

# Global $\beta$ Limits with Non-Optimal Current Profiles

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# Global $\beta$ Limits with Non-Optimal Current Profiles

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## 1. SUMMARY

The dependence of JET  $\beta$  limits on the plasma current distribution is investigated. Faraday rotation measurements are used to infer the internal poloidal magnetic field. In some operating regimes, notably during PEP H-modes and in high bootstrap current operation, the maximum  $\beta$  achieved is much lower than the Troyon expression but close to the Wesson-Sykes limit. This may be attributable to a dependence of the  $\beta$  limit on the edge current density.

## 2. INTRODUCTION

The study of global  $\beta$  limits in tokamaks normally makes use of the Troyon expression [1]:  $\beta_{\max} = C_T \epsilon / q_a$ , that results from a numerical optimization of plasma equilibria. The Troyon expression has been a surprisingly good guide to the achievable  $\beta$ , both in its magnitude and in its scaling with plasma parameters, even though the experimental current and pressure profiles sometimes bear little resemblance to those resulting from the numerical optimisation. However the coefficient  $C_T$  does show some variation from one plasma regime to another. See figure 1. This implies that dependences other than those contained in the Troyon expression cannot be neglected.

On the basis of the theory of high- $n$  ballooning modes and low- $n$  kink modes, a dependence of the  $\beta$  limit on the current profile is expected. Recent investigations in D-III-D over a wider range of  $q_a$  and during transient modifications of the current profile have revealed such a dependence and suggest that the  $\beta$  limit scales with the internal inductance  $l_i$  [2]. While an expression of the form:  $\beta_{\max} = 4 l_i I_p / B_T a$  goes some way toward providing a more general description of the  $\beta$  limit in JET, it is not entirely satisfactory. See figure 2.

Another approach to describing the dependence of the  $\beta$  limit on the current profile makes use of the Wesson-Sykes expression [3]:  $\beta_{\max} \propto \epsilon \int (q/q^3) \rho^3 d\rho$ . This is an approximate, high aspect ratio condition for marginal stability to ideal, high- $n$  ballooning modes and it is therefore not applicable to low aspect ratio

plasmas or plasmas in which low  $m,n$  modes appear to play a role in limiting  $\beta$ . However  $\beta$  limits in TFTR current ramp experiments on TFTR [4] have been shown to be in good agreement with the Wesson-Sykes expression, and it is of interest to verify whether this formulation also gives a good description of the  $\beta$  limit in JET plasmas.

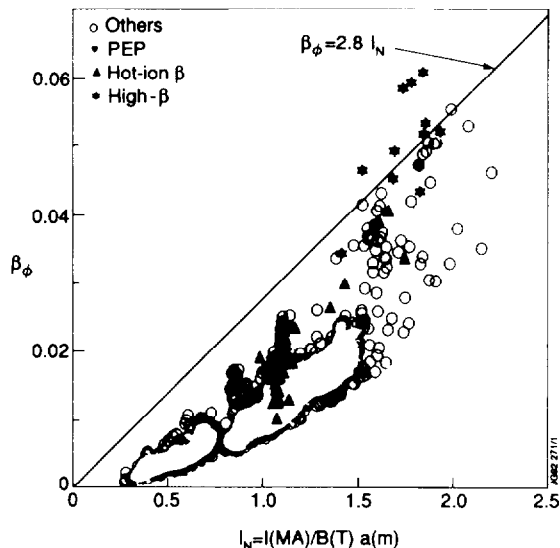


Figure 1:  $\beta$  versus the Troyon parameter. See reference [11].

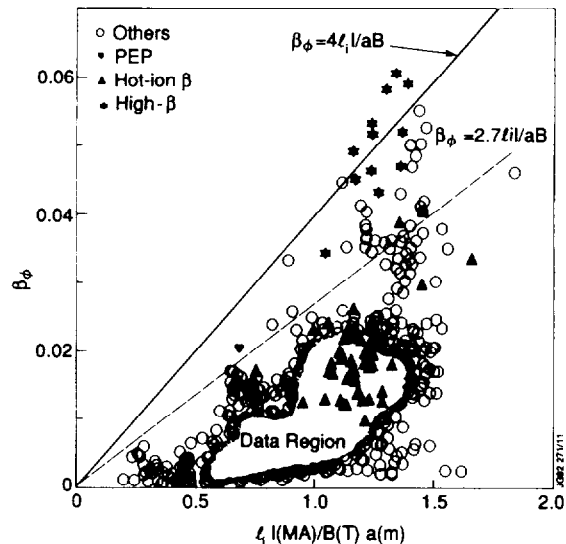


Figure 2:  $\beta$  versus  $l_i I_p / aB T$ . See reference [11].

