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H.P. Summers, K.H. Behringer and B Denne

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H.P. Summers<sup>1</sup>, K.H. Behringer and B Denne

*JET-Joint Undertaking, Culham Science Centre, OX14 3DB, Abingdon, UK*

<sup>1</sup>*Department of Physics, University of Strathclyde, John Anderson Building,  
107 Rottenrow, Glasgow G4 0NG, UK*

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**ABSTRACT.**

The importance of having reliable estimates of electron-ion recombination coefficients for species such as  $\text{Ni}^{+17}$  in tokamaks is pointed out. The state of theoretical calculation of recombination rates in thermal plasma is then briefly reviewed.

A new treatment is summarised which unites detailed computation of the direct recombination reactions populating excited ionic energy levels (emphasising dielectronic and radiative recombination) with a careful study of the effects in a plasma of subsequent collisions on the initial direct capture distributions. This includes both singly excited and doubly excited (resonance) state distributions

Illustrations are drawn from fairly ionised ions of interest in laboratory plasmas and include some specific results on recombination of neon-like and adjacent ions of iron, nickel and selenium. these ions either are strongly radiating species in tokamak plasmas such as JET or important X-ray studies

Some results are shown of modelling of nickel ion radiation in JET using the new rate coefficients.

## INTRODUCTION

The local densities of ions in a plasma are determined by the balance of ionisation, recombination and diffusion. For an impurity species such as nickel in the JET tokamak, modelling of the ion distribution for a typical pulse incorporating these processes is shown in figure 1.  $Ni^{+25}$ , in the central plasma, is determined by the balance between ionisation and recombination, while  $Ni^{+1}$  in the edge plasma is determined by the balance between ionisation and inward diffusion. An intermediate ion such as  $Ni^{+17}$  is influenced by all three processes. Such intermediate ions are important radiators. Routine measurement of the spatial distribution of spectral radiation from ions such as  $Ni^{+17}$  and  $Ni^{+25}$  is made on JET. We seek a consistent model from which absolute nickel density and radiated power may be deduced. The impurity diffusion coefficient in JET is inferred empirically to be approximately  $1 \text{ m}^2 \text{ sec}^{-1}$  independent of ion charge. This is from the observed positions of radiating shells (eg figure 2). Ionisation and recombination coefficients by contrast are sought from theoretical atomic physics calculation.

For plasma electron densities  $N_e \lesssim 10^{14} \text{ cm}^{-3}$ , an impurity ion is almost certainly in its ground energy level (or lowest metastable level, if present). The effective ionisation coefficient is the ground level, two body electron-ion ionisation rate coefficient and is negligibly dependent on plasma density. The effective recombination coefficient for an ion such as  $Ni^{+17}$  at densities  $N_e \sim 10^{14} \text{ cm}^{-3}$  on the other hand is sensitive to density. It is also markedly sensitive to the recombining ion structure. This paper reviews briefly calculations of recombination. A new general calculation for effective recombination coefficients is then described and illustrated. Finally, we return to the modelling of nickel in JET.

## 2. AN OVERVIEW OF THE RECOMBINATION COEFFICIENT

The effective electron-ion recombination coefficient characterises the growth of the ground level population density of ions  $A^{+Z}$  due to free electron capture by ions  $A^{+Z+1}$ . It is a composite coefficient composed of direct electron captures into any accessible energy level of  $A^{+Z}$  (which is stable to Auger breakup, and called a singly excited state), followed by additional processes which influence or interrupt the captured electron's random walk to the ground level. The relevant direct capture reactions are radiative recombination, dielectronic recombination and three-body recombination. Restricting to electron densities  $N_e \lesssim 10^{14} \text{ cm}^{-3}$  and temperatures  $T_e$  at which the ion  $A^{+Z}$ 's fractional abundance in equilibrium is large, only the first two direct processes are important. The direct dielectronic recombination process is itself a composite process composed of a radiationless transition to a resonant state (the resonant states which are not stable to Auger breakup are called doubly excited) followed by a radiative stabilisation. The doubly excited states can be influenced by additional interactions competing with Auger breakup and radiative stabilisation in finite density plasma. A complete recombination calculation has therefore three steps, namely deduction of direct dielectronic recombination coefficients to singly excited states (including determination of doubly excited state populations), deduction of direct radiative recombination coefficients to singly excited states and deduction of singly excited state populations. In the limit of zero density, only the first and second steps are required since the final effective recombination coefficient is merely the sum of direct captures to all levels. Also, the doubly excited state populations in the first step reduce to simple branching ratio expressions. Most studies have been of these first two steps. An additional layer of complexity is added if consistent inclusion of metastable states is sought. Thus the recombining ion  $A^{+Z+1}$  may have populated metastable states. The ground and metastable populations of  $A^{+Z}$  evolve

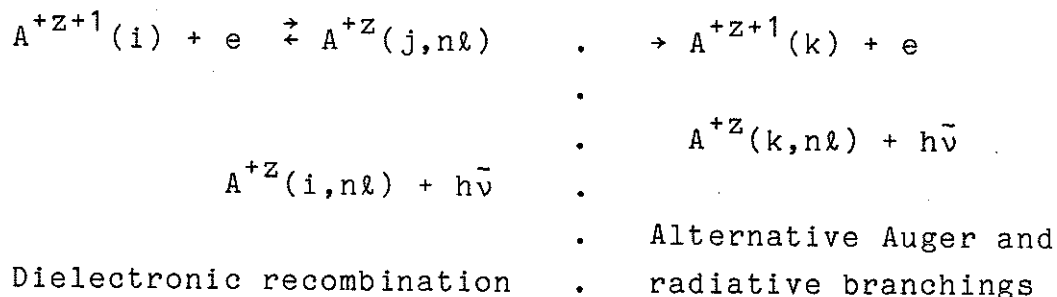
with comparable time constants but are weakly coupled to each other. Separate effective recombination coefficients are required for each recombined and recombining ion metastable pair. Also in the derivation of the generalised coefficients, there is blurring of the distinction between singly excited and doubly excited states.

