JOINT EUROPEAN TORUS

JET–P(87)13

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JET-Joint Undertaking, Culham Science Centre, OX14 3DB, Abingdon, UK

Preprint of Paper presented to the 7th International Conference on Plasma Physics
Kiev, USSR, April 1987

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HEAT TRANSPORT IN PARTIALLY STOCHASTIC TOKAMAK MAGNETIC FIELDS*

M. HUGON, M. BRUSATI, P.P. LALLIA, P.H. REBUT

JET Joint Undertaking, Abingdon, Oxfordshire, OX14 3EA, United Kingdom

ABSTRACT

The anomalous confinement in tokamaks could be explained by a model which takes explicitly into account the magnetic field line topology (equilibrium between islands and a stochastic region). It is shown how the properties of this topology affect the particle motion. This topology could be self-sustained by the difference of conductivity between the stochastic region and the islands, which depends on the temperature gradient. The resulting particle and heat flows are calculated by recurrence analysis between island chains. Finally, a preliminary comparison between the model and JET data is given: the poloidal mode number M associated to the perturbation should be between 10 and 20.

1. INTRODUCTION

The anomalous electron thermal transport in tokamaks is still a major problem, which remains to be solved. Among the various explanations proposed so far, the possible existence of micro-tearing modes driven by the electron temperature gradient deserves a particular attention. In this approach, the overlapping between neighbouring island chains leads to the formation of stochastic field lines. The electron heat flow along the stochastic field lines is responsible for the anomalous thermal losses and allows to sustain the islands. Calculations have been performed both in the linear case[1,2] and including non-linear effects[3,4], but without taking explicitly into account the magnetic topology.

In the model presented here, the difference of behaviour of the field lines between the inside and the outside of the islands is intrinsically non linear. Two resistive effects directly linked to the presence of a temperature gradient are proposed for the

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*Preprint of an Invited Paper presented to the 7th International Conference on Plasma Physics (Kiev, USSR. April 1987) self-sustainment of this magnetic topology. In the calculations, the plasma is assumed to be weakly collisional in a steady state.

2. MAGNETIC CONFIGURATION

The magnetic configuration consists of N_o chains of magnetic islands in a slab geometry, where 2ε is the virtual island width and Δ is the distance between two chains (see Fig.1). $\gamma = 2\varepsilon/\Delta$ is the overlapping parameter. The associated magnetic field equations are given by Eqs.(1)[5]. A field line computation indicates that: (a) for $\gamma < 0.8$, laminar magnetic surfaces still exist between island chains; (b) for $0.8 \lesssim \gamma \lesssim 1.5$, islands are embedded in a stochastic region; (c) for $\gamma > 1.5$, the islands are completely destroyed and the interaction region is fully stochastic. This paper deals only with case (b), which allows to distinguish between two regions in the magnetic topology: inside the islands where the field lines are generating closed magnetic surfaces and outside the islands where they have a stochastic behaviour.

