v_{HP} in much the same manner. Especially in the mixing region an enhancement of χ_e leads to a more rapid flattening of the initial dipole perturbation. This explains the hump in the weight functions in the inversion region (Fig. 5).

Second, if the time constant τ associated with the sink is larger than the time constant τ_{HP} of the relaxation of the $\tilde{T}_e$-profile, the effect of the sink is global rather than local. In those cases the radial distribution of the sinks is not very important and the approximation τ = constant is justified. So we get the ordering: $\tau > 5\tau_{HP}$: damping term negligible; $\tau_{HP} \leqq \tau \leqq 5\,\tau_{HP}$: extended model adequate approximation; $\tau < \tau_{HP}$: strong local effects of sinks, the radial profile of τ should be taken into account.

The extended diffusive model is found accurate if compared with full transport simulations. However, some assumptions have been made in the transport code as well as in the diffusive models, ie. that χ_e does not depend on T_e or ∇T_e. What would be the effect of such dependences on heat pulse studies? As is easily checked, a ∇T_e-dependence does not change the linearised diffusion equation. It only replaces χ_e by $\chi_e' = \chi_{e,eq} + (\frac{\partial}{\partial \nabla T_e} \chi_e)\, \nabla T_{e,eq}$ where the subscript eq. refers to the unperturbed equilibrium situation. The analysis presented in section 2 and 3 is still fully applicable. The χ_{HP} value derived from heat pulse propagation now corresponds to χ_e'. Hence a ∇T_e dependence in χ_e does not invalidate the analysis, it only affects the physical interpretation of

χ_{HP}(see [10]). A T_e-dependence in χ_e is formally equivalent to a convective term in the heat transport. Such a term does not affect v_{HP}, but changes the pulse shape and the radial damping rate. To properly account for this term a third parameter should be extracted from the experimental data. Inspection of pulse shapes so far has not shown any strong non-diffusive effect [6], which is why in heat pulse analysis χ_e is assumed independent of T_e. Also the effect on the damping rate alone can be used to reject the possibility of a significant T_e-dependence on the basis of experimental data, as is done in ref. [10].

In conclusion, we have shown that the extended diffusive model has reduced systematic errors in heat pulse analysis to order 10% and strongly improved the localization of the measurement. Thus the accuracy of the analysis is in line with the present measuring accuracy. If we want to extend the analysis to comprise non-diffusive heat transport, the pulse shapes need to be taken into account.

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REFERENCES

1. Callen J D, Jahns G L, Phys Rev Lett 38 (1977) 971.

2. Bell J D, Dunlap J L, Paré V K, Callen J D, Howe H C, Lazarus EA, Murakami M, Thomas C E, Nucl Fusion 24 (1984) 997.

3. Soler M, Callen J D, Nucl Fusion 19 (1979) 703.

4. Sillen R M J, Piekaar H W, Oyevaar Th, Gorunov E P, Bagdasarov A A, Vasin N L, Nucl Fusion 26 (1986) 303.

5. Alladio F, Mazzitelli G, Tuccillo A A, Vlad G, in Controlled Fusion and Plasma Physics (Proc 12th Europ Conf, Budapest, 1985), 9F-I (1985) 138.

6. Fredrickson E D, Callen J D, McGuire K, Bell J D, Colchin R J, Efthimion P C, Hill K W, Izzo R, Mikkelsen D R, Motincello D A, Pare V, Taylor G, Zarnstorff M, Nucl Fusion 26 (1986) 849.

7. T-10 Group, in Controlled Fusion and Plasma Physics (Proc 12th Europ Conf, Budapest, 1985) 9F-I (1985) 38.

8. Piekaar H W, Goedheer W, Niestadt R M, Stringer T E, Tubbing B J D, 27th meeting of the American Physical Society, Div of Plasma Physics, paper 6P49, in Bull of American Phys Soc, 30, No 9, San Diego (1985).

9. Goedheer W, Nucl Fusion 26 (1986) 1043.

10. Tubbing B J D, Lopes Cardozo N J, van der Wiel M J, submitted to Nucl Fusion.

11. Tubbing B J D, Barbian E, Campbell D J, Hugneholtz C A J, Niestadt R M, Oyevaar Th, Piekaar H W, in Controlled Fusion and Plasma Physics (Proc 12th Europ Conf Budapest, 1985) 9F-I (1985) 215.

