Parametric Dependencies of JET Electron Temperature Profiles

B Schunke, K Imre[1], K Riedel[1].

JET Joint Undertaking, Abingdon, Oxon, OX14 3EA.
[1] New York University, New York, USA.

The JET Ohmic, L-Mode and H-Mode electron temperature profiles obtained from the LIDAR Thomson Scattering Diagnostic [1] are parameterised in terms of the normalised flux parameter ψ and a set of the engineering parameters like plasma current I_p, toroidal field B_T, line averaged electron density $\bar{n}$,... We intend to use the same model to predict the profile shape for D-T discharges in JET and in ITER.

1. The log-additive temperature profile model

Assuming that the electron temperature profile in a tokamak depends only on the global parameters, we adopt a generalised log-additive model to describe the electron temperature profile [2] $\ln\left[\theta\left(\psi,\bar{u}\right)\right] = f_0(\psi) + \sum_{l=1}^{L} f_l(\psi) h_l\left(\bar{u}\right)$ with $h_l\left(\bar{u}\right) = (\ln[I_P], \ln[B_t], \ln[\bar{n}], \ln[\kappa], .)$,

where all variables are normalised to their mean values in the data set.

To estimate the unknown $f_l(\psi)$ we expand in B-splines: $f_l(\psi) = \sum_{k=1}^{K} \alpha_{lk} \beta_k(\psi)$ using cubic spline functions $\beta_k(\psi)$. All spline coefficients are estimated simultaneously with a penalised least square regression by minimising $\sum_{i,j} \left(\frac{\ln[T_i(\psi_j^i)] - \theta(\psi_j^i, \overline{u_i})}{\sigma_{i,j}}\right)^2 + \sum_l \lambda_l \int_0^1 \mid f_l''(\psi) \mid^2 d\psi$.

$T_i\left(\psi_j^i\right)$ is the jth radial measurement of the ith measured temperature profile and $\sigma_{i,j}$ is the associated error. The second term is the smoothness penalty function, which damps down artificial oscillations in the estimated $f_l(\psi)$. We use the Rice criterion C_R to estimate the total predicted error, which consists of variance plus smoothing bias plus the model bias (the error arising from the use of an incorrect model):

$$C_R = \frac{\sum_{i,j}\left(y_{ij} - \theta_{ij}\ ) / \sigma_i\right)^2}{(N - 2 \times \text{degrees of freedom})}$$ where $y_{i,j} = \ln[T_i(\psi)]$.

The Rice criterion differs from a least square fit by the denominator and enables us to compare models and optimise a given model with respect to the smoothing parameters, choosing λ_l, adding one parameter at a time during a sequential selection procedure and minimising C_R.

2. Advantages of the log-additive model:

- Discharge specific phenomena are eliminated by fitting all profiles simultaneously.
- Physics insight into which global variables influence temperature profile.
- Compact representation for a class of discharges.
- The fitted profiles may easily be input into analysis codes.

- Extrapolation to new values of engineering parameters possible.
- Self consistent errors, including discharge variability, are estimated using repeated measurements.

3. JET electron temperature profile parameterisation

We have compiled and, using the fitting method described above, statistically analysed a 43-profile Ohmic data set, a 55-profile L-Mode data set and a 51-profile H-Mode data set. The data were taken from experimental campaigns from 89 to 92 (see Figure 1 for examples of the fit obtained). Our parameter range is: I_p = 1.-5. MA, B_T= 1.1-3.4 T, q_{95}=2.8-17. Each profile is measured at 50 locations along the mid-plane of the JET vessel. The raw profile data show much radial structure and vary slowly in parameter. We remove the outermost points near the inner wall, where the dumping of the laser light causes a spurious spike on the profile.

Ohmic temperature analysis: Table 1 visualises the selection procedure of the dominant covariants for the Ohmic data set. Fitting all candidates in a one variable fit, one finds that Ip minimises C_R. This parameter is then selected and paired with all other variables in a two variable fit. The variable that minimises C_R in combination with Ip is then selected as second parameter, and we continue to add another variable to minimise CR.

Table 1: Rice Table for Ohmic data set

Vars	1 Var	2 Var	3 Var	4 Var	5 Var
$\ln[\bar{n}]$	8.58	3.07	**1.90**	seed	seed
$\ln[q_{95}]$	5.70	3.23	2.42	**1.50**	seed
$\ln[I_p]$	**3.49**	seed	seed	seed	seed
$\ln[B_T]$	7.20	**2.81**	seed	seed	seed
$\ln[\kappa]$	7.92	3.49	2.76	1.86	1.49
a	8.48	3.44	2.79	1.88	1.46
V_{loop}	8.51	3.45	2.71	1.85	1.46
Z_{eff}	8.01	3.24	2.78	1.87	1.43
ℓ_i	7.79	2.91	2.61	1.77	1.47

The sequential selection procedure shows that Ip is the most important variable in determining the plasma temperature, followed by the toroidal field B_T, the line average density $\bar{n}$ and q_{95}. Adding a fifth variable does not appreciably decrease C_R, so we choose the four variable model. The Rice table shows that the plasma inductance is not particularly influencing the temperature.