### 3. EXPERIMENTAL CONSIDERATIONS

We use the term  $\beta$  “limit” in the operational sense of a value that cannot be increased solely by the application of a larger quantity of heating power and that is not the direct result of a catastrophic change in the plasma-wall interaction. Transport gives rise to a  $\beta$  limit only if the incremental confinement time is less than or equal to 0. The experimental determination of such a  $\beta$  limit is made difficult by the fact that the studies are necessarily conducted near the limits of available heating power as well as the thermal limits of plasma facing components. Therefore we initially confine ourselves to the determination of the existence domain of JET plasmas in some appropriate parameter space. Then we examine whether the discharges at the high- $\beta$  boundary of this domain appear to be at a  $\beta$  limit.

For this study, we use the code IDENTD [5]. This uses external measurements of the poloidal flux and the tangential magnetic field at the vacuum vessel and internal, line-integrated, measurements of the poloidal field from the multi-channel polari-interferometer [6] in the reconstruction of the plasma equilibrium. The lines of sight of the latter diagnostic are shown in figure 3. The equilibrium reconstructions are considered acceptable if the input data are fitted within their error bars and if the calculated  $\beta$  agrees (within 15%) with the independent, diamagnetic, measurement of this quantity. See figure 4.

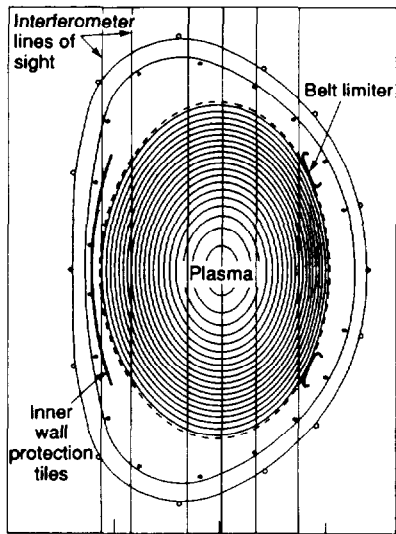


Figure 3: Position of the JET polarimeter / interferometer chords.

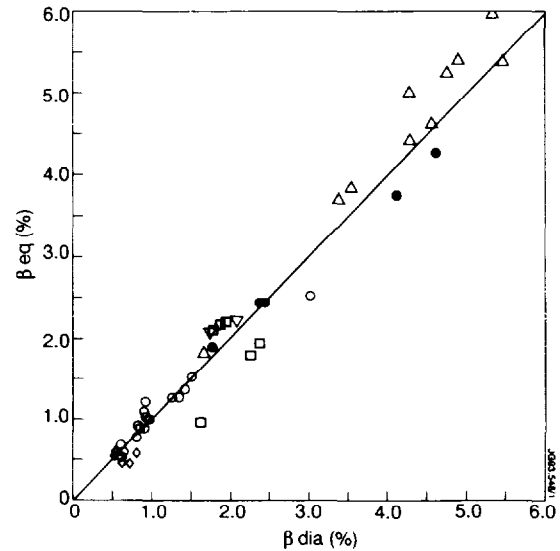


Figure 4:  $\beta_{eq}$  versus  $\beta_{dia}$

Since our aim is to obtain the dependence of the  $\beta$  limit on the current profile, we have selected for study high-performance discharges in regimes where a wide range of current distributions is expected. Thus, in addition to high- $\beta$  discharges obtained at high power and/or low toroidal field, we have also considered discharges at moderate  $\beta$  obtained in:

- large bootstrap current fraction experiments [7], or
- current-rise heating experiments (especially PEP H-modes [8]).