Turning firstly to the direct radiative recombination coefficient, table 1 summarises calculations up to about one year ago. The use of hydrogenic formula for capture to whole principal quantum shells and semi intuitive adjustments to them are prominent. There is some use of experimental and refined calculated photoionisation cross-section data for capture to the ground levels. Figure 3 shows some comparisons for the radiative recombination coefficient to  $\text{Fe}^{+23}$  over all levels. More details are in Summers (1986). From a practical point of view, the large number of states accessible to recombination and the variety of ions precludes the general use of the most sophisticated modern theoretical photoionisation techniques for direct recombination coefficients. For intermediate accuracy results for complex ions, a hierarchy of algorithmic and numerical methods has been established recently (Summers, 1986b; Burgess & Summers, 1987). These use numerical procedures based on observed quantum defects and a variational effective central potential for ground state, metastable state and low angular momentum excited state capture, linked to hydrogenic methods for high angular momentum capture. The specification includes arbitrary LS coupled resolution. The formulation is in terms of a generalisation of bound-free Gaunt factors. Figure 4 shows variation of the bound-free Gaunt factor,  $g^{\text{II}}$ , with  $z$  for recombination to form Na-like states. Figure 5 shows some recombination coefficients to a variety of states of  $\text{Fe}^{+21}$ . More details are contained in Burgess & Summers (1987).

Turning to a dielectronic recombination, table 2 summarises zero density limit calculations of the dielectronic coefficient up to about one year ago. Figure 6 shows some



zero density comparative results for  $Fe^{+20}$ . These should be viewed in the light of the basic reaction sequence relevant at zero density, namely



We also draw attention to the seminal part played by the Burgess General Formula (Burgess, 1965). More details are in Summers (1986a). The coefficient is sensitive to the 'parent ion' transition  $i \rightarrow j$ . Distinction is useful between cases when the transiting parent electron between states  $i$  and  $j$  does not change its principal quantum number ( $\Delta n=0$  transitions) and those when it does. For  $\Delta n=0$  transitions, the General Formula tends to be quite accurate, and very high  $n$  shells are strongly populated. For  $\Delta n > 0$ , the General Formula is less reliable and fairly low  $n$  shells are populated. Alternative branchings, if energetically accessible, can cause significant disturbance of the main dielectronic pathway. The outer electron-orbital,  $n\ell$ , is approximately hydrogenic except when  $n$  belongs to, or is close to the set of quantum numbers represented in the parent states  $i$  or  $j$ . Thus the most relevant considerations for calculation improvement are

- (a) The parent ion transition type (the  $\Delta n$  problem)
- (b) Capture to the lowest accessible levels (the low level problem)
- (c) Competing branching with radiative stabilisation (the alternative branching problem)
- (d) Disturbance of the states  $j, n\ell$  at finite density by collisions (the doubly-excited state redistribution problem)

We return to the calculation of the direct dielectronic coefficients in the next section. Presupposing these are correctly determined, the effective recombination coefficient (properly called the 'collisional-dielectronic recombination coefficient',  $\alpha_{cd}$ ) is obtained in the course of solution for the singly-excited populations  $A^{+Z}(i, n\ell)$ . It is density sensitive because of reionisation of such excited states before the  $n\ell$  electron reaches the ground level. Figure 7 illustrates the behaviour of  $\alpha_{cd}$ .

### 3. A NEW RECOMBINATION CALCULATION

We have established chained semi-automatic computational procedures for a complete recombination calculation. An initial description has been given by Summers et al (1987) and a full description is to be published elsewhere. Here we wish to illustrate the steps only. The first step is calculation of direct dielectronic recombination coefficients to the lowest accessible levels. For example, in  $\text{Se}^{+25} + e \rightarrow \text{Se}^{+24}$ , the initial fluorine-like parent ion term is  $2s^2 2p^5 \ ^2P$ . Capture is possible into  $n=3$  levels of the form  $2s^2 2p^5 \ 3\ell$  and  $2s 2p^6 \ 3\ell$  via doubly excited states  $2s^2 2p^4 \ 3\ell' 3\ell''$  and  $2s 2p^5 \ 3\ell' 3\ell''$  for all  $\ell, \ell', \ell''$ . We include all terms of these configurations. The results are shown in figure 8. Auger and spontaneous emission coefficients are computed using a multielectron, multiconfiguration structure code extended to include distorted free waves (Badnell, 1985) and then composed into recombination coefficients automatically. At usual densities, doubly excited ( $n=3$ ) state redistribution is negligible. The next step is calculation of direct dielectronic recombination coefficients to all higher quantum shells for the captured electron. For example, for  $\text{Ni}^{+7} + e \rightarrow \text{Ni}^{+6}$ , the initial parent ion term is  $2s^2 2p^6 3s \ ^2S$  and possible  $\Delta n=0$  parent transitions ( $i \rightarrow j$ ) are  $2s^2 2p^6 3s - 2s^2 2p^6 3p$  and  $2s^2 2p^6 3s - 2s^2 2p^6 3d$ . The relevant doubly excited states of  $\text{Ni}^{+6}$  are therefore of the form  $2s^2 2p^6 3p \ n\ell$  and  $2s^2 2p^6 3d \ n\ell$ . In thermal plasmas, the populations are usually expressed in terms of Saha-Boltzmann deviations

$b(j, n\ell)$  (the b-factors). In the zero density limit these might be expected simply to be branching ratios of the form  $Aa/(Aa+Ar)$  and tend to 1 for  $Aa \gg Ar$ . The b-factors for states  $2s^2 2p^6 3p n\ell$  which actually result are shown in figure 9. Necessary Auger rates are obtained by firstly calculating threshold partial collision strengths in the distorted wave approximation for the parent excitations and then projecting these below threshold. We include allowed and forbidden excitations. The  $\ell$ -redistribution of doubly excited populations is by positive ion collision. The complete high level solution includes also  $\Delta n=1$  parent excitations and all alternative branchings. The direct recombination coefficients to levels of the form  $2s^2 2p^6 3s n$  which result from all the direct processes are shown in figure 10. The last step is the calculation of the singly excited populations and the deduction of  $\alpha_{cd}$ . Continuing with the  $Ni^{+16}$  illustration, the populations of singly excited states of the form  $2s^2 2p^6 3s n$  are shown in figure 11 and also the resultant  $\alpha_{cd}$ . In presenting these illustrations we have avoided cases with the added complexity of metastables and angular resolution of singly excited state populations. These details are nevertheless amenable to calculation by our computational scheme. More details are contained in Summers et al (1987)

#### 4. THE ABUNDANCE OF NICKEL IN JET

The new recombination rate coefficients for  $Ni^{+17}$  and  $Ni^{+16}$  described here were incorporated in the transport model used to provide the radial nickel distribution in figure 1. Some further details, including specification of the ionisation rate coefficients, are in Summers et al (1987). Agreement of nickel abundances derived from spectrum lines of different ionisation stages would give added confidence to our transport model and its associated atomic rate coefficient data. The resonance lines of  $Ni^{+25}$  and  $Ni^{+16}$  are particularly suitable. The helium-like and lithium-like stages are in the near coronal regime and their ionisation

and recombination rate coefficients are amongst the best established. Also they are not strongly density sensitive. The  $\text{Ni}^{+16}$  stage on the other hand is influenced by transport and its recombination coefficient is density sensitive. Table 3 shows a comparison of nickel abundances from  $\text{Ni}^{+17}$ ,  $\text{Ni}^{+24}$  and  $\text{Ni}^{+25}$  lines for a JET pulse and tends to indicate broad agreement. Evidently such results do not provide a definitive test on any single rate coefficient but nonetheless give some encouragement that the theoretical atomic rates are approaching the true values.

