The SOL Width and the MHD Interchange Instability in Tokamaks

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1. INTRODUCTION

The hot core plasma in tokamaks is surrounded by the scrape-off-layer (SOL), a region of relatively cold plasma with open field lines intersecting the limiter or the divertor target plates. Previous theoretical work has revealed that instabilities in the SOL plasma can strongly influence the SOL plasma behaviour and, in particular, the SOL width, x_0 [1-4]. The SOL stability analysis (resembling that of open traps) shows that there exists a critical ratio of the thermal energy and the magnetic energy, β_{cr}. If the SOL beta is greater than this critical value, β_{cr}, the magnetic field cannot prevent the plasma displacement and a strong MHD instability in the SOL occurs. In the opposite case only slower resistive instabilities can develop. The resistive instabilities depend strongly on both the volume (Spitzer) conductivity and the sheath conductivity near the end plates. In this paper a theoretical investigation of the SOL plasma stability is presented for JET single-null and double-null divertor configurations. The dependence of the stability threshold on the SOL beta and on the sheath resistance is established. Applying a simple mixing length argument gives the scaling of the SOL width.

2. DISPERSION RELATION

The plasma is described by the macroscopic single-fluid MHD model. For the case of a strong magnetic field a simplified system of reduced equations can be applied [3]. The behaviour of linearised perturbations around an equilibrium state are studied by means of the Fourier Ansatz $e^{i\underline{k}\cdot\underline{r}}$ leading to the following dispersion relation for the growth rate γ.

$$\left[\gamma + k_\perp^2\mu_\perp\right] - \frac{G(\ell)}{\left[\gamma + k_\parallel^2\cdot\chi_\parallel + k_\perp^2\cdot\chi_\perp\right]} + \frac{k_\parallel^2\cdot C_A^2}{\left[\gamma + k_\perp^2\cdot D_M\right]} = 0, \qquad (1)$$

where
$$G(\ell) = \omega_g^2\cdot\left(k_y^2 / k_\perp^2\right), \quad \omega_g^2 = \frac{1}{R(\ell)M}\cdot\frac{dP_0}{n_0 dx} \equiv \frac{C_s^2}{x_0 R(\ell)},$$

$$\frac{1}{x_0} = -\frac{dP_0}{P_0 dx}, \quad C_A^2 = \frac{B_0^2}{4\pi M n_0}, \quad D_M = \frac{c^2}{4\pi\sigma_s}, \quad C_s = \left(P_0 / (M\cdot n_0)\right)^{1/2},$$

$\mu_\perp$ denotes the transverse viscosity, $\chi_\parallel$ ($\chi_\perp$) the parallel (perpendicular) thermal conductivity and σ_s the Spitzer conductivity. A new coordinate system with ℓ the coordinate along the

$G(\ell)$ is adopted.

The eigenfunction in the unstable region I (see Fig. 2) is given by the ansatz

$$\phi_1(s) = C_1 \cdot [k \cdot \cos(k \cdot (1-s)) + \alpha \cdot \sin(k \cdot (1-s))], \quad k^2 = \beta^* \cdot (\bar{\gamma} + \delta) \cdot (1/\bar{\gamma} - \bar{\gamma}). \tag{4}$$

For the stable region II the eigenfunction has the following dependence

$$\phi_2(s) = C_2 \cdot [\kappa \cdot \mathrm{ch}(\kappa \cdot (1-s)) + \alpha \cdot \mathrm{sh}(\kappa \cdot (1-s))], \quad \kappa^2 = \beta^* \cdot (\bar{\gamma} + \delta) \cdot (1/\bar{\gamma} + \bar{\gamma}). \tag{5}$$

It is easily verified that these eigenfunctions satisfy the boundary conditions at both target plates. Using the matching conditions between the two regions $\phi_1(0) = \phi_2(0)$, $\partial\phi_1(0)/\partial s = \partial\phi_2(0)/\partial s$ yields the dispersion relation

$$\frac{k + \alpha \cdot \tan(k)}{k \cdot (k \cdot \tan(k) - \alpha)} = \frac{k + \alpha \cdot \tanh(\kappa)}{\kappa \cdot (\kappa \cdot \tanh(\kappa) + \alpha)}, \tag{6}$$

The stability threshold is given by $\beta^*_{cr} = (2.365)^2 \approx 5.6$. The growth rate for arbitrary β^* is displayed in Fig. 2.