#### 4. DATA

Figure 5 shows  $\beta$  as a function of the Wesson-Sykes expression. It can be observed that at all values of  $\beta$  the discharges tend to cluster below the  $\beta = 1.2 \beta_{WS}$  line. Figure 6 compares the current distribution in 2 pulses. One is at the Troyon limit, with  $\beta = 5\%$  attained by applying 14.6 MW of NBI power into a 2 MA, 1.1 T plasma. The other is a PEP H-mode at one half of the Troyon limit, with maximum  $\beta = 1.3\%$ , attained by injection of a 4 mm pellet in the current rise of a 3.1 MA, 2.9 T plasma, followed by 11.4 MW of NBI heating. Both discharges are near the Wesson-Sykes limit. Their shear profiles are almost identical, but the PEP discharge has a much larger safety factor. Consequently the Wesson-Sykes integrand is much smaller for this discharge.

Figure 7 shows the evolution of the PEP discharge. As  $\beta_{WS}$  is approached, ELM activity begins and  $\beta$  saturates.

## 5. DISCUSSION

Following Freidberg [9], we parametrize the  $q$ -profile as  $q = q_0 + (q_a - q_0)\rho^\delta$ . Applying the Wesson-Sykes marginal stability criterion  $s = 1.67\alpha$  gives

$$\beta_{\max} \propto (\epsilon / q_0^2)(q_0 / q_a)^\gamma (1 - q_0 / q_a) \quad [1]$$

where  $\gamma \sim 0.2(\delta + 2)$ . When  $q_0 \sim \ll q_a$  and  $\delta \sim 3$ , this expression reduces to the Troyon expression. Equation 1 underlines the importance of the axial safety factor in determining the Wesson-Sykes limit: as  $q_0$  approaches  $q_a$ , the shear vanishes and the plasma is unstable to any pressure gradient.

More complete numerical calculations [10] that include finite aspect ratio effects show that stability to ideal ballooning modes is, on the contrary, *improved* as  $q_0$  is raised. This is because raising  $q_0$  (a) decreases the average shear (destabilizing), but (b) also increases the Shafranov shift, which increases the shear in the region of unfavourable curvature (stabilizing). Effect (b) is dominant, so that the overall stability limit is raised as  $q_0$  is increased. In fact, this is one mechanism for entry into the 2nd stability region.

Figure 8 shows the  $s$ - $\alpha$  diagram for a discharge close to the Wesson-Sykes limit. Also shown are the stability boundary derived using the Wesson-Sykes high aspect ratio formulation and the stability boundaries (for various radii) computed numerically. Although the global  $\beta$  coincides with the Wesson-Sykes limit, the stability criterion is exceeded in the inner portion of the plasma and not reached in the outer portion. More importantly, the more complete numerical calculation shows that the discharge is everywhere far from marginal stability.

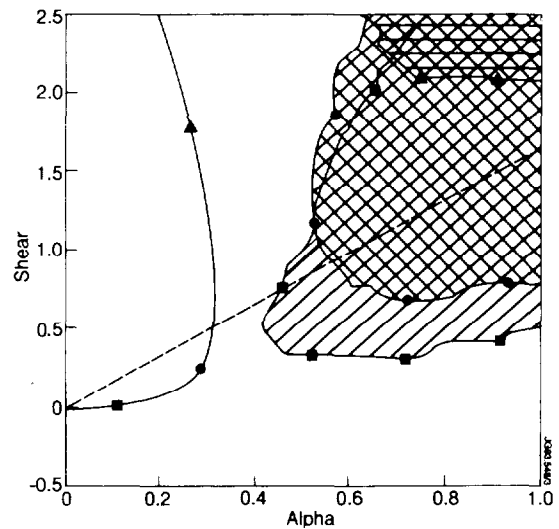


Figure 8:  $s$  -  $\alpha$  diagram for pulse 26710. Also shown are the marginal stability curves for  $R = 3.4$  m (squares),  $R = 3.7$  m (circles), and  $R = 4$  m (triangles), and the Wesson-Sykes approximation  $s = 1.67\alpha$  (dashed line)

We are forced to conclude that the agreement between the Wesson-Sykes  $\beta$ -limit and the observed boundary of the existence domain in the  $\beta_{WS}$  -  $\beta$  plane is fortuitous. The reason for this coincidence may