12. Cenacchi G, Taroni A, Proc 8th Conf on Computational Physics, Eibsee, 1986, 10D (1986) 57.

13. Taroni A, Abels van Maanen A E P M, Core W, Duchs D F, Hellsten T, Muir D G, Stubberfield P M, Tibone F, Watkins M L, in Controlled Fusion and Plasma Physics (Proc 12th Int Conf, Budapest 1985) 9FI (1985) 34.

Appendix

In this appendix we will show that χ_{HP} can indeed be written as a weighed line integral of $\chi_e(r)$ (Eq.7), provided the dynamic range of $\chi_e(r)$ is limited to a factor 10, and we describe how w(r) is calculated.

We linearise the problem by taking the constant χ_e-profile $\chi_e(r) = \chi_{e,o}$ as the unperturbed profile, for which we know $\chi_{HP} \equiv \chi_{e,o}$ since χ_{HP} is specifically derived for this profile, and analyse how small perturbations of this profile affect χ_{HP}. By writing $\delta\chi_{HP} = \chi_{HP} - \chi_{e,o}$ and $\delta\chi_e(r) = \chi_e(r) - \chi_{e,o}$ Eq.(7) yields:

$$\delta\chi_{HP} = \int_o^a w(r)\ \delta\chi_e(r)\ dr \tag{A1}$$

where $\delta\chi_e(r)/\chi_{e,o}$ should be small everywhere .

To determine w(r), we use the test profile $\delta\chi_e(r) = \chi_{e,H}\ H(r-r_H)$, where $H(r-r_H)$ is the Heavyside function, stepping up at $r=r_H$. The cylindrical diffusion code was run for a large number of values of r_H. We determined χ_{HP}^{EXT} and χ_{HP}^{DIF} as a function of r_H, for a chosen measuring region and fixed initial T_e-perturbation. The weight functions are then calculated according to

$$w(r_H) = \frac{1}{\chi_{e,H}} \frac{\partial}{\partial r_H}[\chi_{HP}(r_H)] \tag{A2}$$

The relative amplitude $\chi_{e,H}/\chi_{e,o}$ of the step was varied to test the range of applicability of the analysis. It was found that virtually identical weight functions were obtained for $\chi_{e,H}/\chi_{e,o}$ ranging from -0.7 to +3.0

In a separate test the diffusion code was run with a χ_e-profile which was constant except for two rectangular perturbations of width a/15 (see Fig.A1). The amplitudes A and B of the perturbations were varied indepently, and the resulting variation of χ_{HP}, $\delta\chi_{HP}$, was determined. It was established that the linearity condition:

$$\delta\chi_{HP} = w_1 A + w_2 B \qquad \text{(A2)}$$

holds fairly well for $A/\chi_{e,o}$ and $B/\chi_{e,o}$ ranging from −0.7 to +3.0. The constants w_1 and w_2 are the integrals of w(r) over the width of the perturbations. This result implies that the above analysis is applicable to χ_e-profiles spanning a χ_e-range of up to a factor ten, ie. all profiles that are interesting from the point of view of tokamak physics.