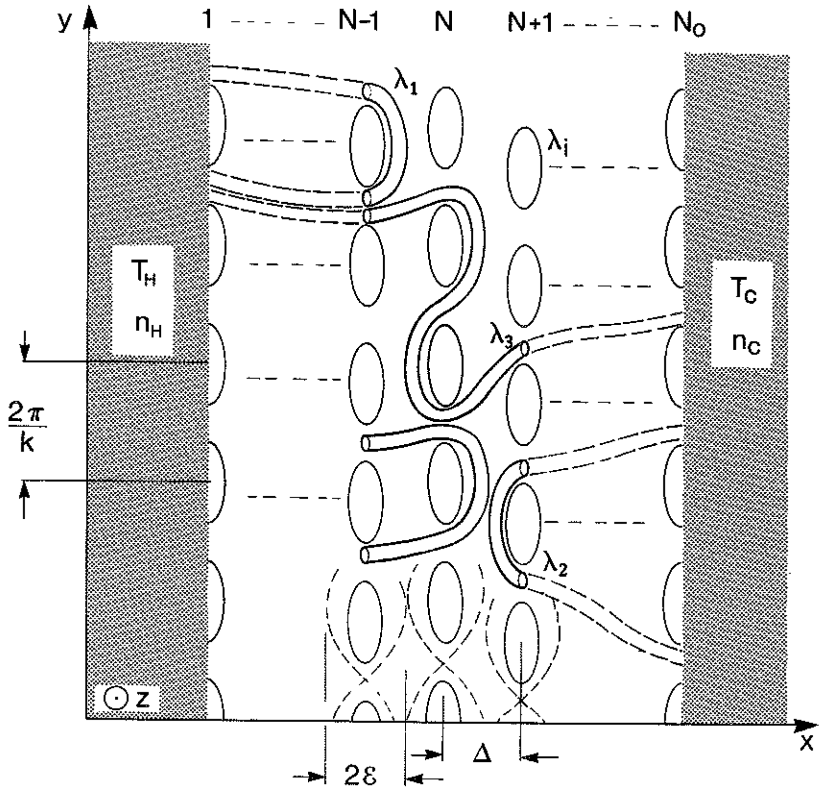

Fig.1 Schematic diagram of the slab model in the *x-y* plane.

Under these conditions and with reference to Fig.1, the normalised cross-sectional area $\sigma(\gamma)$ of the flux tube associated with a stochastic field line going from one island chain to the next one can be defined as:

$$\sigma(\gamma) = \frac{2}{S} \iint \frac{Bx(\gamma)}{B_z} H(B_x) H_{cross} \, dy \, dz; \; S = \iint dy \, dz \qquad (1)$$

where H(Bx) is the Heaviside function for the radial field B_x and

$$H_{cross} = \begin{cases} 1, & \text{when the field line reaches the island chain} \\ 0, & \text{otherwise.} \end{cases}$$

The probability for a stochastic field line to cross one island chain, $\alpha(\gamma)$, is defined as the fraction of $\sigma(\gamma)$, which reaches a third island chain. The computed values for $\alpha(\gamma)$ and for the quantity $\alpha(\gamma)\sigma(\gamma)/(1 - \alpha(\gamma))$ are indicated in Fig.2.

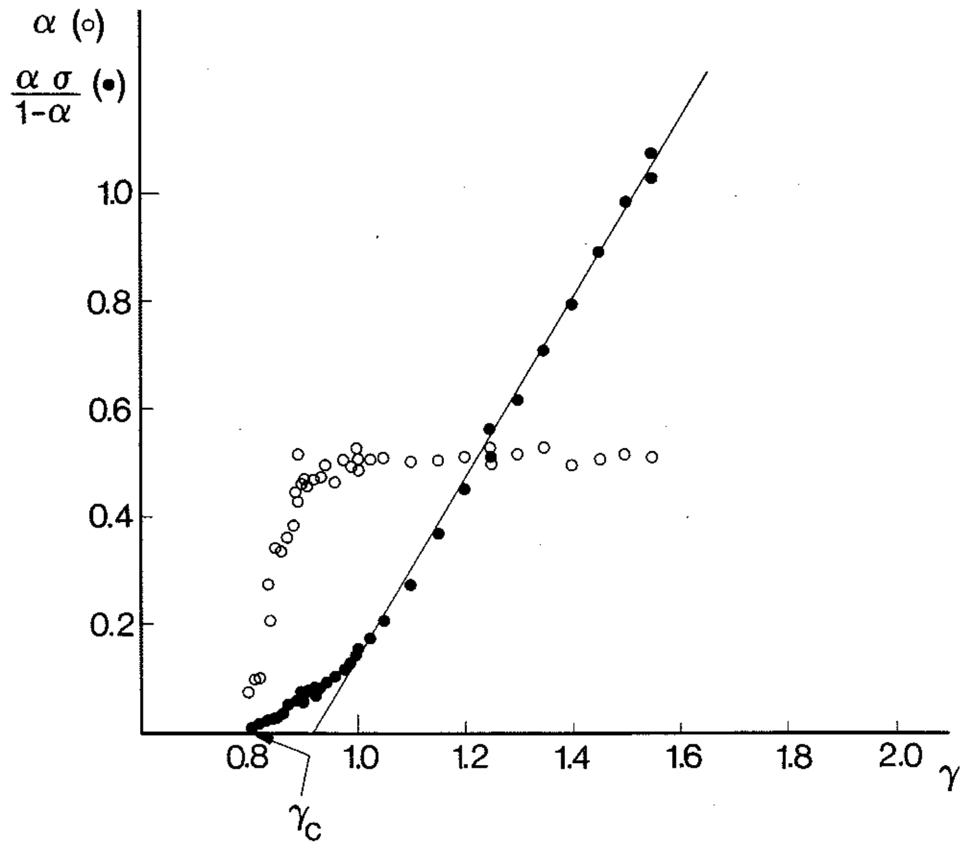

Fig.2 Variation of α and $\alpha\sigma/(1-\alpha)$ as a function of γ.