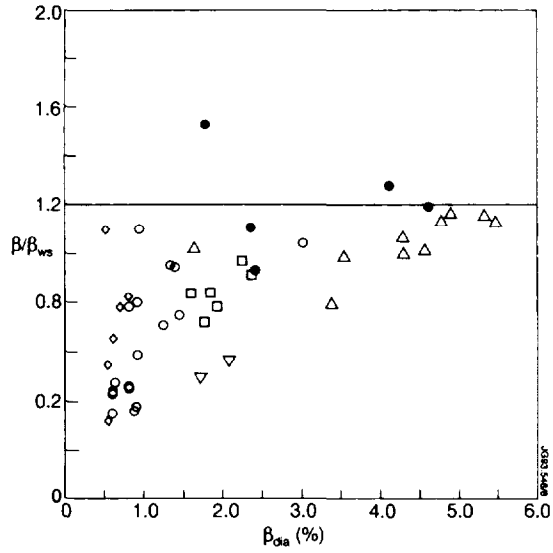


Figure 5:  $\beta$  versus  $\beta_{WS}$   
 Triangles—lower  $B_T$ , open circles—PEP, closed circles—other current rise heating, inverted triangles—hot ion, squares—other high power, diamonds—high  $\beta_p$ .

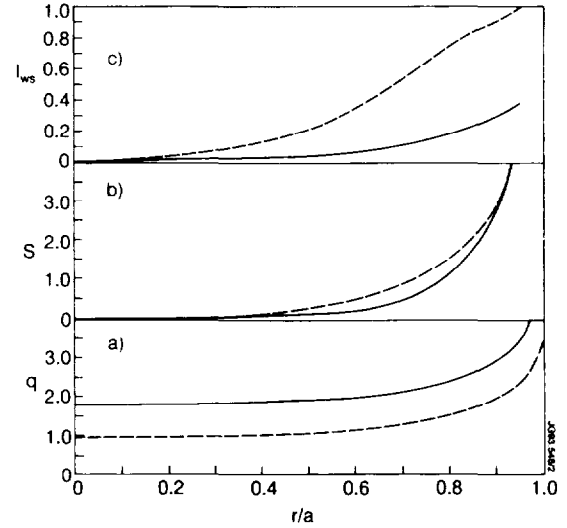


Figure 6: Profiles of (a) safety factor, (b) shear, and (c)  $q'\rho^3/q^3$  for 2 discharges at the Wesson-Sykes limit. Pulse 26236,  $\beta = 0.05$ , dashed line. Pulse 26710,  $\beta = 0.013$ , solid line.

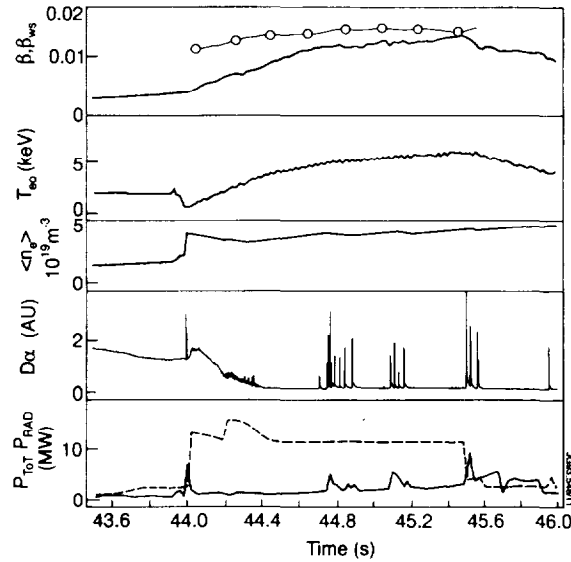


Figure 7: Evolution of plasma parameters in pulse 26710.

be that the Wesson-Sykes integral is strongly weighted toward the plasma edge, and therefore highly sensitive to variations of the current distribution in this region. Thus the Wesson-Sykes criterion, like the DIII-D expression  $\beta_{\max} = 4 I_i I_p / a B_T$  may approximate the stability boundary to modes that are sensitive to currents near the plasma edge, such as external kink modes [2]. The role of ELMs in limiting  $\beta$  lends support to this conjecture. The better agreement of the JET data presented here with the Wesson-Sykes limit (compared with the “4I<sub>i</sub>” limit) may lie in the fact that the Wesson-Sykes expression is more sensitive than I<sub>i</sub> to edge currents. However, since in this context the Wesson-Sykes expression does not correspond to a rigorously calculated stability condition, it must be regarded as an empirical expression.

