Nonlinear Collisionless Magnetic Reconnection and Fast Relaxations

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ABSTRACT

We analyse the nonlinear stage of magnetic reconnection in collisionless and weakly collisional regimes. The reconnection time turns out at least an order of magnitude shorter than the Sweet-Parker-Kadomtsev time for values of the skin depth and of the magnetic Reynolds number typical of the core of large tokamaks.

INTRODUCTION

Laboratory plasmas close to thermonuclear conditions exhibit a variety of relaxation phenomena involving strong magnetic activity, the best studied being probably sawtooth oscillations[1]. A common feature of these phenomena is the fact that they become sharper in the largest, hottest devices like JET[2].

Renewed interest in the sawtooth crash problem was sparked by the observation that at the high plasma temperatures of these experiments sawteeth can occur on a time scale shorter than the electron-ion collision time. For example, Fig. 1 shows the position of the peak of the soft X-ray emissivity during a typical sawtooth crash in JET. By interpreting the position of the peak as the position of the magnetic axis, one can see that the displacement behaves roughly exponentially with time, covering a large fraction of the plasma radius in a timescale of order of $100\mu s$.

Since the sawtooth phenomenon is initiated by a reconnecting mode, the m=1 internal kink, those experimental findings have generated considerable interest in the problem of magnetic reconnection in collisionless or weakly collisional regimes, where electron inertia is responsible for the decoupling of the plasma motion from that of the magnetic field (See Refs.3-4 for an extended list of references).

The timescale predicted by the linear theory of m=1 kink-tearing modes is in good agreement with that observed in the experiments[3]. However the validity of the linear theory is limited by the condition that the displacement of the magnetic axis does not exceed the width of the reconnecting layer, in practice the electron skin depth or the ion Larmor radius, whichever is larger. In order to explain experimental results like those shown in Fig. 1, a nonlinear analysis is required. In particular one needs to understand the dynamics in the early nonlinear stage, $\delta_{linear} << \lambda << a$, where λ is the displacement, δ_{linear} the width of the reconnection layer as given by linear theory and a is the plasma radius.

MODEL AND RESULTS.

Our study of collisionless reconnection employs an extension of reduced MHD on a two-dimensional slab, where the electron inertia terms, proportional to the square of the electron skin depth $d_e = c/\omega_{pe}$ is added in Ohm's law. Larmor radius terms, although formally of the same order as the skin depth terms, are initially neglected as they are not sufficient to decouple the motion of the plasma from the magnetic field lines. We therefore consider the coupled equations:

$$\partial_t U + [\varphi, U] = [J, \psi] + \mu_i \nabla^2 U, \tag{1}$$

$$\partial_t F + [\varphi, F] = \eta \nabla^2 (\psi - \psi_o) - \mu_e \nabla^4 \psi, \tag{2}$$

where $[A,B] \equiv e_z \cdot \nabla A \times \nabla B$, with e_z the unit vector along the z direction. $U = \nabla^2 \varphi$ is the fluid vorticity, φ is the stream function, $J = -\nabla^2 \psi$ is the current density along z, ψ is the magnetic flux function, ψ_o is the equilibrium flux function, $F \equiv \psi + d^2 J$ is the total electron canonical momentum in the toroidal direction and d is the skin depth. The dissipative effects we have included are the ion viscosity μ_i, the electrical resistivity η and the electron viscosity μ_e. Since the equations are normalised these dissipation coefficients must be interpreted as the inverse of Reynolds-like numbers.

The co-ordinates x and y vary in the intervals $x \in [-L_x, L_x]$ and $y \in [-L_y, L_y]$, with the slab aspect ratio $\varepsilon \equiv L_x / L_y < 1$ (Here we choose $\varepsilon = 1/2$). Periodic boundary conditions are imposed at the edge of these intervals. Length and times are normalised to the slab width and to the (poloidal) Alfven time respectively.

The initial condition is a tearing-unstable equilibrium without flow, $J_o = \psi_o = \cos x$ ($L_x = \pi$) with the addition of a small $m = 1$ perturbation in the unstable direction. Most of our studies were carried out with $d/2L_x = 0.04$ so that the logarithmic jump Δ' of the linear eigenfunction across the reconnecting layers is such that $\Delta' d \geq 1$. In this *large-*Δ' regime the mode structure has global character, resembling the eigenfunctions of the $m = 1$ internal-kink, $\varphi_L \approx \varphi_\infty \operatorname{sign} x$ everywhere except in narrow layers near the reconnecting surfaces. Moreover, with our choice of ε, only the m=1 mode is unstable.