The corresponding diffusion coefficient obtained from the mixing length argument is in physical variables

$$\chi_\perp = D_\perp = c^2 / \omega_{pi}^2 \cdot (\beta^* / \beta^*_{cr})^2 \cdot C_s / L_\parallel. \tag{7}$$

In comparison with the linear dependence on β^* for the double-null case a quadratic dependence occurs. Making use of the diffusion equation the SOL width for the single-null divertor is estimated as

$$x_0 = (\beta^* / \beta^*_{cr}) \cdot c / \omega_{pi} \quad \text{with the scaling } x_0 \sim L_\parallel \cdot R^{-1/2} \cdot n^{1/4} \cdot T^{1/2} \cdot B^{-1}. \tag{8}$$

The final result states that x_0 cannot exceed the value c / ω_{pi}.

DISCUSSION

The stability analysis concerning interchange modes in SOL plasmas has been carried out for single-null and double-null divertor geometry. Applying a mixing length argument scalings for the transport coefficients and the SOL width were derived in different regimes. The longitudinal loss mechanisms take into account the hydrodynamical regime $(\lambda_i < L_\parallel)$ with $\tau_\parallel = L_\parallel^2 / \chi_\parallel$ and the kinetic regime $(\lambda_i > L_\parallel)$ with $\tau_\parallel = L_\parallel / C_s$. The ideal MHD interchange instabilities define a stability limit for the SOL plasma beta, β^*_{cr}. At lower beta values $(\beta^* < \beta^*_{cr})$ resistive interchange instabilities occur and the resulting turbulence determines the SOL width. It is shown that for the volume (Spitzer) resistive being the dominant dissipation process the perpendicular transport $\chi_\perp \sim \chi_\perp^s \sim D_M \; \beta^* / \beta^*_{cr}$ is the same for single-null and double-null divertor configurations. For the sheath resistance the transport coefficient scales as $\chi_\perp^{sh} \sim \frac{c^2}{\omega_{pi}^2} \left(\frac{\beta^*}{\beta^*_{cr}}\right)^2 \frac{C_s}{L_\parallel}$ for the single-null divertor and $\chi_\perp^{sh} \sim \frac{c^2}{\omega_{pi}^2} \left(\frac{\beta^*}{\beta^*_{cr}}\right) \frac{C_s}{L_\parallel}$ for the magnetic field line and the local orthogonal coordinates (x, y) in the transverse (R, Z) plane is introduced. In this system all quantities have only weak dependence on ℓ and do not change sign except the quantity R(ℓ) which is defined by $\left(\left[\vec{k}\times\nabla(1/B)\right]\cdot\left[\vec{k}\times\nabla P_0\right]\right)\cdot B_0/n_0 \equiv \frac{1}{R(\ell)}\cdot\frac{dP_0}{n_0 dx}\cdot k_y^2$, and does change its sign along ℓ as indicated in Fig. 1. Since R(ℓ) is the most important term, the weak dependence of the other quantities on ℓ can be neglected. This ordering is suitable for taking into account small-scale transverse perturbations, where the transverse wave length is smaller than the SOL width. From Eq. (1) we can restore the differential equation. In particular, the component along the magnetic field line, i.e. along ℓ, is given by

$$\frac{\partial}{\partial\ell}\frac{C_A^2}{\left[\gamma+k_\perp^2\cdot D_M\right]}\frac{\partial}{\partial\ell}\Phi' - \left[\gamma+k_y^2\cdot\mu\right]\cdot\Phi' + \frac{G(\ell)}{\left[\gamma+1/\tau_\parallel+k_\perp^2\cdot\chi_\perp\right]}\Phi' = 0, \quad (2)$$

where Φ' is the perturbed electrostatic potential, $\left(\vec{b}\nabla\right)\equiv\nabla_\parallel\equiv\partial/\partial\ell$, and the operator $k_\parallel^2\cdot\chi_\parallel$ is replaced by the constant $1/\tau_\parallel$. This allows to reduce the order of the differential equation but still accounts for longitudinal losses. In the case of free-streaming losses onto the target plates at ion sound velocity, $C_s,\tau_\parallel$ is estimated as $\tau_\parallel = L_\parallel/C_s$, where $2L_\parallel$ is the distance between the target plates along $\vec{B}$. The hydrodynamic region is characterised by $\tau_\parallel = L_\parallel^2/\chi_\parallel$. The differential equation needs to be completed by an appropriate boundary condition as done in Ref. [5]. Neglecting small dissipative terms, i.e. $\mu_\perp$, $\chi_\perp$ and $1/\tau_\parallel$, equation (2) and the boundary condition are rewritten in more convenient, dimensionless variables

$$\frac{\partial}{\partial s}\frac{1}{\beta^*\cdot\left[\bar{\gamma}+\delta\right]}\frac{\partial}{\partial s}\phi - \bar{\gamma}\cdot\phi + \frac{\bar{G}(\ell)}{\bar{\gamma}}\cdot\phi = 0, \quad \left.\frac{\partial}{\partial s}\phi\right|_{s=\pm1} = (\mp1)\cdot\alpha\cdot\phi. \quad (3)$$