The behaviour of the field lines in the stochastic region corresponds to a diffusion process through neighbouring island chains according to the equation:

$$L_o \frac{\partial F}{\partial L} = \frac{1}{2} \Delta^2 \frac{\partial^2 F}{\partial x^2} \qquad \left(\text{valid for } \alpha = \frac{1}{2}\right) \tag{2}$$

where F(L,x) is the probability of finding a field line at x after a path length L. L_o is the required path length for a field line to cross an island chain:

$$L_o = \frac{S_c(\gamma)}{\sigma(\gamma)} \frac{4\pi R}{3} \Delta M \tag{3}$$

$S_c(\gamma)$ is the normalised cross-sectional area of the stochastic zone and $\Delta M = \mu\bar{M}$ is the considered range of poloidal mode numbers around an average value $\bar{M}$.

3. PARTICLE MOVEMENT

The particle motion equations are derived in the reference frame moving with the magnetic islands for a weakly collisional plasma. An electric field, which satisfies the symmetry invariance of the magnetic field, allows to maintain the quasi-neutrality condition.

In the particular case of a constant radial electric field $E_x = E_o$, it has been shown that, for a given parallel velocity $w_\parallel$, the guiding centre trajectories correspond to a motion along the field lines in a frame translated by $E_o/B_o' \, w_\parallel$ in the radial direction[5]. (B_o' is the shear term). The properties of the magnetic topology described in Section 2 then apply to the particle motion. The fast particles, for which $E_o/B_o' \, w_\parallel < \Delta$, follow more closely the field lines than the slow particles ($E_o/B_o' \, w_\parallel > \Delta$).

The conservation of the number of field lines implies that the number of guiding centres is conserved. The total current density $\underline{j}_g$ associated with them is divergence free, when perpendicular diffusion is neglected. In case of quasi-neutrality, the parallel current density $\underline{j}_{\parallel g}$ is divergence free. Assuming that $\underline{j}_{\parallel g}$ is equal to the parallel plasma current density $\underline{j}_\parallel$, it is possible to find a scalar λ such that:

$$\underline{j}_\parallel = \lambda \underline{B} \qquad (4)$$

The divergence of Eq.(4) indicates that λ is constant on each field line.

Ohm's law allows to define λ as:

$$\lambda B \frac{\int_0^L \eta_\parallel \, d\ell}{L} = \frac{V}{2\pi R} \qquad (5)$$

where $\eta_\parallel$ is the resistivity along a field line, L is the field line length and V is the loop voltage.

The value of λ associated to a field line depends on its path and on the boundary conditions. In the stochastic region, the field lines wander about and have therefore different values of λ. Three classes of field lines can be distinguished, those which are connected to each boundary of the slab (λ_1, λ_2) and those which are linking both boundaries (λ_3) (see Fig.1). λ_{st} is defined as the statistical average over λ_1, λ_2 and λ_3 at a given location. In the magnetic islands, the field lines generate closed surfaces, to which a constant λ_i is associated. As the islands considered here are small, λ_i is assumed to be constant inside the islands and to depend on the chain only.

To simulate the real situation, hot and cold "seas" of particles with maxwellian velocity distributions F_H and F_C at temperatures T_H and T_C and densities n_H and n_C respectively are introduced at the slab boundaries (see Fig.1). The resulting particle and energy flows across the island chains maintain the density and temperature gradients.