The collisionless equations ($\mu_i = \mu_e = \eta = 0$) were studied numerically and analytically in Ref.4. Here we only summarize the main results.

Initially, in the linear stage, the system evolves exponentially with a growth rate $\gamma \approx d$ until $\lambda \approx d$. By this time the current density perturbation $\delta J = J - J_{eq}$ around the X-point has become of the order of the equilibrium current density.

In the nonlinear stage a current spike of width *d*develops around the X-point. An analytic calculation based on the conservation of F shows that δJ behaves like $\delta J \approx (\lambda/d)\ln(d/x)$ at a distance x from the X-point, $x < d$. The logarithmic singularity is cut off by a new nonlinear time-dependent microscale $\delta(t) \equiv d \exp[-\lambda(t)/d]$ so that at the X-point $\delta J_X \approx (\lambda/d)^2$. No singularity occurs in the flux function to the leading order: $\delta\psi_X \approx \lambda^2$ for $x < \lambda$.

In the nonlinear stage the reconnection proceeds faster than exponentially as far as numerically observable and arguably until the displacement reaches a macroscopic size. This is confirmed by an analytic equation for λ which predicts an explosive time dependence in the early nonlinear stage. Thus the overall reconnection time in the purely collisionless case scales like $\tau_{rec} \approx d^{-1}$, in substantial agreement with Ref.5.

The nonlinear microscale δ becomes rapidly small. Thus, additional physical effects not included in the simple collisionless model eventually come into play and δ is replaced by some other scalelength as a cutoff. In order to understand the experimental results one must identify which of the many possible cutoff mechanisms is relevant in a particular situation. Moreover it is crucial to verify whether the reconnection can proceed at a fast rate even in the presence of spike-limiting mechanisms.

Here we discuss the role of a small amount of dissipation in the form given in Eqs. (1-2). A variety of cases with $\mu_e \neq 0$ and/or $\eta \neq 0$ have been considered, all of them possessing the same linear eigenfunctions and growth rate but differing in the nonlinear phase (Fig. 2).

A pure resistive case ($\eta = 3.\times10^{-3}$ and $d = 0$) is found to follow the Sweet-Parker scenario. By contrast, a moderate resistive case with finite electron inertia ($\eta = 1.5\times10^{-3}$ and $d / 2L_x = 0.028$) behaves essentially in a collisionless fashion: resistivity (at this value) is ineffective as cut-off mechanism. This is understood by inspecting Ohm's law (2) using the previously given expressions for the flux function and for the current. One can see that the resistive term in the spike region is bounded to $O(\eta\lambda^2 / d^2)$ which is smaller than the l.h.s. of Eq. (2) as long as $\eta < \gamma_{linear} d^2$ (or $\eta^{1/3} < d$) (the resistivity term is a regular perturbation). When this occurs the resistivity can also be neglected in the linear theory and it is never important.

On the other hand, the electron viscosity term is $O[\mu_e(\lambda / d)(1 / x^2)]$ and can balance the leading collisionless terms at a sufficiently close distance from the X-point for any value of μ_e. Thus the electron viscosity is an efficient cut-off mechanism. Note that not only the true (collisional) viscosity but also any process acting as a current hyper-resistivity would be an equally effective candidate. This idea has been confirmed by running simulations with μ_e in the range $4.\times10^{-7} \leq \mu_e \leq 6.4\times10^{-6}$ (with $d = 1/4$). One can see that the microscale (Fig. 2a) is strongly affected by the viscosity. On the other hand the growth of δJ_X is not slower than exponential even when the viscosity is switched on (Fig. 2b). Thus the total reconnection time is substantially unaffected. Therefore the presence of a viscous cutoff (as long as it is smaller than the skin depth) does not alter the conclusion that reconnection continues to proceed at a fast rate in the nonlinear stage.

