A New Approach to the Evaluation of *I-V*-Characteristics and Application to Highly Collisional Divertor Plasmas

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1. Introduction

Langmuir probes are a simple and indispensable tool for the investigation of the edge plasma in magnetic fusion devices. In the form of single probes mounted in the divertor plates, they are widely used in tokamak fusion experiments to monitor the electron density and temperature of the divertor plasma. However, there is growing doubt about the correct evaluation of measured current-voltage-characteristics. In fact, there is little reason to expect that traditional probe theory is still applicable when the probe is operated in a strong magnetic field.

Recent divertor-probe results at JET underline the problem. In the case of a dense and cold divertor plasma close to or in the state of detachment, rather low values of the electron-to-ion ratio of saturation currents, I_p^- / I_p^+, are found, in a number of cases even smaller than unity. Moreover, the electron temperatures derived essentially from the slopes of the characteristics at the floating point according to $T_e = eI_p^+(\mathrm{d}V_p/\mathrm{d}I_p)$ appear to be substantially overestimated in comparison with predictions from SOL code calculations [1] for this type of discharges (e.g., 12 eV contrasted with 2 eV).

The present paper is an attempt to make a first step towards resolving the problem on the basis of non-ambipolar fluid theory applied to the plasma between the probe-tip Debye-sheath and the adjacent and opposite return sheaths. The very simple model used for the "environment " of the probe has already been treated earlier [2], assuming a non-resistive plasma, and this approach is useful in providing a general understanding of, among other things, low I_p^- / I_p^+ ratios in a magnetized plasma. However, with regard to T_e, the downward correction it predicts cannot exceed a factor of two. Clearly, in the particular case of highly collisional cold divertor plasmas the plasma resistivity (finite Spitzer conductivity) must be taken into account. The present paper upgrades the theory in this respect, describes a corresponding routine procedure of data evaluation in order to derive corrected, i.e. lower, values of T_e, and reports first results.

2. Fundamentals

2.1. General aspects of Langmuir probe measurements in strong magnetic fields

(i) Operating a Langmuir probe means applying a voltage to, and measuring the current through, a network of "resistors" of different type: probe-tip and return sheaths (R_{PS} and R_{RS}, respectively), cross-field ($R_\perp$) and longitudinal ($R_{||}$) resistances of the plasma. $R_{||}$ is the only linear one. (ii) Classical single probe theory deals only with the probe-tip sheath, i.e. R_{PS}, assuming $R_{\perp} = R_{||} = 0$. As part of the voltage drops at $R_{\perp}$, $R_{||}$, and R_{RS}, T_e will always be overestimated in this way. (ii) Saturation of the electron current at a low level cannot be caused by R_{RS} alone as long as a *finite* $R_{\perp}$ allows the current to flow more and more away from the probe's flux tube across the field when the voltage is increased. Only a current-limiting $R_{\perp}$, becoming asymptotically infinite, can be responsible.

2.2. Perpendicular electric current in a magnetized plasma according to fluid theory

According to the *total* momentum balance in a magnetized plasma (N = neutral density),

$$\vec{j} \times \vec{B} = \vec{\nabla} p + (k_i + k_{cx}) N m_i n \vec{\upsilon} + m_i \vec{\Gamma} \cdot \vec{\nabla} \vec{\upsilon} + \vec{\nabla} \cdot \vec{\pi} \ , \qquad (1)$$

the cross-field current is a matter of forces that balance the *Lorentz force* $\vec{j} \times \vec{B}$. In the presence of anomalous processes, this holds true provided that fluctuations of the magnetic field are unimportant. Exactly this approach underlies non-ambipolar SOL models [3,4].
The relevant cross-field current $j_{\perp} = e\Gamma_{i\perp} - e\Gamma_{e\perp}$ is determined by the last three terms in Eq. (1) alone, and these depend on the *drift velocity* υ_d (in $\vec{\nabla} p \times \vec{B}$-direction) which can be caused by a perpendicular electric field $E_{\perp}$. In terms of ion and electron fluxes separately, both $\Gamma_{i\perp}$ and $\Gamma_{e\perp}$ contain the same (anomalous) diffusion term (diffusion is ambipolar), and $\Gamma_{e\perp}$ is nothing but diffusive, while $\Gamma_{i\perp}$ depends *additionally* on the three terms mentioned.

3. The Simplest Model: Flush Mounted Probe in a 100% Recycling Scrape-Off Plasma

(i) The scrape-off plasma is sustained only by ionization of neutrals, i.e. $\vec{\nabla}_{\perp} \cdot \vec{\Gamma}_i = 0$, which imposes an integral condition on the density of neutrals.
(ii) $\vec{B} = const.$ (no variation in shape and area of flux tube cross sections).
(iii) Approximations in the fluid equations: $T_e = const.$, $T_i = const.$, neglect of viscosity.

