DIMENSIONAL ANALYSIS OF L-H POWER THRESHOLD SCALINGS

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INTRODUCTION

The region where the physical effects of the open field lines become important includes part of the closed field line edge plasma as well as the scrape-off-layer (SOL) plasma. Due to the fast longitudinal losses just outside the last closed flux surface, i.e. the separatrix, steep gradients in the temperature and density build up. The near separatrix region in tokamaks provides a new length scale parameter $x_0 = (-d \ln p_o/dx)^{-1}$ being of the order of several Larmor radii. Modified drift and interchange instabilities occur in this region due to longitudinal losses. The related turbulence gives rise to perpendicular transport and regulates the width x_o. The H-mode is set up when the shear flow stabilises these instabilities leading to the condition on the normalised gyro-radius $\hat{\rho} = \rho_i/x_0 > c_1$ where c_1 is of order unity.

The ITER data base working group has studied the scaling of the L-H transition power threshold with plasma parameters and has recently presented a reasonable fit to the multimachine threshold data base [1]:

$$P > 0.025\, n_o^{0.75} B_o S \quad \text{(1a)}, \qquad P > 0.4\, n_o B_o R^{2.5} \quad , \quad \text{(1b)}$$

where P is the power in MW, n_o the average density in 10^{20} m^{-3}, $B_o$the toroidal field in Tesla and R the major radius in m. Both forms assume that the linear B_o dependence is correct and take a quadratic dependence on the length scale or a linear dependence on the density as correct and adjust the other parameters. Our model indicates that the edge pressure gradient length x_o is the relevant length scale.

The simplified energy balance near the separatrix ($T_i = T_e = T_o$) reads (under the assumption that the density gradient is comparable or less than the temperature gradient): $\frac{\partial}{\partial t} T_0 = \frac{\partial}{\partial x} \chi_\perp \frac{\partial}{\partial x} T_0 - \frac{1}{\tau_\parallel} T_0$.

Here the $\tau_\parallel$ is the characteristic time of the longitudinal losses. Integrating this equation and inserting the total power which diffuses into the SOL as a boundary condition gives the following balance relations:

$$\frac{P}{n_0 S} \approx \frac{\chi_\perp T_0}{x_0} \approx \frac{x_0 T_0}{\tau_\parallel} \; . \tag{2}$$

Here P denotes the total power which diffuses across the separatrix and S the tokamak surface S.

The scientific method used is dimensional analysis together with the mixing length argument. This enables us to derive the scaling of the perpendicular heat conductivity from the analysis of the edge instabilities without knowing the specific details of the physical processes. For the L-H transition the condition of shear flow stabilisation is applied. Then the product $x_0\,T_0$ can be obtained as a function of P, once $\tau_{\|}$ is known. The longitudinal losses are described by two different models. In the first regime the losses are due to classical longitudinal thermal conduction, i.e. if $\lambda < L_{\|}$, and the second one is due to free-streaming flow, i.e. if $\lambda > L_{\|}$. Here λ is the particle mean free path, $L_{\|}$ the distance along the field line between the target plates and C_s the sound speed.

$$\tau_{\|}^{th} = \frac{L_{\|}^2}{\chi_{\|}} \propto \frac{L_{\|}^2 n_0}{T_0^{5/2}} \qquad (3a) \qquad\qquad \tau_{\|}^{fs} = \frac{L_{\|}}{C_s} \propto \frac{L_{\|}}{T_0^{1/2}} . \qquad (3b)$$

TURBULENT TRANSPORT COEFFICIENT $\chi_\perp$ FOR THE OPEN FIELD LINES REGION

We briefly discuss the linear equations for the mutual drift and interchange modes, from which we define the dimensional parameters of the problem. The ion and electron continuity equation together with matching boundary conditions of the ion and electron current to the corresponding sheath currents can be cast into the form (as explained in detail in Ref. [2]) with ϕ' being the perturbed potential:

$$\begin{aligned}(\omega+\omega_*)\cdot(k_\perp\rho_i)^2\cdot\frac{e\phi'}{T_0}+\omega_{*g}\cdot\frac{n'}{n_0} &= -\,i\nu\cdot\left[(1-\xi)\cdot\frac{n'}{n_0}+\xi\cdot\frac{e\phi'}{T_0}\right]\\ \omega_*\cdot\frac{e\phi'}{T_0}-\omega\cdot\frac{n'}{n_0}+\frac{\omega_{*g}}{2}\cdot\frac{n'}{n_0} &= -\,i\nu\cdot\left[(-\xi)\cdot\frac{n'}{n_0}+\xi\cdot\frac{e\phi'}{T_0}\right],\end{aligned} \qquad (4)$$

where ω_* denotes the electron drift frequency, ω_{*g} the magnetic drift frequency, $\nu = C_s/L_{\|}$ and $\xi = \frac{j_{o\|e}}{j_{o\|i}}$. These equations can be solved analytically for the resistive interchange instability as well as for the drift instability. Inserting the maximum growth rate and the corresponding perpendicular wave vector the mixing length argument determines the transport coefficients:

$\chi_{\perp RI} = \chi_{GB}(L_{\|}/R)(\hat\beta)^{\nu}$ and $\chi_{\perp DR} = \chi_{GB}(L_{\|}/R)^{1/3}\left(\frac{x_o}{R}\right)^{-1/3}$, (with ν = o and 1 in special cases) where $\hat\chi_{GB} = \frac{cT_o}{eB_o}\cdot\hat\rho$ is the gyro-Bohm coefficient and $\hat\beta = \left(L_{\|}^2/R\cdot x_o\right)\cdot\beta$ is the normalised beta. Thus the dimensional analysis together with the mixing length arguments give the following general expression for the turbulent transport for the mutual influence of interchange and drift instabilities:

$$\chi_\perp = \hat{\chi}_{GB} \cdot f\left(\hat{\beta}, \xi, L_\parallel / R, \hat{\rho}, x_o / R\right) \approx \hat{\chi}_{GB} \cdot f\left(\hat{\beta}, x_o / R\right) = \hat{\chi}_{GB}\left(\hat{\beta}\right)^\alpha \left(\frac{x_o}{R}\right)^\delta \quad (5)$$

THE CRITERION FOR STABILITY

As a criterion for stability we apply the condition $V_0' > \gamma_{max}$, where $V_0' = dV_0 / dx$ is the shear velocity. This condition has been derived for both the interchange and drift modes. It indicates that the perturbations have no time to develop fully and are suppressed due to the velocity shear drag. This gives the following estimate for the shear flow $V_0' \approx \frac{C_s}{x_0} \cdot \frac{\rho_i}{x_0}$. Inserting the result for the growth rates yields $\hat{\rho}_{RI} > \left(\frac{x_0}{R}\right)^{1/2}$ and $\hat{\rho}_{Dr} > \left(\frac{x_0}{L_\parallel}\right)^{1/3}$, respectively. As discussed in detail in Ref. [2] the more stringent dimensional criterion $\hat{\rho} > C_1 = 0\,(1)$ need to be applied. Then the marginal condition $\hat{\rho} \approx 1$ can be rewritten as $T_0 \propto x_0^2 B_0^2$.

DISCUSSION

The first striking result is that a cubic dependence of the threshold power on the magnetic field is obtained for the free-streaming case, i.e. $\frac{P}{n_o S} = B_o^3 f(n_o, \alpha, \delta)$

For classical parallel losses, on the other hand, the empirical scalings can be reproduced (with $L_\parallel \alpha R$ i.e. ignoring the q_a dependence). Scaling (1a) is obtained for $\alpha = -1/2$ and $\delta = 1/6$. Then the transport coefficient scales as $\chi_\perp = \hat{\chi}_{GB} \cdot \left(\hat{\beta}\right)^{-1/2} (x_o / R)^{1/6}$. The scaling (1b) requires $\alpha = 1/3$ and $\delta = 0$ and implies $\chi_\perp = \hat{\chi}_{GB}\left(\hat{\beta}\right)^{-1/3}$.

Let us compare these findings with the experimental observations Figure 1 displays the L-H transition power threshold for many JET discharges. The data indicate a stronger than linear dependence on $n_e B_0$, in particular for low density and high power. Considering the case of SOL drift type instabilities the dependence of the L-H threshold on the net current onto the target plates provides the possibility for control. It follows that for an ion current in the instability region exceeding the corresponding electron current, i.e. $\xi < 1$, the plasma becomes more stable. Consequently the condition for L-H threshold is more favourable, i.e. lower. If the ion drift is in the direction towards the X-point then the ion current onto the target plates just outside the separatrix should be larger than in the case when the ion drift direction is away from the X-point. In contrast, the electrons stick close to the separatrix. This fact can explain the difference in the observed L-H transition power threshold.