## REFERENCES

Aldrovandi S M V & Pequingnot D (1973) *Astron. & Astrophys.* 25, 137

Badnell N R (1985) Cambridge University, Applied Mathematics Dept. Report DAMPT/AA-1J01-NRB

Behringer K H, Denne B, Magyar G, Breton C, De Michelis C, Ramette J, Saoutic B (1986) Proc. Workshop on Basic and Adv. Fusion Plasma Diag. Techniques, Varenna, Italy

Beigman I L, Vainshtein L A and Chickov B N (1981) *Sov. Phys. JETP* 53, 490

Barfield W P (1975) *Nucl. Fusion* 15, 1192

Barfield W P (1979) *Astrophys. J.* 229, 856

Bely-Dubau F, Gabriel A H and Volonte S (1979) *Mon. Not. R. Astr. Soc.* 186, 405

Burgess A (196 ) *Mem. R. Astr. Soc.* 69, 1

Burgess A (1965) *Astrophys. J.* 141, 1588

Burgess A & Chidichimo M C (1983) *Mon. Not. R. Astr. Soc.* 203, 1269

Burgess A & Seaton M J (1960) *Mon. Not. R. Astr. Soc.* 120, 121

Burgess A & Summers H P (1976) *Mon. Not. R. Astr. Soc.* 174, 345

Burgess A & Summers H P (1987) *Mon. Not. R. Astr. Soc.* (in press)

Burgess A & Tworkowski A S (1976) *Astrophys. J.* 205, L105

- Dubau J, Gabriel A H, Loulerque M, Steenman-Clark L & Volonte S (1981) Mon. Not. R. Astr. Soc. 195, 705
- Hahn Y, Gau J N, Luddy R & Retter J A (1980) J. Quant. Spect. Rad. Transf. 23, 65
- Jacobs V L, Davies J, Kepple P C, Blaha M (1977) Astrophys. J. 211, 305
- Lee C M & Pratt R H (1975) Phys. Rev. A12, 1825
- Mertz A L, Cowan R D & Magee N H (1976) Los Alamos Report LA-6620-175
- Peach G (1967) Mem. R. Astr. Soc. 71, 13
- Post D E, Jensen R V, Tarter C B, Grasberger W H, Lokke W A (1977) At. Data Nucl. Data Tables 20, 397
- Safronova U I (1975) J. Quant. Spect. Rad. Transp. 15, 231
- Seaton M J (1959) Mon. Not. R. Astr. Soc. 119, 81
- Seaton M J (1964) Planet Space Sci. 12, 55
- Seaton M J & Storey P J (1976) "Atomic Processes & Applications" - ed. Burke & Moiseiwitsch - North Holland
- Shore B (1969) Astrophys. J. 158, 1205
- Summers H P (1986a) JET Joint Undertaking report JET-IR(86)01
- Summers H P (1986b) JET Joint Undertaking report JET-P(86)06
- Summers H P, Behringer K H & Wood L (1987) Physics Scripta (in press)
- Tartar C B (1971) Astrophys. J. 168, 313

Tartar C B (1973) Astrophys. J. 181, 607

Von Goeler (1975) Nucl. Fusion 15, 301







TABLE 3

Comparison of nickel densities derived from various ions JET pulse 4831 at 6.5 secs.

<u>ION</u>	<u>B-C/B-M (cm<sup>-3</sup>)</u>	<u>Present (cm<sup>-3</sup>)</u>
Ni <sup>+17</sup>	6.4 <sup>10</sup>	3.9 <sup>10</sup>
Ni <sup>+24</sup>	2.2 <sup>10</sup>	
Ni <sup>+25</sup>	3.0 <sup>10</sup>	

B-C/B-M denotes the Burgess-Chidichimo (1983) ionisation sites and the Burgess General Formula (1965) for dielectronic recombination, modified by the prescription of Mertz et al. (1976) for  $\Delta n=1$  and adjusted for density following Post et al. (1977).