The resulting model for the JET Ohmic temperature profile is:

$$T(\psi)=\mu_o(\psi)\, I_p^{f_I(\psi)}\, B_t^{f_B(\psi)}\, \bar{n}^{f_n(\psi)}\, q_{95}^{f_q(\psi)} \quad \text{with } \mu_o(\psi)=\exp(f_0(\psi)) \tag{1}$$

Because Eq(1) already contains B_T and Ip , q_{95} seems to account for changes in R, a and κ. Thus we can reparameterise (1) using $q_{95}Ip/B_T$ as the fourth variable. As seen in Figure 2 the electron temperature profile broadens and becomes slightly hollow with increasing current when the other parameters are held constant. The same effect is also seen with decreasing toroidal magnetic field for constant current. Since $f_I(\psi)\neq c-f_B(\psi)$ the profile shape depends primarily but not exclusively on the ratio Ip/B_T. Our best fit has an average error of 183 eV, which is 12.6 % of the typical line average temperature. The error bar for predicting new measurements is larger than the typical residual fit error. With $C_R\approx1.5$ the error bar for

predictions is 22% larger than the typical measurement error.
A similar fit to the Ohmic density profiles shows that $n(\psi)$ depends only on $\bar{n}$ and B_T and that a good approximation is $f_B(\psi) = c - f_n(\psi)$.

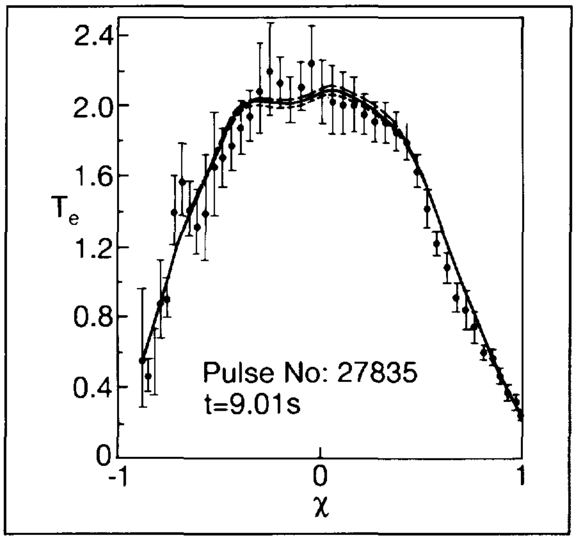

Figure 1: Example of predicted temperature profile from model with raw data

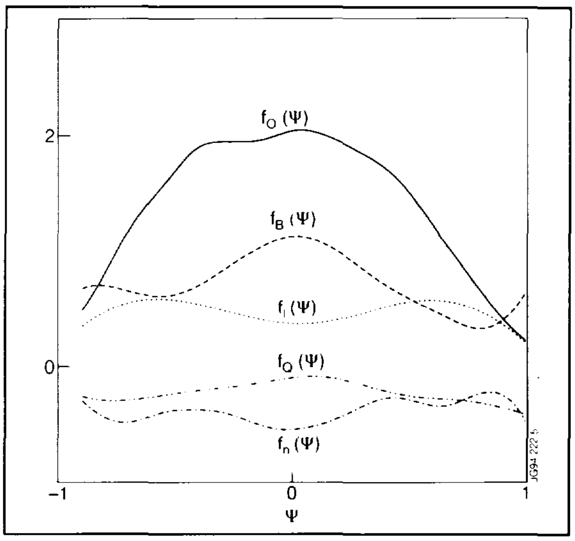

Figure 2: Spline functions for Ohmic electron temperature profile fits

L- Mode temperature fit: Repeating the analysis on the L-Mode data, we find the dependency on B_T to be stronger than in Ohmic discharges. T_e decreases and broadens with $\bar{n}$. The dependency on the current is weaker (~ $I_p^{0.4}$) then in the Ohmic case. ICRH heated discharges are more peaked than NB heated discharges, therefore we introduce the parameter $\gamma = P_{RF}/P_{tot}$ into the set of covariants.