3.1. Basic physical parameters

$$C = \frac{a}{\rho} \frac{1}{\sqrt{Z+\mu}} \quad \text{where} \quad \rho = \frac{\sqrt{m_i T_e}}{ZeB} \quad \mu = \frac{\langle \sigma_{cx} \upsilon \rangle}{\langle \sigma_i \upsilon \rangle} \qquad a = \text{projected probe tip half width} \qquad (2a,b,c)$$

$$\vartheta = \frac{a}{\lambda} \quad \text{where} \quad \lambda = \sqrt{\frac{D_{\perp} L}{f c_s}} \qquad f = f(\mu/Z) < 1 \quad \text{density profile factor} \qquad (3a,b)$$

$$\gamma^2 = \frac{e^2 n_0^* c_s L}{T_e \sigma_{||}} = \frac{L}{1.96\, \lambda_{ei}} \sqrt{\frac{m_e}{m_i}} \left(Z + \frac{T_i}{T_e} \right) . \qquad (4)$$

n_0^* = sheath edge density, probe unbiased
λ_{ei} = mean free path of electrons

L has to be treated as a *reduced* (effective) connection length which excludes that part of the scrape-off layer where the density of neutrals is low and the electron temperature is high.

3.2. Non-resistive case ($\sigma_{||} = \infty$) in slab geometry

With both V (probe voltage) and $\Phi(y)$ (extra plasma potential induced by biasing the probe, y across the field) normalized to T_e/e, the final probe equation reads (for details see [2]):

$$a^2\frac{\mathrm{d}^2\Phi}{\mathrm{d}y^2}-\frac{\vartheta^2}{C^{*2}}\left(a\frac{\mathrm{d}\Phi}{\mathrm{d}y}\right)^2=C^{*2}\begin{cases}1-\frac{1}{2}\left(1+\mathrm{e}^{V}\right)\mathrm{e}^{-\Phi} & \text{for } |y/a|<1\\ 1-\mathrm{e}^{-\Phi} & \text{for } |y/a|\geq 1\end{cases}\qquad C^{*2}=C^2+\vartheta^2\frac{T_i}{ZT_e}. \quad (5a,b)$$

In terms of $en_0^* c_s$-normalized current densities, the floating-point slope, $s_0=\mathrm{d}\bar{j}/\mathrm{d}V$, of the probe characteristic and the saturation-current levels are given by the following formulas:

$$s_0=\frac{1}{2}+\frac{\sinh C^*}{2C^*\exp C^*}\qquad S_0^-=\frac{2}{\vartheta\sqrt{1+2(\vartheta/C^*)^2}}\qquad S_0^+\approx\exp\left[\left(\frac{\vartheta}{C^*}\right)^2\left(\frac{1}{s_0(C^*)}-1\right)\right]. \quad (6a,b,c)$$

Eqs. (6b,c) result from the density reacting to biasing the probe according to the relation $n^*(y)/n_0^*=\exp\left(-(\vartheta/C^*)^2\Phi(y)\right)$. This leads to a *depletion of density* in the probe's flux tube if $\Phi\to\infty$ (electron-current saturation for $V>>0$). The density effect is a matter of $D_\perp$ (cf. Eq. (3)). Pin-plate probe experiments [5] give evidence for this behaviour of both Φ and n.

3.3. Resistive case (linearized treatment) in slab geometry

For reasons of feasibility, the fluid equations relating to Φ have been linearized to describe the *vicinity of the floating point only*, yielding the 2-D probe equation (x along field lines)

$$C^{*2}L^2\frac{\partial^2\Phi}{\partial x^2}+\gamma^{*2}a^2\frac{\partial^2\Phi}{\partial y^2}=0\qquad\qquad \gamma^{*2}=\gamma^2\frac{C^{*2}}{C^{*2}+\vartheta^2}. \quad (7a,b)$$

Its solution by Fourier transformation leads to the normalized floating-point slope $s(C^*,\gamma^*)$:

$$s=1-\frac{2}{\pi}\int_0^\infty \mathrm{d}k\,\frac{\mathrm{sh}\left(\frac{2\gamma^*}{C^*}k\right)+\frac{k}{C^*\gamma^*}\mathrm{ch}\left(\frac{2\gamma^*}{C^*}k\right)}{\left[1+\left(\frac{k}{C^*\gamma^*}\right)^2\right]\mathrm{sh}\left(\frac{2\gamma^*}{C^*}k\right)+2\frac{k}{C^*\gamma^*}\mathrm{ch}\left(\frac{2\gamma^*}{C^*}k\right)}\frac{\sin^2 k}{k^2}\qquad s(C^*,0)=s_0(C^*). \quad (8a,b)$$

Derivation of T_e from measured I-V-characteristics

Using the experimental parameters $T_e^{\mathrm{orthodox}}=eI_p^+\left(dV_p/I_p\right)$ and $\bar{j}_p^+$, assuming $\vartheta=0$, the basic relation $T_e=s\left(C(T_e),\gamma(T_e)\right)T_e^{\mathrm{orthodox}}$ has to be solved for T_e with C given by Eq. (2) and γ determined by (according to Eq. (4), $\ln\Lambda=10$)

$$\gamma^2=5.262\,Z\,(L/\mathrm{m})\,(\bar{j}_p^+/\mathrm{Acm}^{-2})/(T_e/\mathrm{eV})^{5/2}.$$

As $\vartheta=0$, this yields an *upper limit* for T_e.