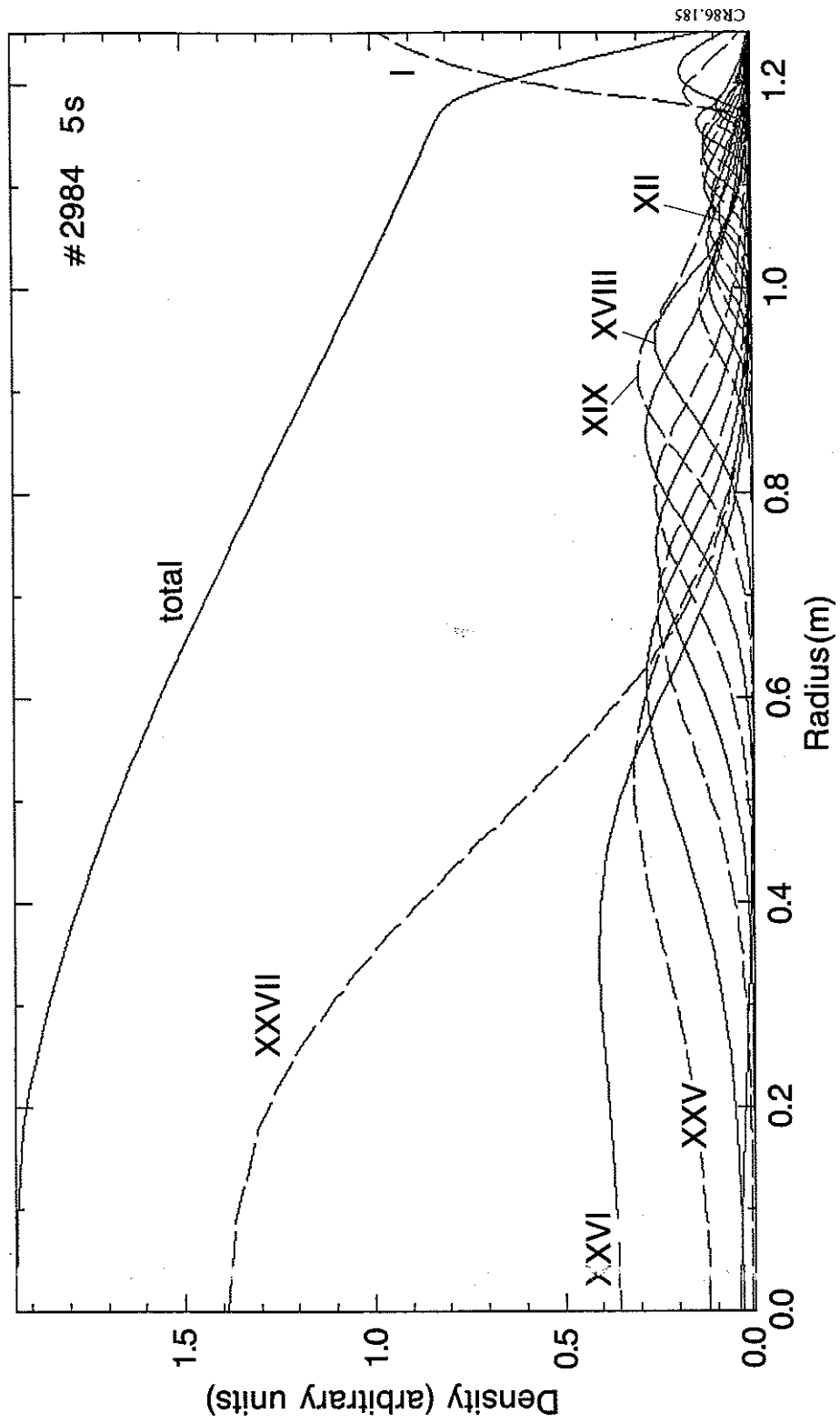
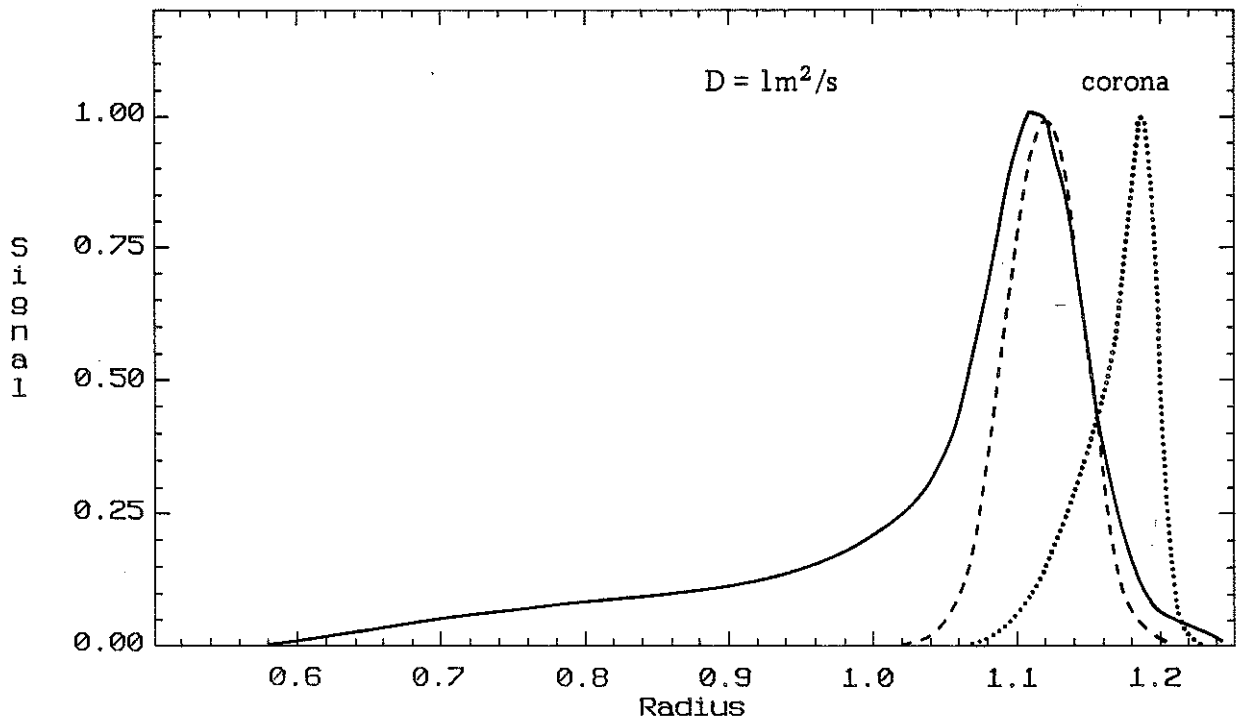
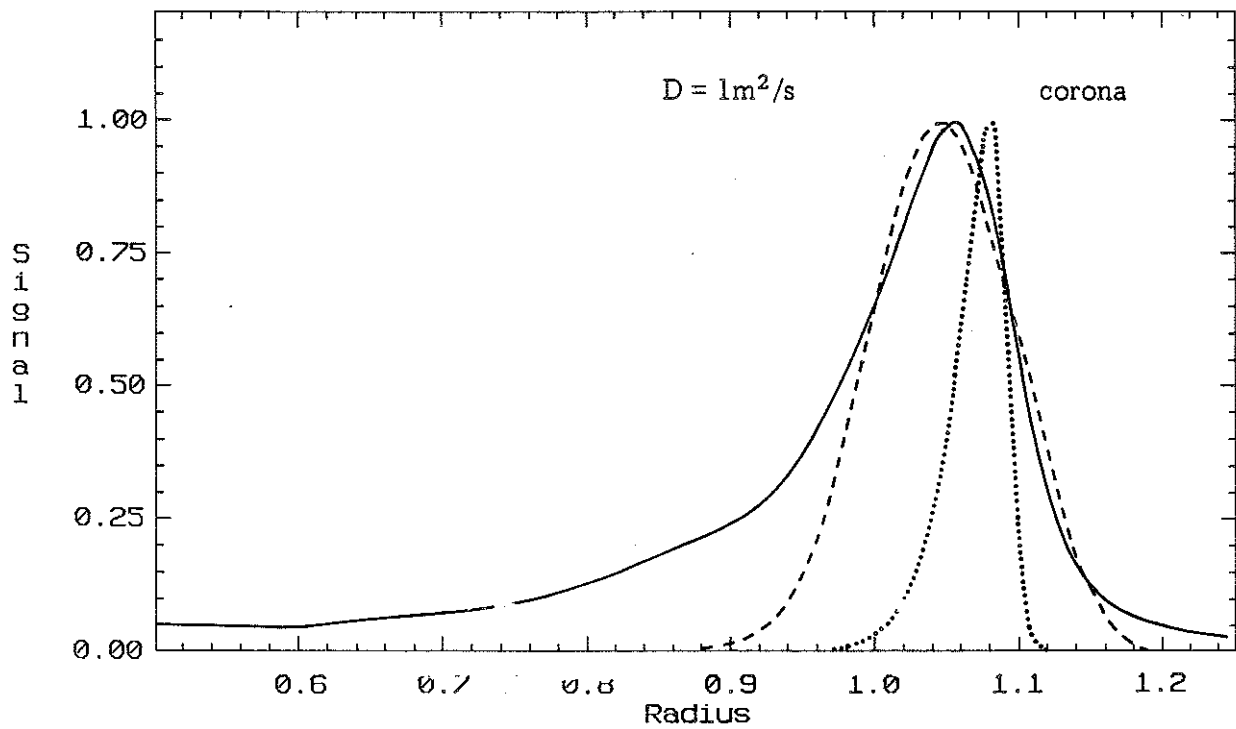


Fig. 1 Distribution of nickel ion densities radially in a JET discharge determined from an impurity transport code.