For bigger resistivity, when $d < \eta^{1/3}$, the electron inertia is negligible in the linear phase and the displacement grows with $\gamma_{linear} \approx \eta^{1/3}$ until it is of order of the width of the linear layer $\lambda \approx \delta_{linear} \approx \eta^{1/3}$. In the nonlinear stage the system is expected to follow the Sweet-Parker scenario with the layer width shrinking as $\delta_{nonlinear} \approx (\eta / \lambda)^{1/2}$ while the displacement grows as a power law $\lambda \approx \eta t^2$. If $d < \eta^{1/2}$ (*strong collisionality*) the displacement reaches the macroscopic size $\lambda \approx 1$, where $\delta_{nonlinear} \approx (\eta)^{1/2}$, in the characteristic Sweet-Parker-Kadomtsev time $\tau_{SPK} \approx \eta^{-1/2} \approx (\tau_{Alfven}\tau_{Resistive})^{1/2}$. If however the skin depth falls in the intermediate range $\eta^{1/2} < d < \eta^{1/3}$ (*moderate collisionality*) the electron inertia becomes again important when $\delta_{nonlinear} \approx d$ [6]. This occurs at some value of the displacement $\lambda^* \approx \eta / d^2$ after a time $t^* \approx d^{-1}$. Afterwards we expect that the reconnection will proceed essentially in a collisionless fashion until $\lambda \approx 1$. The reconnection time is therefore controlled by the electron inertia, $\tau_{rec} \approx d^{-1} << \tau_{SPK}$, as long as $d < \eta^{1/2}$, throughout the collisionless and the moderate collisionality regimes. The borderline between these regimes is typical of large tokamak sawteeth and it is therefore of especial experimental interest. Here the reconnection time turns out at least an order of magnitude shorter than the Sweet-Parker-Kadomtsev time: $\tau_{rec} / \tau_{SPK} \approx \eta^{1/6} < 10^{-1}$.

For typical high temperature JET parameters, $\eta^{1/3} \approx d \approx 3. \times 10^{-3}$ (the skin depth is normalized to the $q = 1$ radius). After a linear phase, where resistivity and electron inertia are of comparable importance, the nonlinear evolution of $m = 1$ modes is controlled by collisionless effects. The reconnection time $\tau_{rec} \approx \tau_A / d$ is of order of 100μs. Ion Larmor radius effects can shorten this time by up to a factor three[3,8,9].

The fluid model we have investigated has a number of limitations, as discussed in Ref.7. However the indications from our analysis are that the occurrence of a rapid nonlinear stage, when the system evolves faster than the Sweet-Parker-Kadomtsev timescale, is a fairly general phenomenon in weakly collisional systems characterised by large values of the Δ' parameter.

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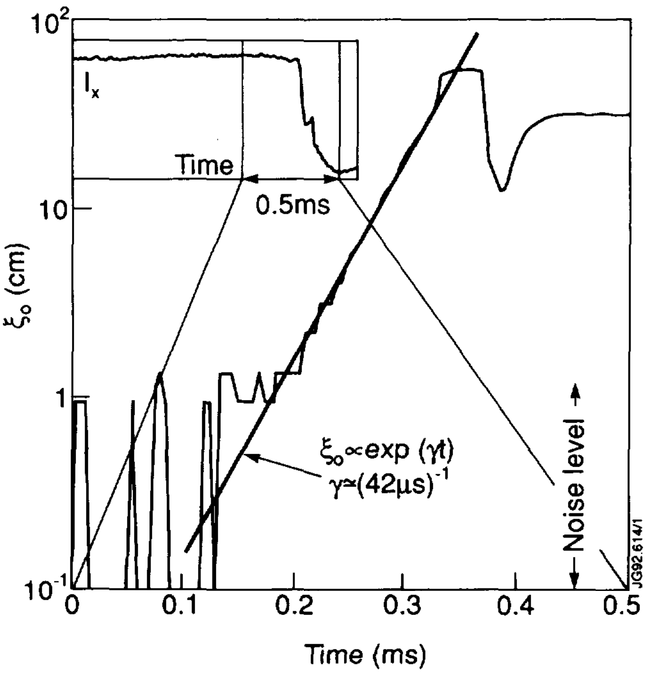

Fig. 1 *Evolution of the position of the peak of the soft X-ray emissivity during a fast sawtooth crash in JET.*

