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G. Hommen, M. de Baar, J. Citrin, J.W. Blokland, R.J. Voorhoeve,
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G. Hommen[1,2], M. de Baar[1,2], J. Citrin[2], J.W. Blokland, R.J. Voorhoeve[1],
M.F.M. De Bock[1], M. Steinbuch and JET EFDA contributors*

JET-EFDA, Culham Science Centre, OX14 3DB, Abingdon, UK

[1]Eindhoven University of Technology, Control Systems Technology Group/Science and Technology of Nuclear Fusion, PO Box 513, 5600 MB Eindhoven, the Netherlands
[2]Dutch Institute For Fundamental Energy Research, Association EURATOM-FOM Trilateral Euregio Cluster, PO Box 1207, 3430 BE Nieuwegein, the Netherlands, www.rijnhuizen.nl
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ABSTRACT.
The flux surfaces' layout and the magnetic winding number q are important quantities for the performance and stability of tokamak plasmas. Normally, these quantities are iteratively derived by solving the plasma equilibrium for the poloidal and toroidal flux functions using a wide range of diagnostics signals.

In this work, a fast, simple, non-iterative numerical method is proposed to estimate the shape of the flux surfaces by an inward propagation of the plasma boundary, as can be determined for example by optical boundary reconstruction described in [5] towards the magnetic axis, as can be determined independently with the Motional Stark Effect diagnostic. The method, called *OFIT+*, is not model based and therefore allows for the assessment of various equilibria. The estimated flux surfaces are compared to results of CRONOS simulations of plasma discharges in the ITER, JET and MAST tokamaks, showing agreement to within 1% of the normalized minor radius for almost all treated plasmas.

The flux surface estimates significantly simplify the calculation of the plasma q-profile, by integrating the magnetic field pitch angle, measured using the MSE diagnostic, over the flux surfaces. Results of this q-profile reconstruction are compared to CRONOS simulations and show agreement to within 10% for all treated plasmas. The impact of the shape of the flux surfaces on the q-profile, particularly the profiles of elongation and Shafranov shift, are assessed.

OFIT+ could easily be made available in real-time, providing the mapping of control actuators and sensors to the minor radius and giving an accurate estimate of the q-profile, thereby providing crucial information for plasma profile control experiments and advanced tokamak operation.

1. MEASUREMENT AND CONTROL OF PLASMA PROFILES

Modern tokamak experiments, especially those that pursue so called Advanced Scenarios, are consistently moving in the direction of integrated control of internal and external plasma parameters. These parameters include profiles of current, temperature and density, plasma shape as well as first-wall heat-loads. The integrated control of these parameters is a Multi-Input, Multi-Output (MIMO), distributed control problem, with time-varying input and output functions. Both sensors and actuators are distributed in the two dimensional poloidal plane, while plasma profiles are defined in the one dimensional normalized minor radius. The mapping of actuators and sensors depends on the shape of the flux surfaces (contours of constant poloidal flux), which vary with the machine state, plasma parameters and the plasma shape. The layout of the flux surfaces can be determined using Grad-Shafranov solvers such as EFIT [1] or EQUINOX [2]. These codes iteratively solve the equilibrium for the poloidal and toroidal flux functions, by fitting a finite set of plasma current filaments to the magnetic measurements and a wide range of other diagnostics. To meet the RealTime (RT) constraints, the RT versions of such equilibrium solvers reduce and limit the number of iterations per equilibrium by using the preceding time step as initial condition.

In this paper, a fast, simple, non-iterative numerical method is proposed to estimate the layout of the flux surfaces. The advantage of this approach, besides being numerically simpler and therefore faster to execute, is that there can be no convergence issues. Iterative non-linear solvers may alternate between different local minima in the solution space for consecutive time steps. The method proposed here provides a one-to-one mapping from inputs to outputs. Additionally, the processing time is stable and predictable and the number of input signals required is greatly reduced.

The method relies on the observation that the shape of the flux surfaces inside the tokamak plasma is strongly constrained by the shape of the plasma boundary and the location of the magnetic axis. The remaining freedom in flux surface shapes depends on machine dimensions, discharge regime and plasma parameters, whose relations can be estimated on the basis of measurements, reconstructions or simulations of tokamak plasmas prior to the experiment.