The aternative, $\vartheta>0$, requires that (i) C, γ be replaced with C^*, γ^* according to Eqs. (5b), (7b) and (ii) both $\bar{j}_p^+$ and T_e^{orthodox} be divided by $S^+(\vartheta,C^*,\gamma^*)$ (approximate S^+ available) to take into account the increase of density as $V_p\to-\infty$. *Overestimated* values of ϑ, yielding a *lower limit* for T_e, are obtained by equating the experimental I_p^-/I_p^+ to S_0^-/S_0^+, Eq. (6), or by using the classical $D_\perp$ yielding $\vartheta=\sqrt{f/(1.96Z)}\,a/(\rho\gamma)$ (Eq.(3)).

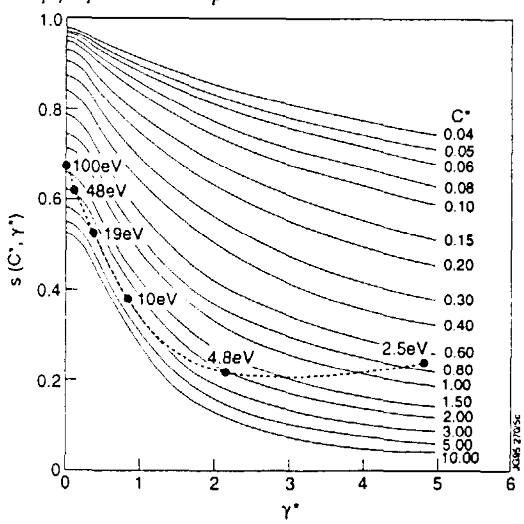

Fig.1: Normalized floating-point slope of the I-V-characteristic as a function of the parameters C^* and γ^*. The dependence of the latter on T_e is exemplified by the broken line which corresponds to the uppermost curve in Fig. 2.

4. Results

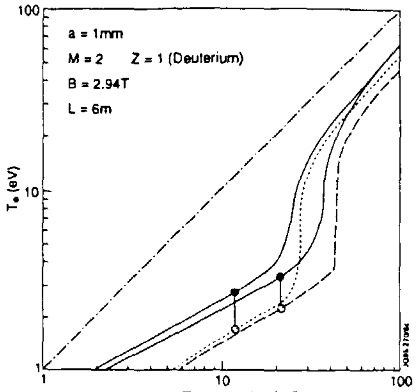

Fig.2: Upper (solid) and lower (dashed) limits for T_e in dependence on $T_e^{orthodox}$. Ion saturation currents 7.3 A/cm² and 18 A/cm² (upper and lower curves).

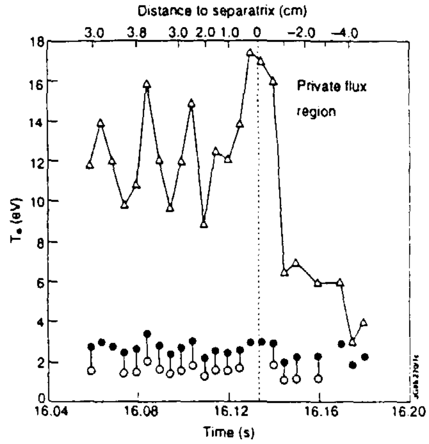

Fig.3: $T_e^{orthodox}$ (Δ) contrasted with upper and lower limits of T_e (● and ○). JET pulse No 31627 during separatrix sweeping prior to detachment.

5. Conclusions

1. Ordinary non-ambipolar fluid theory [3,4] can provide a basic understanding of *I-V*-characteristics from probes in a magnetic field [2] including very small values of I_p^- / I_p^+.
2. Plasma resistivity becomes crucial in dense and cold divertor plasmas which, if ignored, causes too-high electron temperatures to be derived from measured *I-V*-characteristics.
3. The resistive version of the simple 100%-recycling model can deliver drastically reduced electron temperatures which may not be unreasonable in the light of code predictions [1].
4. The quantitative results presented have to be regarded only as preliminary ones since

- retaining the model itself, improvements are necessary with respect to (i) using cylindrical geometry, (ii) finding empirical rules of choosing suitable values of L (effective connection length), (iii) solving the nonlinear version of the model to find S^- and S^+, and
- beyond the 100%-recycling model, which poorly reflects the actual situation in a divertor SOL, it is crucial to take account of (i) magnetic shear, (ii) pre-existing currents and cross-field drift motions in the plasma, (iii) profiles of T_e, n and neutral density along $\vec{B}$.

Ultimately, a biased Langmuir probe ought to be treated as an integral part of the scrape-off layer as a whole, described in terms of non-ambipolar fluid theory.

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