Here $\phi = e\cdot\Phi'/T$, $s = \ell/L_\parallel$, $\bar{G}(\ell) = G(\ell)/G$, where G is a characteristic value such that $-1 < \bar{G}(\ell) < +1$, $\beta^* = L_\parallel^2\cdot G/C_A^2 = L_\parallel^2/(x_0\cdot R)\cdot\beta$, where $\beta = 4\pi P/B_0^2$ is the usual beta for SOL parameters, $\bar{\gamma} = \gamma/(G)^{1/2}$, $\delta = k_\perp^2\cdot D_M/(G)^{1/2}$, $\alpha = \alpha_1\cdot\left(\bar{\gamma}+\delta\right)$ with $\alpha_1 = \left(\frac{L_\parallel^2}{x_o\cdot R}\right)^{1/2}\left(\frac{\omega_{pi}}{k_\perp\cdot c}\right)^2$. It is emphasised that in this dimensionless form β^* appears, which is of order unity, instead of the small β $\left(\approx 10^{-4}\right)$.

The pumped divertor, which was recently installed at JET, is a single-null divertor. For this magnetic field line geometry the curvature term G(ℓ) in Eq. (2), plotted in Fig. 1, changes sign while the average part is approximately zero. Numerical evaluation shows that the closer the field is to the separatrix the more G(ℓ) assumes a step-function dependence. For analytical treatment we assume that $G(\ell) = \theta\left(\ell-\ell_\parallel\right)$ is a step function. However, the simplest situation with constant $G(\ell)\equiv 1$ can be regarded as a suitable model for a double-null divertor. For the **single-null divertor** the step-function model for the curvature term double-null divertor. The dependence on beta is stronger for the single-null geometry due to the partial compensation of regimes with favourable and unfavourable curvature. The basic result is that with increasing SOL beta the difference between single-null and double-null disappears. Near the ideal MHD stability threshold the transport coefficient is $\chi_\perp \sim \frac{c^2}{\omega_{pi}^2} \frac{c_s}{L_{\|}}$. Consequently, the MHD instabilities regulate the SOL width and allow values up to $x_0 \sim \frac{c}{\omega_{pi}}$.

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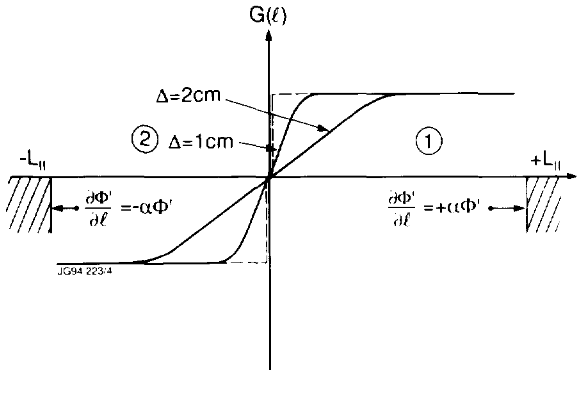

Fig. 1) The dependence of the driving curvature term along the field line for single-null divertor.

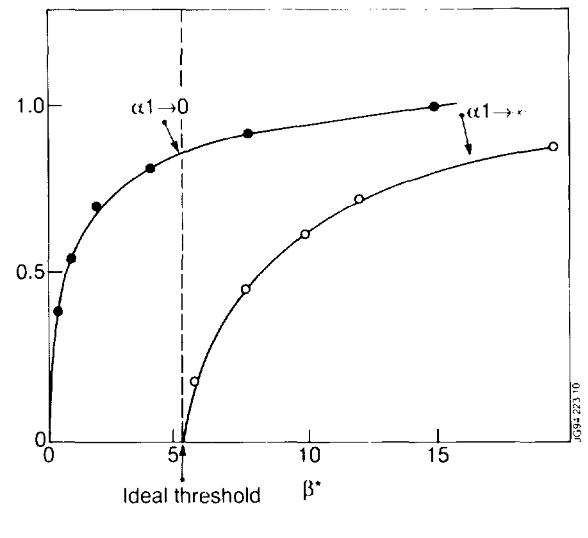

Fig. 2) The growth rate for single-null divertor