4. SELF-SUSTAINMENT OF THE ISLANDS

The condition for the self-sustainment of the islands is found through Ampère's equation. Taking into account the definition of the overlapping parameter γ, this equation can be written for the field component of poloidal mode number M as:

$$2\Delta M \cdot \frac{\delta j}{\langle j \rangle} = \tilde{F}(\gamma) \cdot \frac{B_z}{\mu_0 \langle j \rangle R} \qquad (6)$$

where $\delta j = j_{st} - j_i$ is the current density excess in the stochastic zone with respect to the islands and $\langle j \rangle$ is the average current density parallel to the magnetic field at the minor radius r. $\tilde{F}(\gamma)$ depends on the shape and cross section of the islands. As shown in Fig.3, $\tilde{F}(\gamma)$ presents a minimum close to 0.5 for $\gamma \approx 1.1-1.2$. The minimum value of δj necessary to sustain the islands is obtained by replacing $\tilde{F}(\gamma)$ by 0.5 in Eq.(6). Note that $\tilde{F}(\gamma)$ becomes infinite when γ tends towards 0.8^+ and 1.5^-, which indicates the stability of the equilibrium between two phases in the magnetic topology: islands and stochastic region.

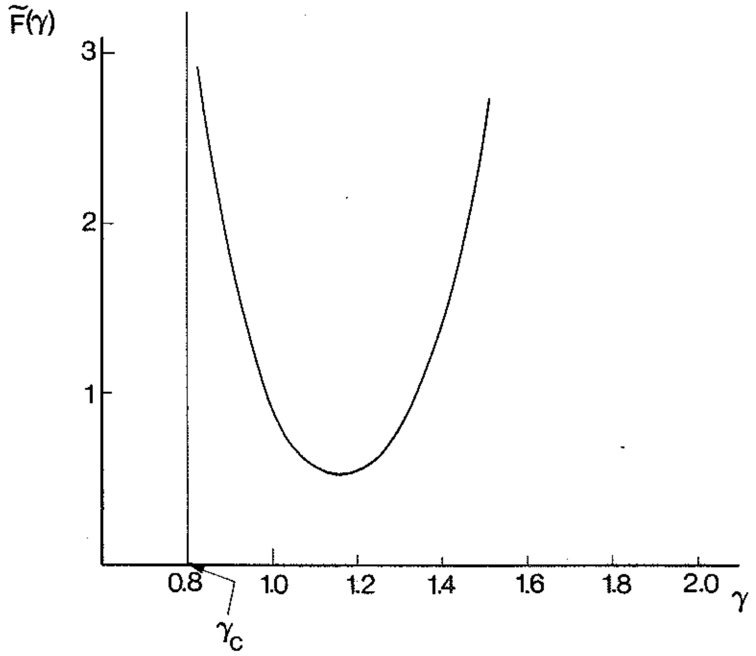

Fig.3 Variation of $\tilde{F}(\gamma)$ as a function of γ.

From Eq.(4), it follows that $\delta j / \langle j \rangle = (\lambda_{st} - \lambda_i)/\langle \lambda \rangle$, the λ value being related to the average electrical resistivity along a field line by Eq.(5). The current density $\delta j = j_{st} - j_i$ necessary to sustain the islands could be produced by two resistive effects:

a) - In the stochastic region at the N^{th} island chain, λ_{st} depends not only on the relative lengths of the field line paths in the different plasma regions including the two "seas", but also on the local electron distribution function F_N. For infinite "seas" and $N_o >> 1$, it can be shown by recurrence analysis that F_N is a linear combination of the maxwellian distribution functions describing the hot and cold "seas". Inside the N^{th} island chain, there is only one maxwellian electron distribution function at the temperature T_N between T_C and T_H. The presence of fast electrons coming from the hot "sea" in the stochastic zone leads to a larger conductivity in the stochastic region than inside the island. This implies that j_{st} is larger than j_i.

b) - The slow electrons can cross (enter or leave) the islands because of the translation due to the electric field as discussed in Section 3. In a steady state, the number and the energy of these electrons is conserved. The slow electrons coming from the cold "sea" then cool the islands. This effect is calculated using a Krook collisional model. The positive difference between the current density flowing in the stochastic region and that flowing inside the island is derived from Ohm's law and Spitzer conductivity.

For small temperature and density differences between the hot and the cold "sea", it is found that:

$$\frac{\delta j}{\langle j \rangle} = \nu \frac{(T_H - T_C)^2}{T_N^{\,2}} \qquad (7)$$

where $\nu \approx 0.1$ for case a), 0.2 for case b) and 0.3 when both cases are considered.