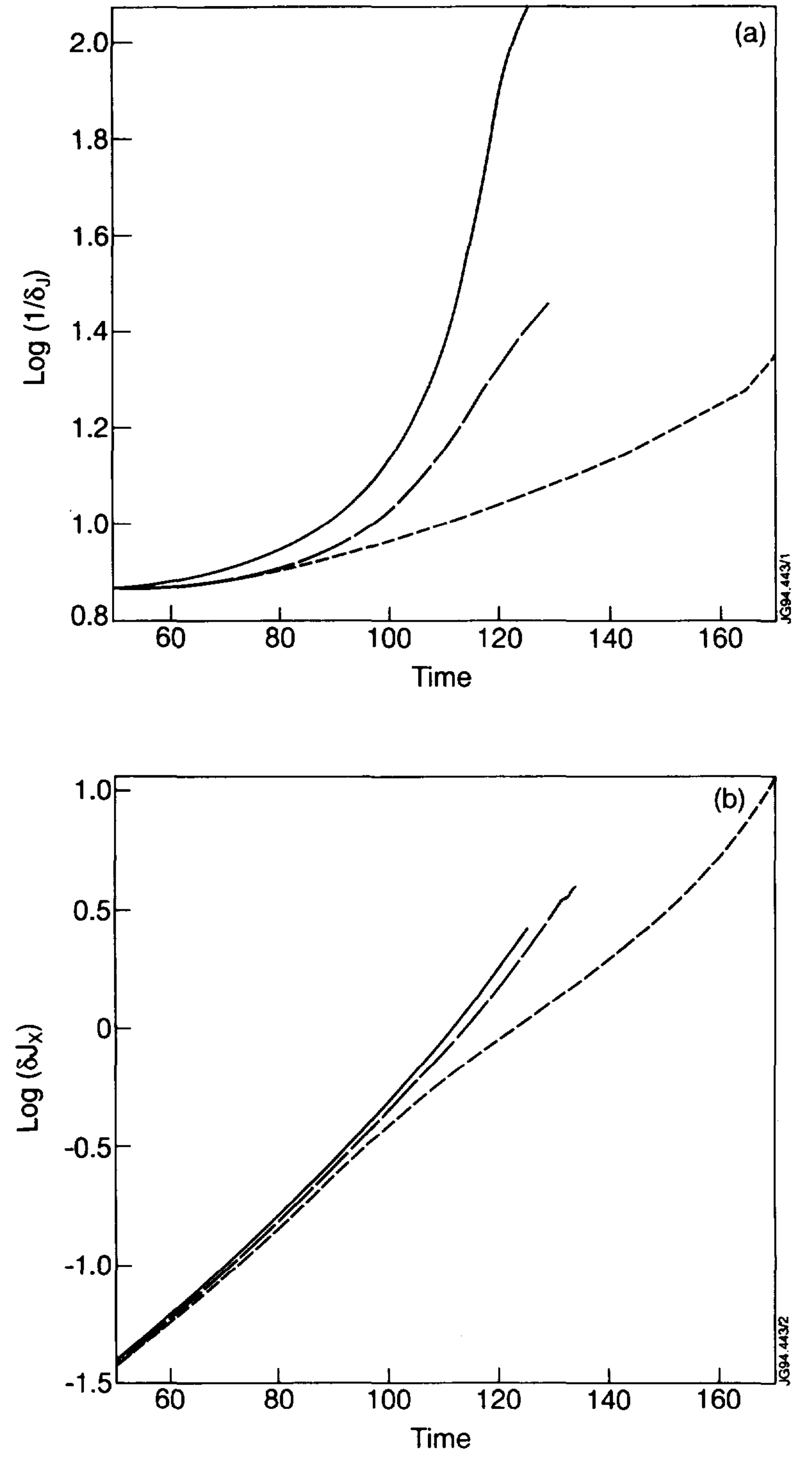

Fig. 2 *Decimal logarithm of a)* $\delta_J^{-1} = (\partial_x^2 \delta J)^{1/2}$ *and b)* δJ *at the X-point vs time. Solid lines: collisionless case. Long dash: with electron viscosity* $\mu_e = 6.4 \times 10^{-6}$. *Short dash: pure resistive case* $\eta = 3. \times 10^{-3}$.