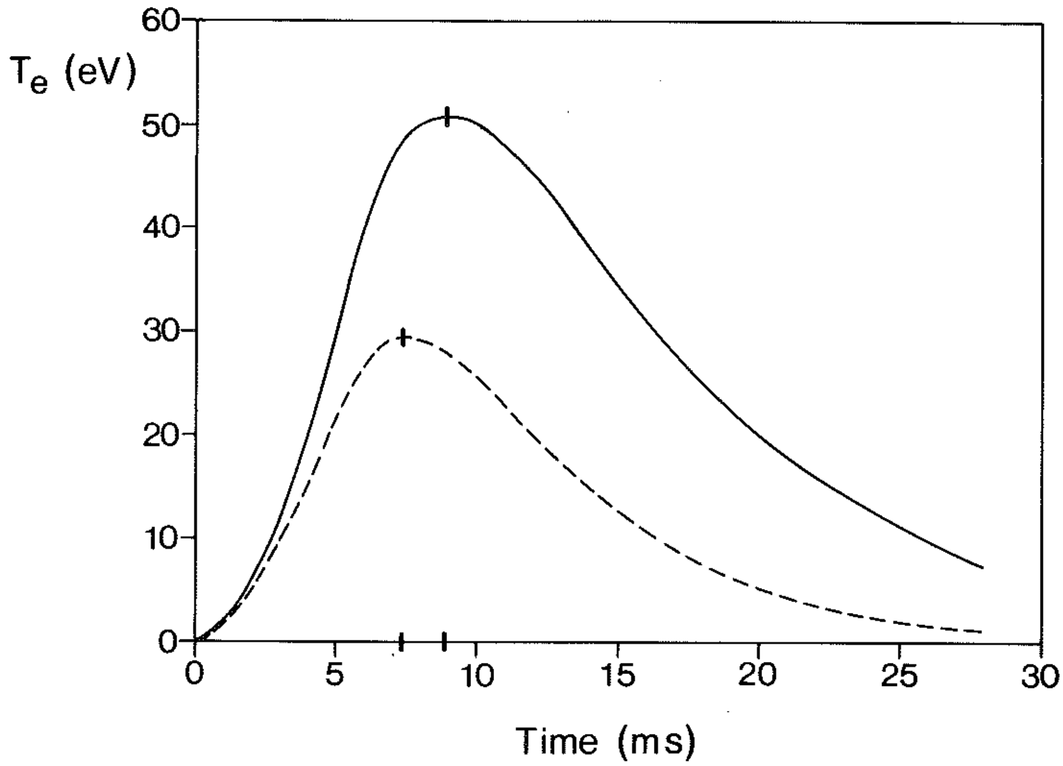

Fig. 1 The effect of the damping term on the shape of the heat pulse. Shown is the evolution of the temperature perturbation at a radius outside the mixing radius. The full line represents the solution of the diffusion equation without damping, the broken line is the solution if damping is included. Clearly the effect of the damping term is to reduce the maximum and the time at which the maximum is reached.

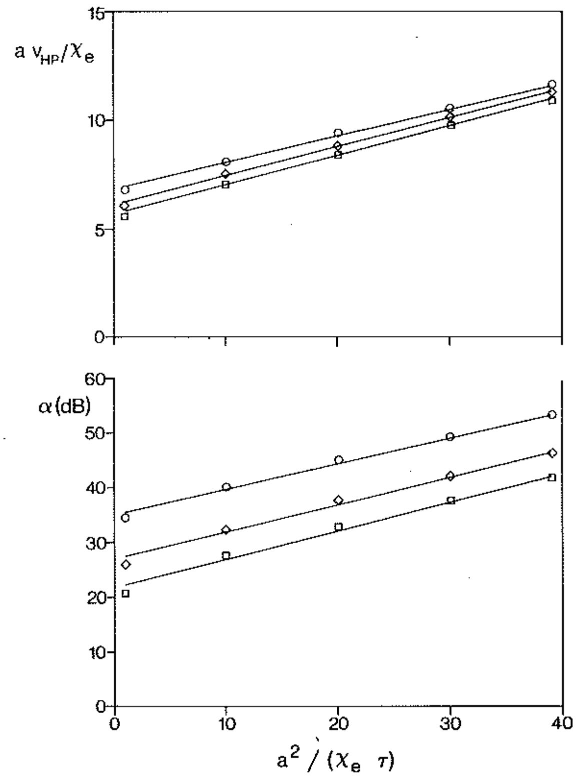

Fig. 2 The effect of the damping term is parametrized by the dimensionless quantity $K = a^2/\chi_e\tau$. Shown are the normalised heat pulse velocity and damping rate as a function of K, calculated at three distances from the mixing radius: O:$(r-r_{mix})/a = 0.10$; ◇:$(r-r_{mix})/a = 0.15$ and □ :$(r-r_{mix})/a = 0.20$. In this example $r_{mix} = 0.5a$.

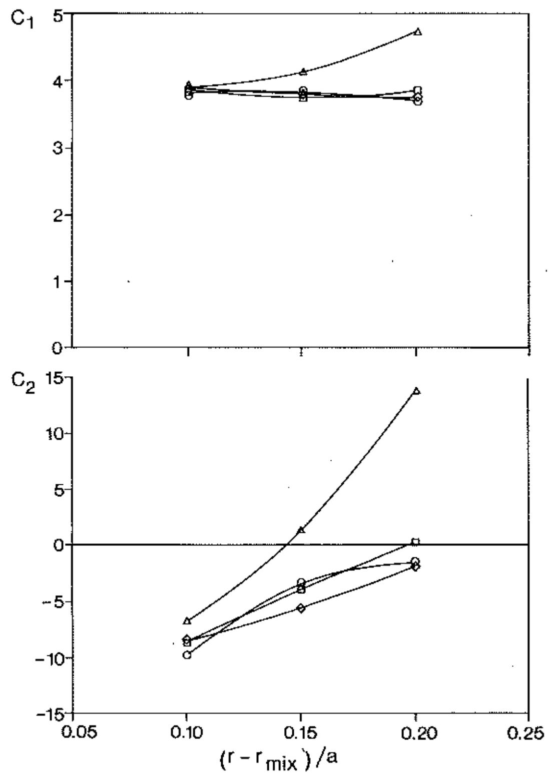

Fig. 3 The constants c_1 and c_2 as a function of $(r-r_{mix})/a$, for $r_{mix} = 0.3a$(o); $r_{mix} = 0.4a$ (◇); $r_{mix} = 0.5a$ (□) and $r_{mix} = 0.6a$ (Δ). The important constant c_1 is nearly independent of both r_{mix} and r_{mix}, while c_2, which describes a correction of order 10% to χ_e, does show a dependence of $r-r_{mix}$. For $r > 0.8a$ the boundary condition begins to have significant effects.