Both mechanisms proposed here for the self-sustainment of the islands depend on the temperature gradient but not on its sign.

5. RESULTING PARTICLE AND HEAT FLOWS

The electron and ion flows through N_o island chains are calculated by recurrence analysis taking into account the symmetry properties of the islands and the particle adiabatic invariants[5]. Both flows are equal to satisfy the quasi-neutrality condition. When they are assumed to be negligible, the radial electric field can be written as:

$$E_o = \frac{KT_H}{q_e}\left(\frac{\nabla_x n}{n_H} + \frac{1}{2}\frac{\nabla_x T}{T_H}\right) \quad (8)$$

Eq.(8) shows that the drift velocity due to E_o follows the electron diamagnetic velocity. This result has already been obtained for microtearing modes[4].

The calculation of the electron heat flow in the case of infinite seas and of $N_o >> 1$ leads to:

$$P_{Heat\ e^-} = \frac{\alpha(\gamma)\sigma(\gamma)}{1-\alpha(\gamma)}\,(2\pi r K\nabla T)\,\left(\frac{3}{2\mu}\right)^3 \frac{1}{\bar{M}^5}\left(\frac{q^2}{q'}\right)^2 n_c v_c;\ v_c = \sqrt{\frac{8KT_c}{\pi m_e}} \quad (9)$$

It follows from Eq.(9) that the average poloidal mode number $\bar{M}$, which defines the number N_o of chains to be crossed, has a strong influence on the heat flow. $P_{Heat\ e^-}$ is also dependent on the temperature and density of the cold "sea" and on the sign of the temperature gradient.

The quantity $\alpha(\gamma)\sigma(\gamma)/(1-\alpha(\gamma))$ shown in Fig.2 can be approximated by 5.17 $(\gamma-0.9)$ when $\gamma > 0.96$. Thus Eq.(9) can be written as:

$$P_{Heat\ e^-} \approx \left(1 - \frac{\gamma_c}{\gamma}\right) S\chi\nabla T \quad (10)$$

where χ is the thermal conductivity and S is the plasma area. Eq.(10) has a form similar to the one suggested by JET experimental results[6].

6. PRELIMINARY COMPARISON WITH JET EXPERIMENTAL DATA

A rough estimation of the quantities used in the model is obtained by assuming that $P_{Heat\ e^-}$ is equal to the input power in JET. A more careful analysis should also take into account the ion losses and the neoclassical terms. The confinement region is assumed here to extend from $q = 1$ to $q = 2$.

In JET, the poloidal mode numbers M are expected to cover the range 10 to 20. The islands could be self-sustained by current densities $\delta j = j_{st} - j_i$ larger than $2.5\times10^{-2}\langle j\rangle$. The corresponding ratio $(T_H - T_C)/T_N$ deduced from Eq.(7) is around 30%. The distance between two island chains Δ would be about 8mm, which is slightly larger than the ion Larmor radius. This corresponds to roughly fifty island chains between the $q = 1$ and $q = 2$ surfaces. The magnetic perturbation to poloidal field ratio could be about 2.5×10^{-4}.

The electric field E_o calculated from Eq.(8) is in the range of 5 to 7 kV/m, and the drift velocity due to E_o is close to 2 to 3 km/s. This result is consistent with magnetic measurements at the JET plasma edge. The corresponding particle confinement time is expected to be around 2s.

7. CONCLUSION

In a tokamak, small magnetic islands could be in equilibrium with a stochastic region. The thermal transport along field lines linking different plasma regions would then explain the anomalous confinement.

Two mechanisms have been proposed for the self-sustainment of this topology. They result from the presence of hot and cold electrons in the stochastic zone. Both mechanisms lead to a difference of conductivity between the stochastic region and the islands, which depends on the square of the temperature gradient.

The model allows to account for the heat flows observed in JET by values of the magnetic perturbation to poloidal field ratio of the order of 2.5×10^{-4} and of the poloidal mode number M between 10 and 20.

More work is needed to better describe the self-sustainment of this magnetic topology by taking into account other effects, neglected so far, like perpendicular diffusion, finite Larmor radius and field curvature.

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