OVI 1032      SHOT 6291



Ni XVII      SHOT 6289

CR 86.206/2

Fig.2 Abel-inverted profiles and transport code predicted ones in JET discharges.  
 ——— Abel inverted measured profile; - - - - - transport code predicted profile;  
 ..... coronal model profile (see Behringer et al, 1986).

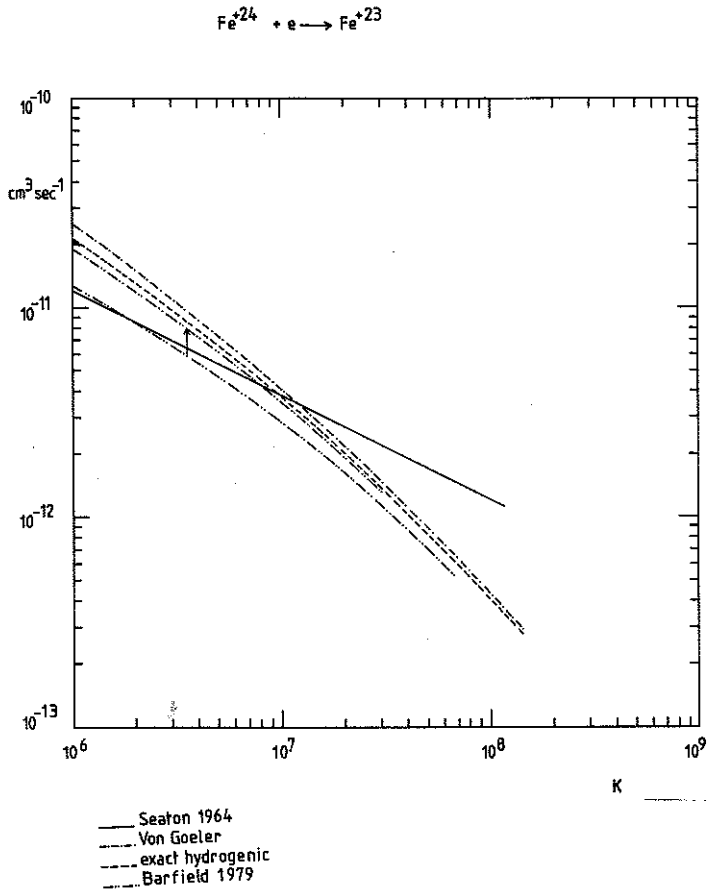


Fig. 3 Comparison of net radiative recombination coefficients for  $\text{Fe}^{+24} + e \rightarrow \text{Fe}^{+23}$ . The data of Barfield includes levels up to  $n = 6$ . This gives the lower curve. It has been corrected upwards by including the exact hydrogenic values for higher  $n$ 's.

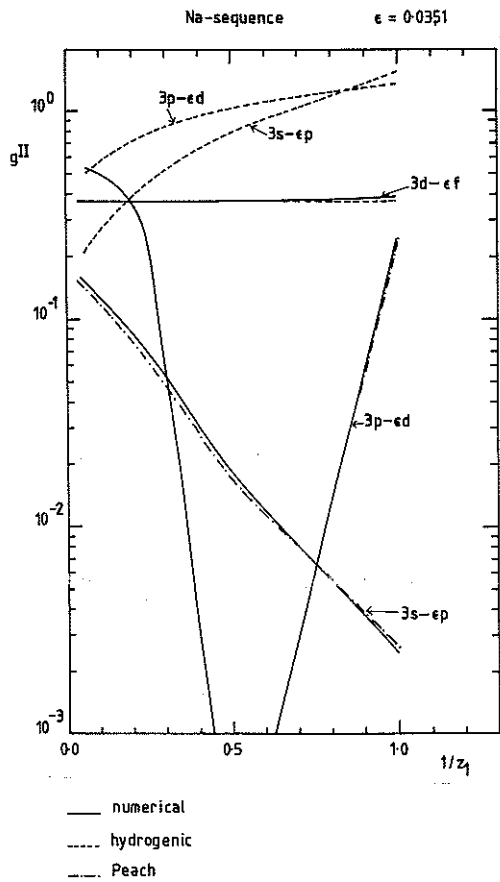


Fig. 4 Variation of the bound-free Gaunt factor ( $g^{\text{II}}$ ) with ion charge for the Na-sequence.  $z_1$  is the recombining ion charge.  $\epsilon$  is the reduced free electron energy (Summers, 1986b).

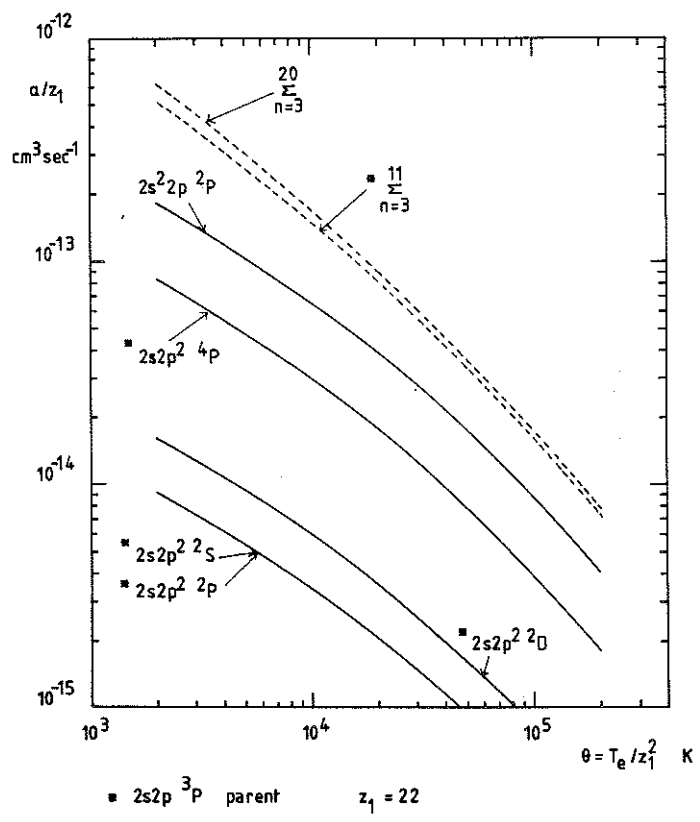
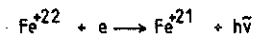


Fig. 5 Scaled radiative recombination coefficients to selected levels for  $\text{Fe}^{+22} + e \rightarrow \text{Fe}^{+21}$ .  $z_1$  is the recombining ion charge.  $\theta$  is the reduced electron temperature.