More recent threshold data are shown in Figure 2. The best fit is obtained for a slightly nonlinear dependence $P/S \;\alpha\; n_e^{0.75} B_0^{1.20}$. Obviously, the cubic dependence on B_0 in the free streaming limit (expected for low density and high temperature) is not observed. The presented model allows to derive conclusions on detached divertor plasmas. In this case a cold layer develops in front of the target plates, which suppresses the currents onto the target plates. It is concluded that the SOL drift instability will be absent but the SOL interchange instabilities become more severe due to the absence of line-tying of the magnetic field. As a consequence the L-H threshold should increase.

CONCLUSIONS

The near separatrix region in tokamaks provides an important length scale parameter $x_0 = (-d \ln p_0/dx)^{-1}$ being of the order of several ion gyro radii. In the L-H transition both resistive interchange and drift instabilities are stabilised with increasing edge temperature thereby decreasing x_0. The H-mode is set up when the shear flow stabilisation of drift waves and of interchange modes becomes effective. The results indicate that the length parameter x_0 prompts new dimensionless quantities such as $\hat{\rho}$, x_0/R and $x_0/L_\parallel$, which should be taken into account in the empirical scalings for the transport coefficients. The other important result is that finite beta physics, in form of the parameter $\hat{\beta} = (L_\parallel^2 / x_0 R) \cdot \beta$ being of order unity, plays an important role for electrostatic SOL turbulence. Classical parallel thermal heat conduction provides a threshold power scaling similar to the experimentally observed scaling.

[1] H-mode Data Base Working Group, presented by F. Ryter, 21st EPS Montpellier 1994, Vol. I p.334.
[2] J.G. Cordey, W. Kerner and O. Pogutse to appear in Plasma Phys. Contr. Fus.

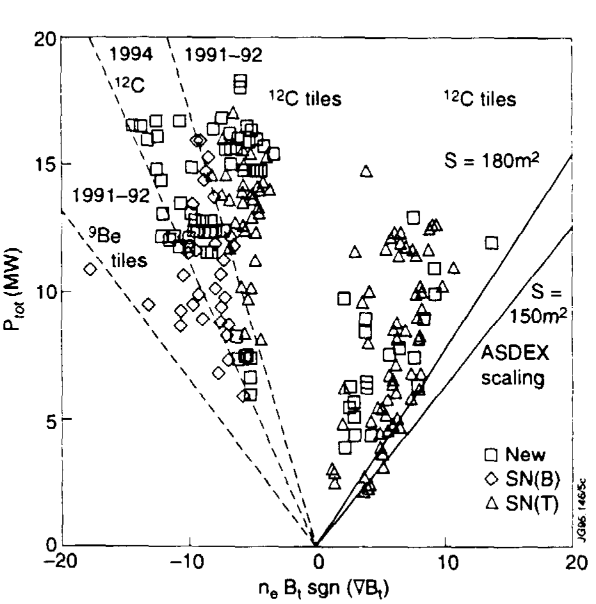

Fig. 1: H-mode operational diagram for both ion ∇B directions

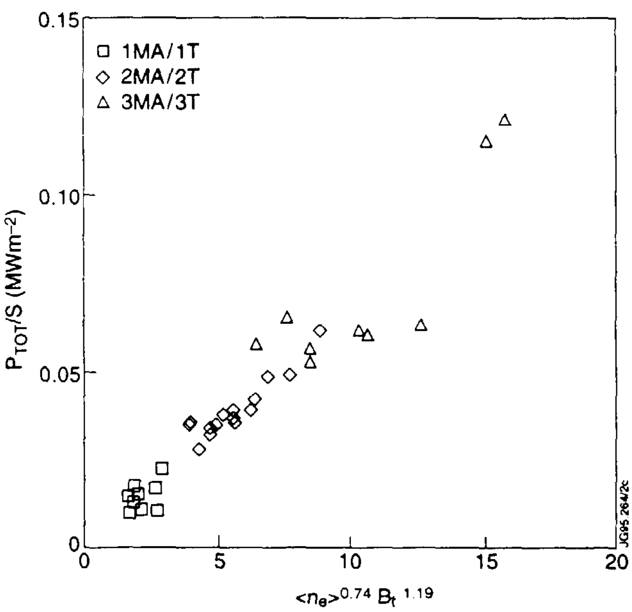

Fig. 2: Power threshold dependence on density and magnetic field; $<n_e>$ in 10^{19} m^{-3}, B_t in Tesla.