Formally, the difference between the Wesson-Sykes limit and the “4I<sub>i</sub>” limit arises principally because, for a flat current profile, the Wesson-Sykes integral goes to 0 while I<sub>i</sub> goes to 0.5. Figure 9 shows that substitution of 4I<sub>i</sub> with 12(I<sub>i</sub> - 1/2) produces a limit similar to the Wesson-Sykes limit.

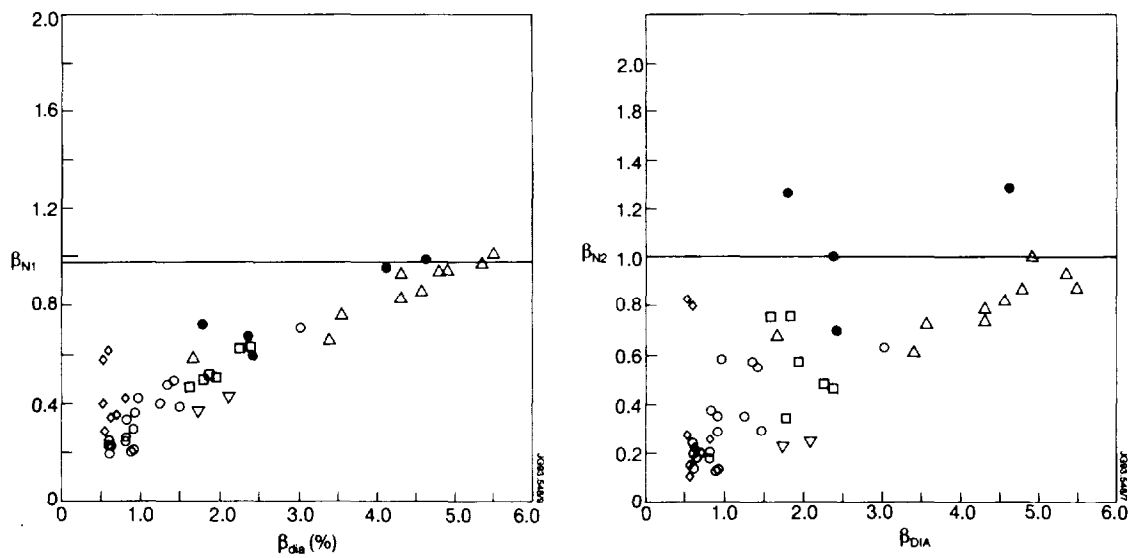


Figure 9:  $\beta_N$  versus  $\beta_{dia}$  (a)  $\beta_{N1} = \beta / (4 I_i I_p / a B_T)$ , (b)  $\beta_{N2} = \beta / (12(I_i - 1/2) I_p / a B_T)$ .

## 6. IMPLICATIONS

These results are of particular importance for conceptual reactors with high bootstrap current fractions. They imply that with pressure profiles typical of the H-mode, and correspondingly broad current profiles with  $q_0 \gg 1$ , the  $\beta$  limit may be very low. For example, a JET discharge with 70% bootstrap current is considered. See figure 10. If the pressure profile near peak  $\beta$  is assumed to remain constant and the evolution of the current profile is extrapolated using TRANSP (see figure 11), then even the modest  $\beta$  achieved transiently in this discharge cannot be sustained in steady-state. See figure 12.