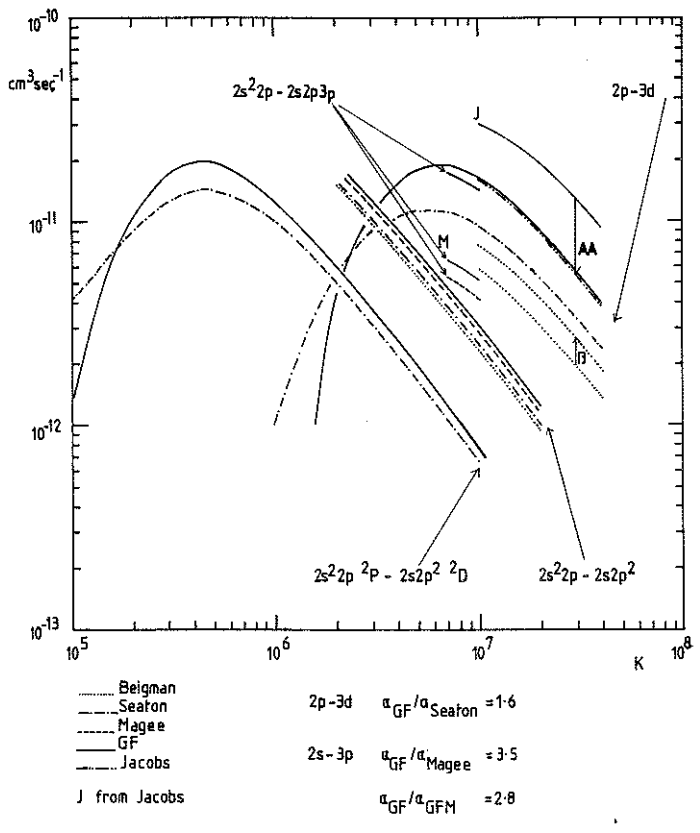
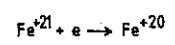


Fig. 6 Comparison of net dielectronic recombination coefficients at zero density for  $\text{Fe}^{+21} + e \rightarrow \text{Fe}^{+20}$  for various core (parent) transitions. Short curves are determined at  $T_e = 10^7 \text{ K}$  only. A temperature correction  $\beta$  is made to the results of Beigman (see Summers, 1986b). Seaton and Storey values for the whole core excitation  $2s^2 2p - 2s2p^2$  have been determined by scaling up with oscillator strengths. The Jacobs estimate for  $\alpha_{GF}$  for the  $2p - 3d$  core excitation is very high. Also, Jacobs shows a large alternative Auger channel correction. Beigman indicates that there is no significant alternative Auger channel correction. The Mertz reduction (Magee et al) is good for the more unusual  $\Delta n = 1$  inner shell core excitation but less good for the usual outer shell excitation.

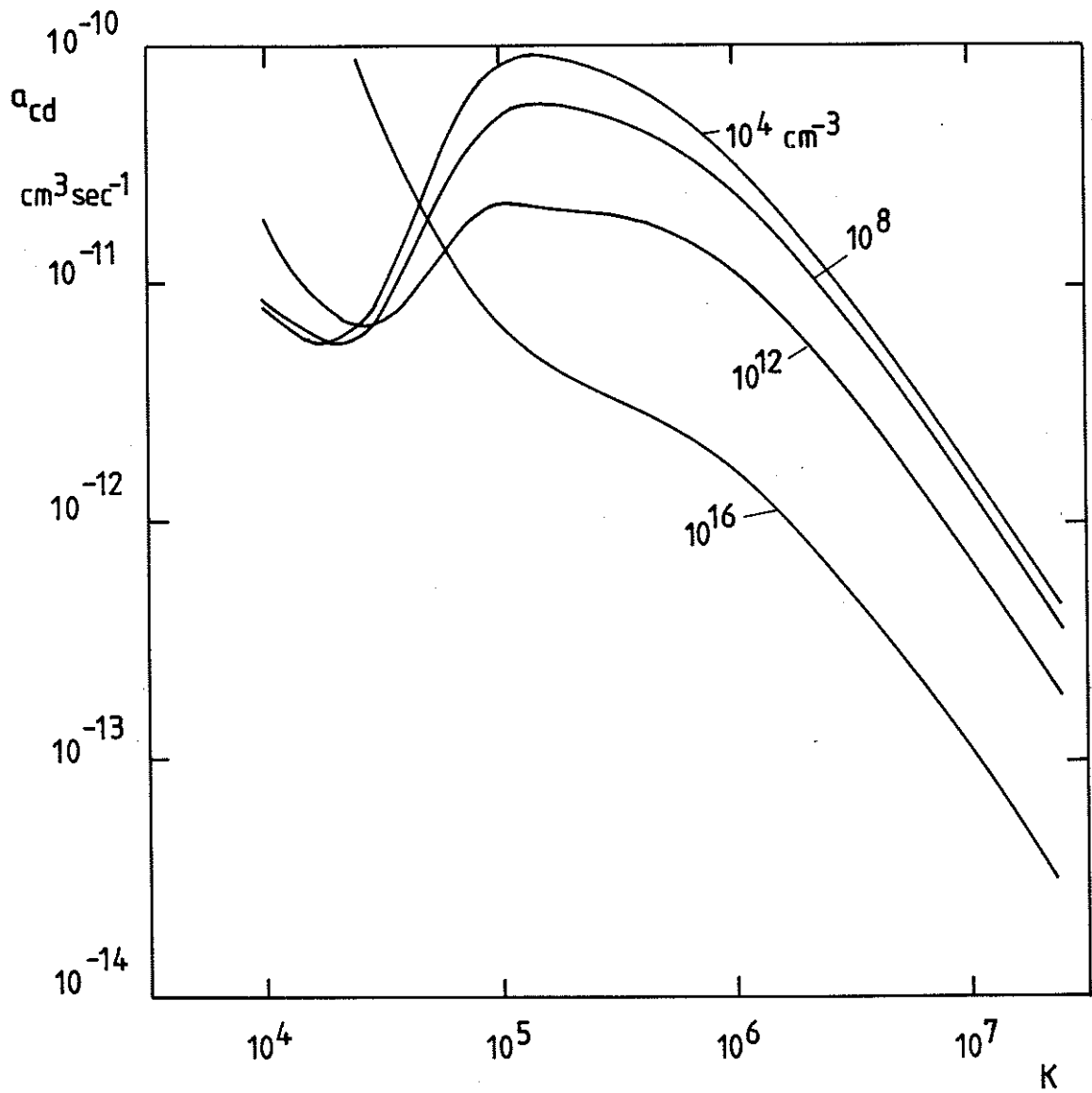
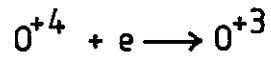


