H-mode profile parameterization for extrapolation and control

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Abstract

Abstract. The advantages of a parameterization of the plasma profiles in terms of a semi-parametric representation, $T(\rho, I_p, \bar{n}, B_t, P_L)$, are described. By using a log-additive model on $\partial T/\partial\rho$, a diffusive-like parameterization can be achieved. An initial comparison of the profiles on DIII-D and JET shows that the normalized profiles are fit with an error of 7%. Regression results on $\bar{n}(I_p, B_t, R)$ and $\bar{T}(\bar{n}, I, B_t, R)$ are described. The preliminary analysis indicates $\bar{n}$(ITER)$\simeq$1/2 Greenwald limit in contrast to the design value of 2 × Greenwald limit. In our preliminary study, $\bar{T}$ scales as $\bar{n}^{-.23}$ instead of $\bar{T} \sim \bar{n}^{-1.0}$ as assumed in the ITER design. As a result, the predicted ITER temperature is much hotter than the design value.

1. Introduction

Our goal is to parameterize the observed temperature and density profiles as a function of the normalized toroidal flux, ρ, and the control variables such as $I_p, B_t, \bar{n}, P_L$ and R: $T(\rho, I_p, B_t, \bar{n}, P_L)$. We neglect the time evolution and assume that the steady state profile is uniquely determined by the control variables in the model. In effect, we are determining a large dimensional response surface.

There are at least six advantages of profile shape scalings. First, the shape scaling summarizes the characteristic profile shapes over an operating period. Second, these expressions can be used in transport, stability and heating codes as realistic temperature and density shapes. In particular, our models can be used for real time burn control. Third, the fitted profiles serve as a benchmark against which new classes of discharges may be compared. Fourth, by fitting many discharges simultaneously, the signal to noise ratio is enhanced, and we average over effects which are not reproducible from discharge to discharge. Fifth, in many cases, physics insight can be gained from examining the profile parameterizations. In particular, we are sometimes able to isolate similarity variables in the profile shape dependencies. Finally, in multi-machine databases, we can determine a size scaling and extrapolate the profile shape to new experiments. Thus, we can predict the peaking factors and resulting fusion power production for the International Tokamak Experimental Reactor (ITER).

Not all of the potential control variables influence the profile shape. We add control variables to the model one variable at a time choosing from a list of candidate variables. At each stage, we add the control variable which reduces the fit error the most. This procedure is called sequential variable selection.[1] By only using the important variables in the model, new insight into the underlying physics often results. We denote the number of control variables by K. The set, $\{I_p, B_t, P_L, \bar{n}, R\}$, corresponds to $K = 5$. In Ref. 1, we proposed two semi-parametric families of response surfaces: log additive temperature models:

$$\ln[T] = f_0(\rho) + f_1(\rho)\ln[\bar{n}] + f_2(\rho)\ln[I_p] \ , \tag{1}$$

and log-additive diffusivity models:

$$\ln[\chi] = g_0(\rho) + g_1(\rho)\ln[\bar{n}] + g_2(\rho)\ln[I_P] \ , \tag{2}$$

where χ is the diffusivity. For (1), we find that the profile shape is almost uncorrelated with its line average. Therefore, we parameterize the normalized profile shape:

$$T_e(\rho)/\bar{T} \text{ or } n_e(\rho)/\bar{n} = \mu(\rho)\ I_p^{f_I(\rho)}\ P_L^{f_P(\rho)}\ R^{f_R(\rho)}\ \ldots \tag{3}$$

We have analyzed a number of different profile data sets from JET and DIII-D [1,2,3]. Typically, the accuracy is 7 % for the normalized profiles. For both (1) and (2), we replace an unknown function of K variables with $(K+1)$ unknown functions of one variable. This log-additive assumption greatly reduces the class of possible models. As a result, the functional form ansatz of (1) or (2) introduces a systematic error. By specializing to (1) or (2), we reduce the number of free parameters, and thereby reduce the variance of the estimate. The usefulness of the log-additive ansatz occurs when the reduction in variance outweights the introduced systematic error.

2. Log-additive gradient models

The log-additive models may be interpreted as the leading order term in a Taylor series expansion of the true function $T(\rho, I_p \ldots)$. Our experience is that (3) adequately describes the profile response in a single tokamak, but that (3) can yield unphysical extrapolations. In general, more peaked profiles are associated with smaller values of size, R. The difficulty with the representation of (3) is that (3) leads to extrapolations of the form:

$$\left(\text{Large gradient} \overset{R\uparrow}{\Longrightarrow} \text{Small gradient}\right) \overset{R\uparrow\uparrow}{\Longrightarrow} \text{Negative gradient} \ . \tag{4}$$

In (4), " $\overset{R\uparrow}{\Longrightarrow}$ " means that as R increases, the temperature gradient decreases. As R increases further, (3) predicts the extrapolation to negative gradients. This is nonphysical since it corresponds to negative diffusion:

$$\left(\text{Small } \chi \overset{R\uparrow}{\Longrightarrow} \text{Large } \chi\right) \overset{R\uparrow\uparrow}{\Longrightarrow} \text{Negative } \chi \ . \tag{5}$$