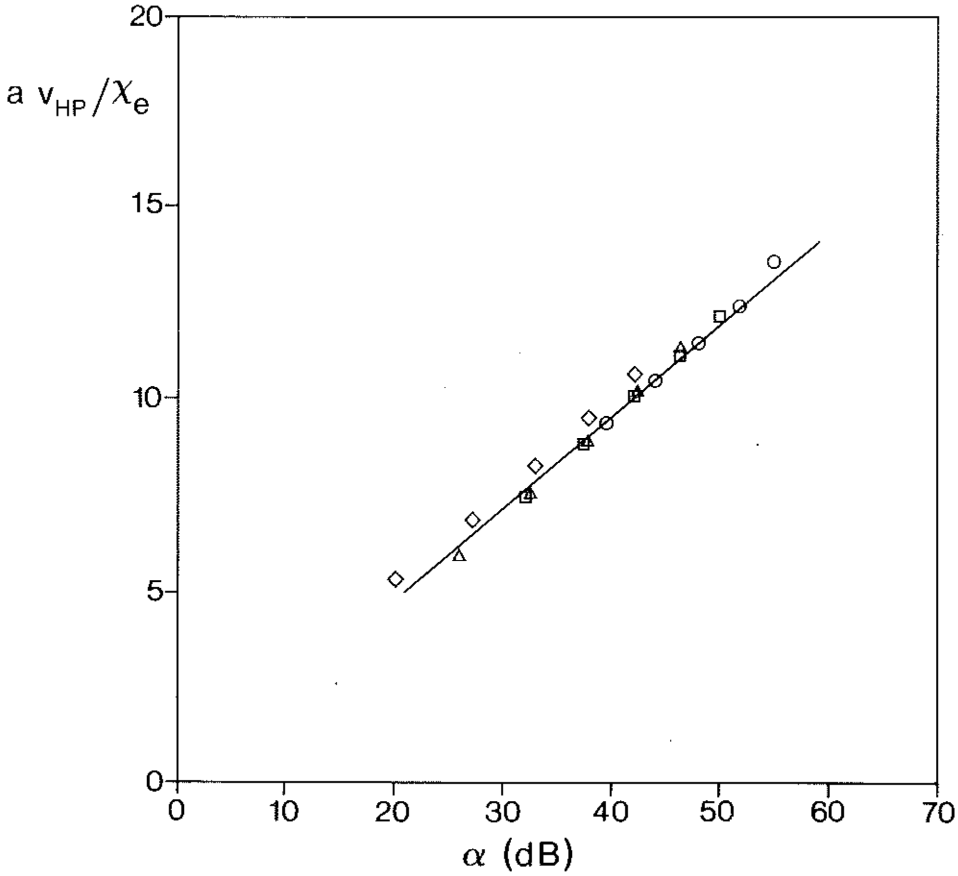

Fig. 4 The normalised heat pulse velocity is proportional to α at $r-r_{mix} = 0.15a$. The proportionality is independent of r_{mix} (plotted are results for $r_{mix} = 0.3a$(o); $r_{mix} = 0.4a$ (◇); $r_{mix} = 0.5a$ (□) and $r_{mix} = 0.6a$ (Δ) and does not depend on K (total K range: K = 0 to 40).