Fig.7 Variation of collisional-dielectronic recombination coefficients with temperature and density for  $\text{O}^{+4} + e \rightarrow \text{O}^{+3}$ .

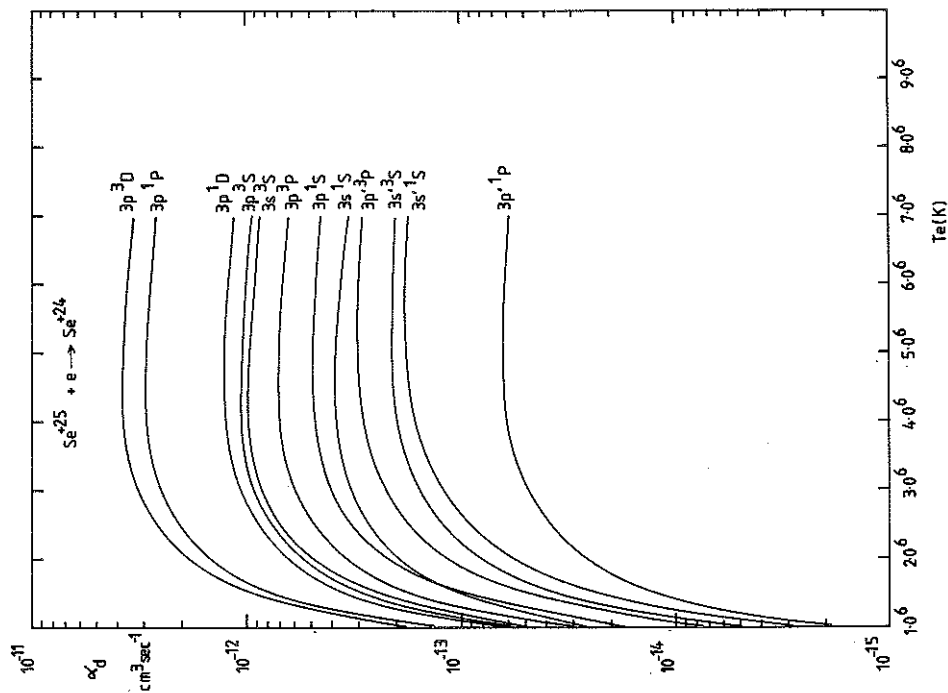


Fig. 8 Direct dielectronic recombination coefficients to selected  $n = 3$  terms via  $n = 3$  doubly excited states for  $\text{Se}^{+25} + e \rightarrow \text{Se}^{+24}$ . Unprimed terms have  $2s^2 2p^5$  parent, primed terms have  $2s 2p^6$  parent.

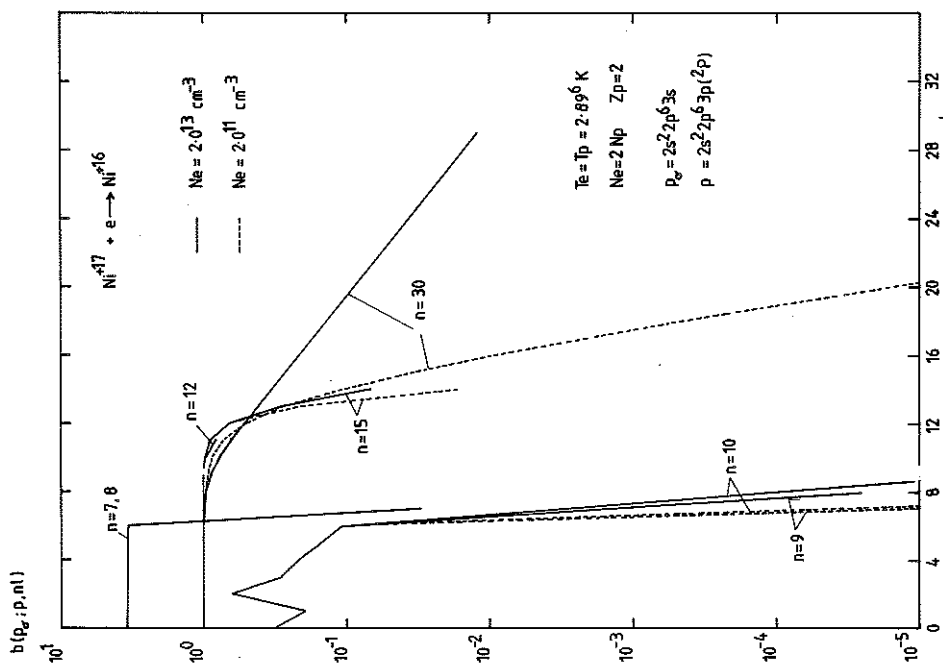


Fig. 9  $b$ -factors for doubly capturing states of  $\text{Ni}^{+16}$ .  $p_e$  is the initial capturing parent ion state,  $p$  is the parent state on which the doubly excited states are based. Parent configurations included are  $2s^2 2p^6 3s$ ,  $2s^2 2p^6 3p$  and  $2s^2 2p^6 3d$ .