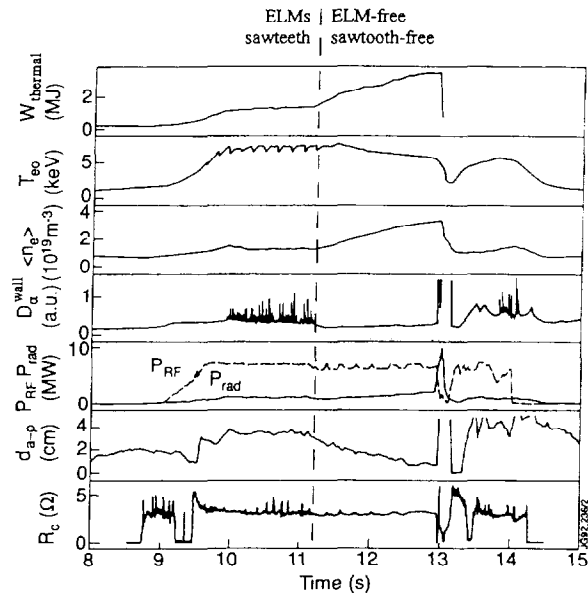


Figure 10: Evolution of plasma parameters in pulse 25264. 70% of the plasma current is driven by the bootstrap effect at  $t = 52.5$  sec.

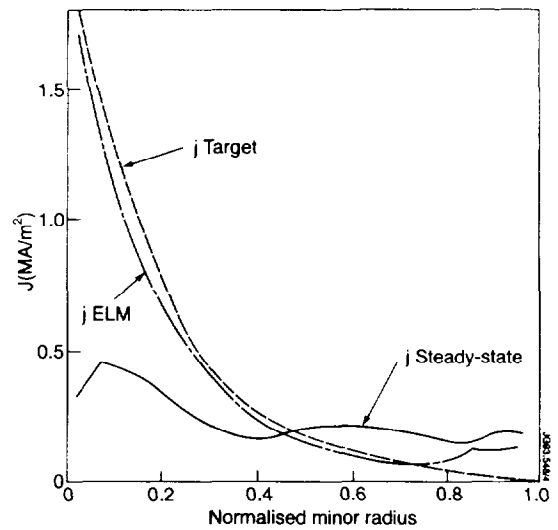


Figure 11: The current profile in pulse 25264. (a) beginning of H-mode, (b) peak  $\beta$ , (c) extrapolated to steady-state assuming neo-classical resistivity and constant pressure profile after (b).

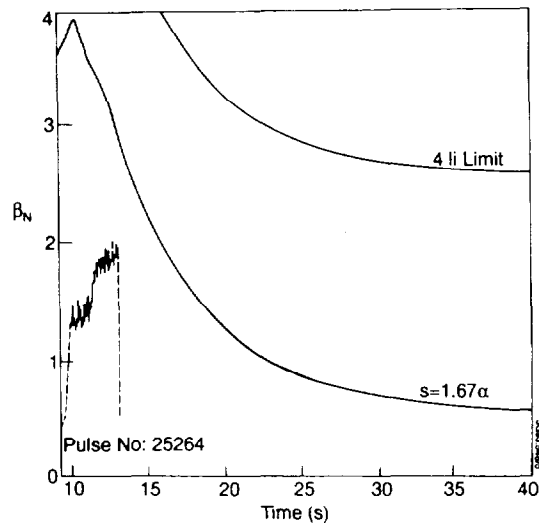


Figure 12: Evolution of the Wesson-Sykes and "4Ij" limits corresponding to the extrapolated evolution of the current profile.

## 7. FUTURE WORK

The results presented here come out of a rather small database. Future work will concentrate on a more systematic mapping of the  $\beta$ -limit in  $q_0 - q_a$  space.

$\beta$ -limit studies often concentrate on H-mode discharges because the improved confinement of H-modes reduces the power required to reach the  $\beta$ -limit. Current profile modification offer another approach to the study of  $\beta$ -limits. The different boundary conditions which obtain in L-mode discharges could give further information about the dependence of the  $\beta$ -limit on edge currents.

## 8. CONCLUSIONS

The  $\beta$ -limit in JET discharges with a wide variety of current distribution has been studied. Empirically, the Wesson-Sykes criterion gives a good description of the achievable  $\beta$ , but a local analysis does not allow the conclusion that  $\beta$  is limited by high- $n$  ideal ballooning modes. Indeed the time evolution of these discharges provides only weak evidence of a  $\beta$  limit of any kind.

The form of the Wesson-Sykes expression suggests that the  $\beta$ -limit may be sensitive to edge current density. This could limit the performance of tokamaks operating with large bootstrap current fraction.

## 9. ACKNOWLEDGEMENT

Enlightening discussions with T C Hender and J A Wesson are gratefully acknowledged.

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