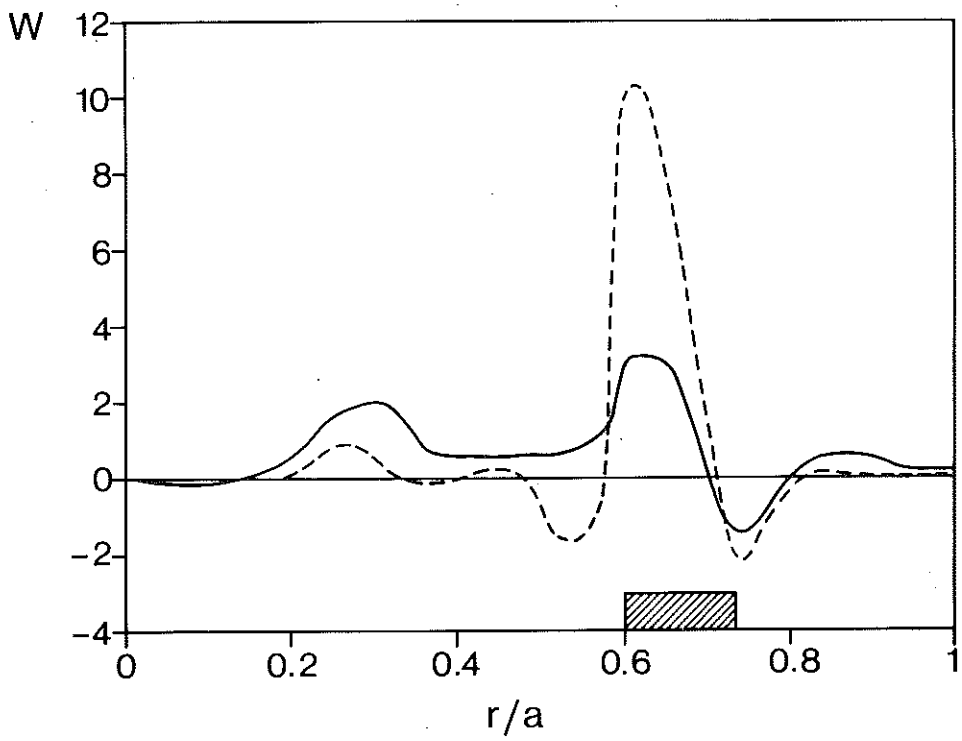

Fig. 5 The weight function w calculated for the diffusive model (full line) and the extended model (dashed line), for $r_{mix} = 0.5a$; α and v_{HP} are averaged over the radial interval $0.6a < r < 0.73a$. Clearly the extended model yields a local value of χ_e, the weight function being strongly peaked in the measuring interval. The diffusive model yields a broader line-averaged value of χ_e.

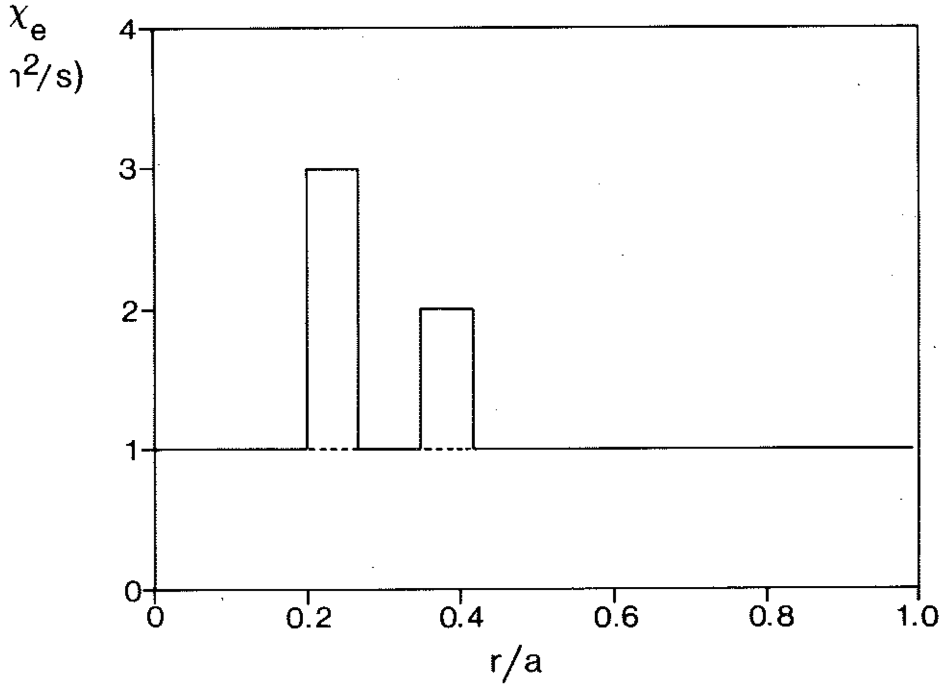

Fig.A1 To test the linearity of the response of χ_{HP}^{EXT} and χ_{HP}^{DIF} to a perturbation of the χ_e-profile, the plotted profile shape was assumed. The sign and amplitude of the two rectangular perturbations were varied independently, as a check of the validity of Eq. (A2).