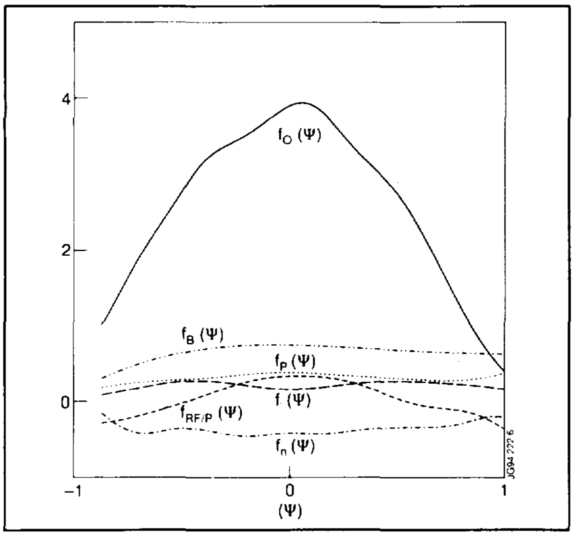

Figure 3: Spline functions for L-Mode temperature profile fits

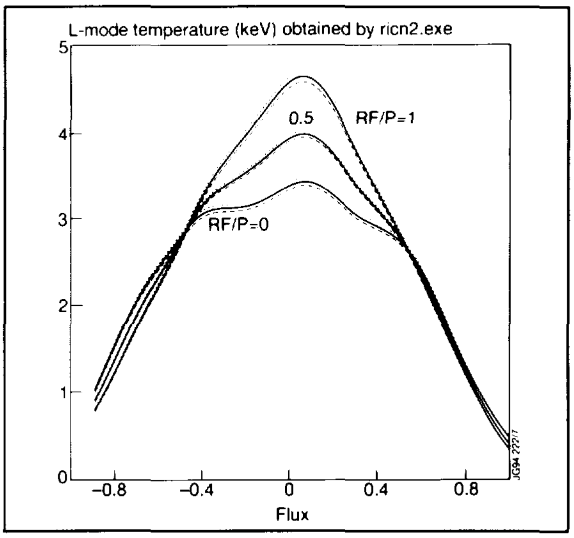

Figure 4: L-Mode temperature profiles varying γ

Best fit model: $T(\psi) = \mu_o(\psi)\, P_{tot}^{f_P(\psi)} \exp\{\gamma^{f_\Gamma(\psi)}\}\, \bar{n}^{f_n(\psi)}\, B_t^{f_B(\psi)}\, I_p^{f_I(\psi)}$ (Figure 3)

Figure 4 shows how the L-Mode temperature profiles change according to our model when varying γ. Regardless of the value of γ the profile shape stays the same outside 0.5 χ, inside 0.5 χ the application of radio frequency heating alone produces a more peaked temperature profile than the application of neutral beam heating.

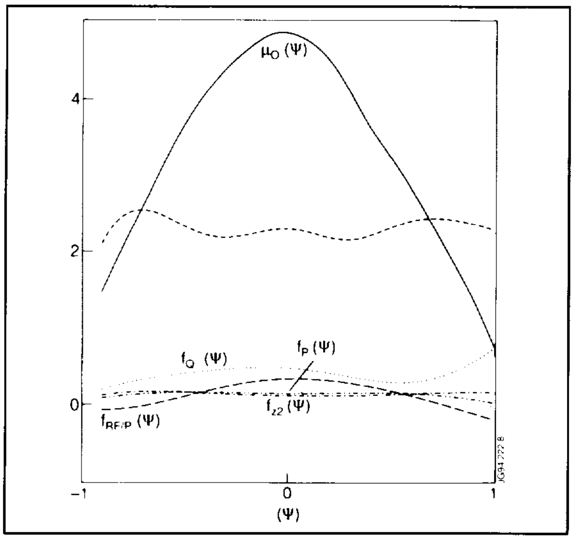

Figure 5: Spline functions for H-Mode temperature profile fits

H-Mode temperature fit: Our best fit parameterisation shows a strong dependency on the current (~ I_p^2). We found that q_{95} has an influence on the electron temperature profile in the H-Mode case, but there is no significant dependency on the density as in the L-Mode data set. Again we find ICRH heated discharges to be more peaked than NB heated discharges, and the parameter $\gamma = P_{RF}/P_{tot}$ appears also in the parametric description of the H-Mode profile as a covariant. We also found that Z_{eff} strongly influences the H-Mode temperature.

Best fit model: $T(\psi) = \mu_o(\psi)\, z_{eff}^{f_z(\psi)}\, P_{tot}^{f_P(\psi)}\, I_p^{f_I(\psi)}\, q_{95}^{f_q(\psi)} \exp\{\gamma^{f_\gamma(\psi)}\}$ (Figure 5)

Conclusions

The statistical analysis of JET Ohmic, L- and H-Mode data sets shows that the electron temperature profiles fit a log-additive model well. The further analysis of JET L- and H-Mode discharges, e.g. the inclusion of parameters describing the sawtooth phase or the power deposition profile - if feasible, will clarify the parametric dependencies. The parameterisation and characterisation of the corresponding density profiles is nearly completed and will be reported later. A Multimachine database for extrapolation to ITER performance seems possible.

[1] H.Salzmann, J.Bundgaard, A.Gadd, et.al., Rev.Sci.Instrum.**59**, 1451(1988)

[2] McCarthy, P.J., Riedel, K.S., Kardaun, O.J.W.F., Murmann, H., Lackner, K., Nuclear Fusion, **31**, 1595 (1991)