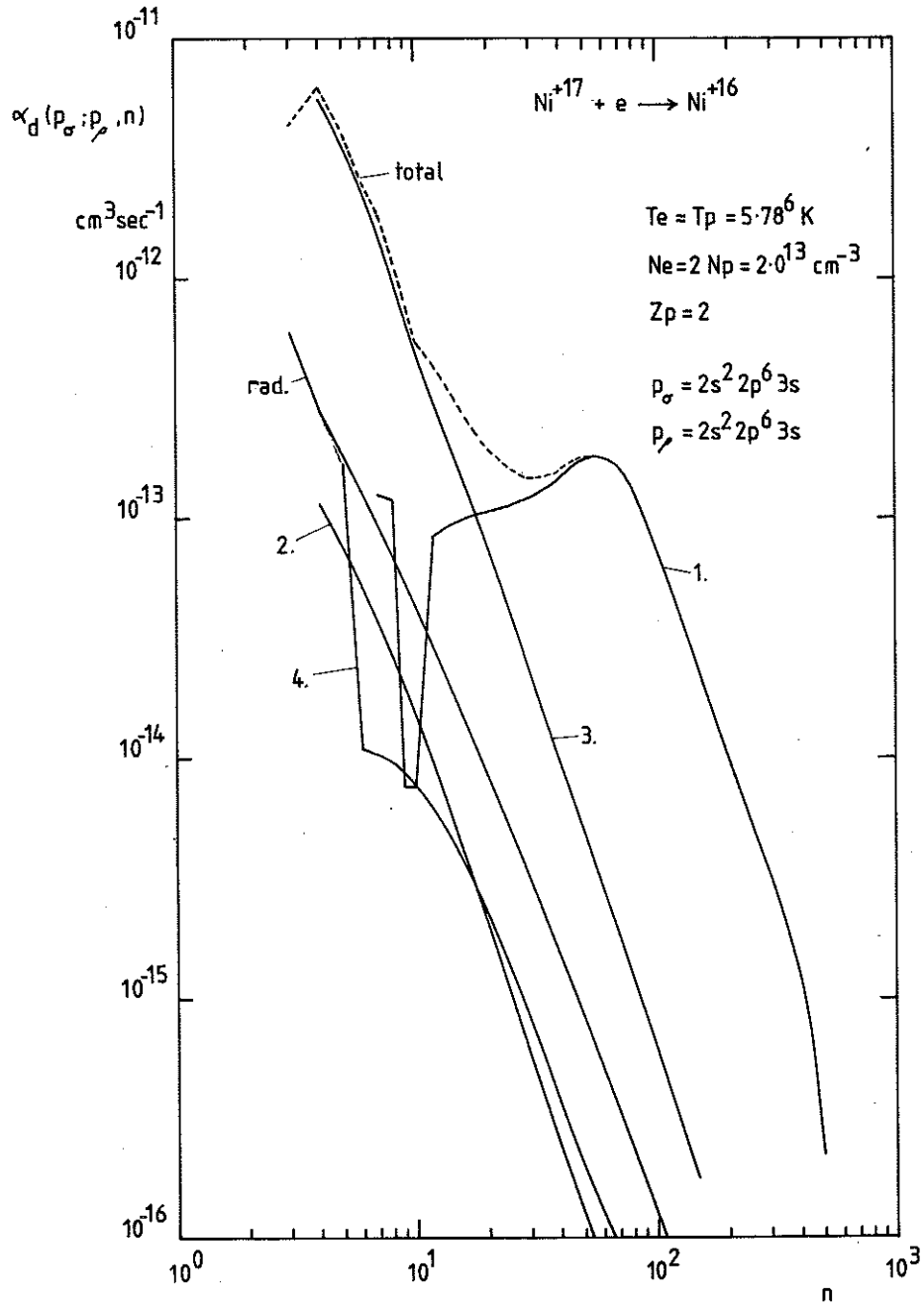


Fig. 10 Direct dielectronic coefficients to  $n$  shells for  $\text{Ni}^{+17} + e \rightarrow \text{Ni}^{+16}$ .  $p_\sigma$  is the initial capturing parent ion state,  $p_\rho$  is the final parent state.

Contributions to the total arise from

1.  $2s^2 2p^6 3s, 2s^2 2p^6 3p, 2s^2 2p^6 3d$
2.  $2s^2 2p^6 3s, 2s^2 2p^5 3s^2$
3.  $2s^2 2p^6 3s, 2s^2 2p^5 3s 3d, 2s^2 2p^5 3s 3p, 2s^2 2p^6 3d$
4.  $2s^2 2p^6 3s, 2s 2p^6 3s 3p, 2s^2 2p^5 3s 3p, 2s 2p^6 3s^2$

Radiative recombination coefficients are also shown.

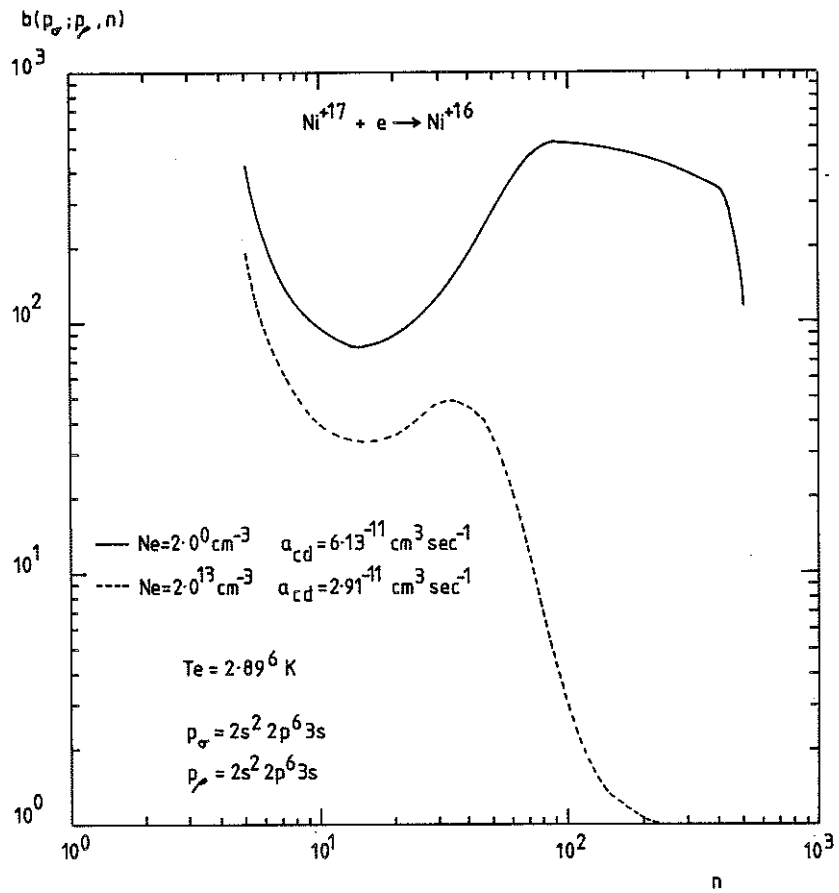


Fig. 11 Singly excited population structure of  $n$  shells for  $\text{Ni}^{+16}$ .  $\alpha_{cd}$  is the collisional-dielectronic recombination coefficient.