To remedy this problem, we propose a third class of models:

$$\ln\left[-\frac{dT}{d\rho}\right] = h_0(\rho) + h_1(\rho)\ln[\bar{n}] + h_2(\rho)\ln[I_p] \ldots \tag{6}$$

We relate (2) and (6) by integrating the radial transport equation. Thus the log-temperature gradient models resemble the log diffusivity models. The advantage of the log-temperature gradient model is the more physical extrapolation:

$$\left(\text{Large gradient} \overset{R\uparrow}{\Longrightarrow} \text{Small gradient}\right) \overset{R\uparrow\uparrow}{\Longrightarrow} \text{Near zero gradient} \; .$$

The log-gradient model assumes and imposes that the profile gradient is monotonically decreasing. This assumption is violated in many JET density profiles. Nevertheless, we constrain the fit to be monotonically decreasing. We do this by preprocessing the data and replacing the data with the best fit under the assumption of monotonicity [4]. To fit the log-profile gradient model, we follow a two-stage procedure. 1) Fit each profile separately with a monotonic spline. Evaluate the inferred gradient at each point and its error bar. 2) Use our log-additive smoothing spline code on the multi-profile data consisting of the gradients. After fitting, we must integrate to find the predicted temperature. The predicted temperature has a free constant of integration. To estimate the constant of integration, we regress the line average temperature. This algorithm is being implemented at the present time.

3. Initial DIII-D-JET Comparison

To determine the size dependence of temperature and density profiles and to test the validity of (3) on multi-machine data bases, we have begun a comparison of H-mode profiles on DIII-D and JET [3]. Our profile parameterization fits most of the normalized profiles quite well but a few of the profiles differ considerably from our predictions The average residual error is 7.3% for the normalized density profiles and 7.5% for the normalized temperature profiles. The fit to the unnormalized profiles is less accurate, indicating that the mean temperature is more difficult to predict than the profile shape.

Our preliminary data set compared ELMy DIII-D discharges with ELM-free JET discharges. The DIII-D profiles were significantly more peaked than the JET profiles. The difference was the largest of the plasma edge, where the *normalized* temperature profiles are three times larger on JET than DIII-D. Since ELMs tend to expel plasma at the edge this data mismatch tends to bias our inferred size dependence on the profile fits. The nonstationary nature of the ELM-free discharges violates our steady state assumptions and introduces another source of systematic error. We have compiled an improved database consisting of only ELMy steady-state discharges.

Our preliminary study predicts that profiles for ITER are strongly hollow and peak near the edge. The JET temperature profiles are very broad, but not hollow. In contrast, many of the JET density profiles are slightly hollow. Thus, predicting a hollow density profile for ITER is not unreasonable, but extrapolation to a hollow ITER temperature profile seems unphysical. This result occurs because the JET profiles are much broader than those of DIII-D. As described in Sec. 2, switching from the log-additive profile model to the log-additive profile gradient model should make the extrapolations more physically reasonable.

We use only the control variables which are important in reducing the residual fit error. For the H-mode density profiles, the total plasma current, I_p, and the size, R, are the dominant variables in determining the density profile shape. As the current increases, the density profile broadens and the edge gradient steepens. The H-mode temperature profile depends weakly on a number of control variables. The shape is more peaked for ion cyclotron heating than for neutral beam heating, probably due to the difference in heating profile. As the device size increases, our fit predicts that the edge temperature will rapidly increase, resulting in hollow profiles.

4. Line averages

The data from our initial analysis was taken before the addition of cryopumps to DIII-D and JET. As a result, the line average density was hard to control, and the steady state value of $\bar{n}$ varies only weakly for fixed values of the other control variables. Thus, we examine the parameterization of $\bar{n}$ in terms of I_p, B_t, P_L and R. Assuming a power law form yields

$$\bar{n} = 5.75 I_p^{.21} P_L^{.24} R^{1.01} \ , \tag{7}$$

and no significant B_t dependence is observed. The fit error for (7) is large ($> 30\%$), indicating that $\bar{n}$ is not well described as a dependent variable with a power law dependence. For ITER, (7) predicts a line average density of $4.4 \times 10^{20}/m^3$ which is 1/2 times the Greenwald limit [5]. In contrast, the ITER design value is $2 \times$ Greenwald limit. Thus, the design goal of ITER is four times the prediction of our log-linear regression.

We have also regressed the line average temperature versus the control variables:

$$\bar{T} = c_0 I_p^{.9} R^{-.24} P^{.34} \bar{n}^{-.23} \ . \tag{8}$$

The scaling $\bar{T} \sim \bar{n}^{-.23}$ is much weaker than the ITER design value of $\bar{T} \sim \bar{n}^{-1.0}$. As a result we predict a much larger temperature and smaller density for ITER.

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