In [3] a similar observation is made, and a three parameter description of the flux surfaces is used, a function only of Shafranov shift, elongation and triangularity, and an analytical point-to-point relation is derived linking the magnetic pitch angle measured by Motional Stark Effect (MSE) diagnostic [4] to the safety factor q. Such an approach is valid in the bulk of moderately shaped plasmas. The work was done using a database of discharges from which statistical relations were derived for the flux surface layout parameters. The spread of discharge types in the database and its effect on the statistical regression were not discussed. Also, the progression of errors in the measurements and the shapeparameters to the q-profile was not analyzed.

In the present work, the layout of the flux surfaces is derived directly from the plasma boundary shape, such that also strongly shaped plasmas can be treated, and the flux surfaces near the plasma boundary can be accurately estimated. The shape of the flux surfaces is estimated by propagating the shape of the plasma boundary to the magnetic axis. The plasma boundary can be measured directly by optical boundary reconstruction as described by [5], or can be reconstructed from magnetics. The location of the magnetic axis can be determined independently using MSE, or estimated on the basis of plasma parameters. The resulting flux surface layout provides the mapping of distributed actuators and sensors to the normalized minor radius, effectively fixing the input and output functions of the distributed plasma profile control problem, and significantly simplifies the calculation of the plasma q-profile. The q-profile is calculated by numerically integrating the pitch angle around each flux surface, allowing for arbitrary flux surface shapes. This method is independent of the type of discharge, plasma profiles and the shape of the plasma boundary. The fast calculation of the q-profile also benefits the control and suppression of tearing modes by providing the location of the rational surfaces, where such modes are likely to appear.

This paper is organized as follows. Firstly, the newly developed flux surface estimation algorithm is introduced and the resulting flux surface layouts are compared to results of full equilibrium solvers in chapter 2 for plasma discharges in the MAST, JET and ITER tokamaks. Chapter 3 demonstrates how to calculate the plasma's safety factor q profile using only the shape of the flux surfaces and the magnetic pitch angle as measured by the MSE diagnostic in the plane of the magnetic axis. Resulting q-profiles are shown and compared to results from full equilibrium solvers. Additionally, the sensitivity of the obtained safety factor profile to inaccuracies in the plasma edge, magnetic axis and flux surface shapes is analyzed.

2. FLUX SURFACES ESTIMATES

An algorithm is developed that evolves the shape of the flux surfaces from the plasma boundary to the magnetic axis. The plasma boundary and the location of the magnetic axis may be obtained in arbitrary fashion.

For this work, tokamak simulations run using the CRONOS suite of codes [6] of MAST [7], JET [8] and ITER [9] discharges can provide the plasma boundaries and magnetic axis for a large dataset, and provides flux surfaces calculated using the HELENA [10] Grad-Shafranov solver, embedded within CRONOS, for comparison.

2.1 FLUX SURFACE ESTIMATION ALGORITHM: OFIT+

The flux surface estimation algorithm, called *OFIT+*, that provides the shape of the flux surfaces using the plasma boundary, the magnetic axis and a small number of shaping parameters is described hereafter in enough detail to determine exactly the shape of the flux surfaces for a given set of plasma boundary, magnetic axis location and shape parameters.

The plasma boundary and the magnetic axis data to be used has the form of a set of points in tokamak coordinates (R,Z). The routine produces flux surface shapes for each normalized minor radius ρ, as defined in the equatorial midplane by the major radius R_{eq}, between the magnetic axis R_m and the outer plasma boundary R_{max}:

$$\rho\,(\,R_{eq}\,) = \frac{R_{eq} - R_m}{R_{max} - R_m}$$

The output of the algorithm will consist of a two dimensional set of coordinates (R,Z) that span the minor radius and a full poloidal revolution.

To evolve the shape of the plasma boundary to the magnetic axis, the boundary shape is decomposed into a set of orthogonal low order shape components and a 'Higher Order Terms' (HOT) component that contains the remainder of the shape information. Each of these shape components, including the HOT component, is appointed a function (profile) that describes its propagation from the plasma boundary to the magnetic axis. An additional function describes the Shafranov shift of each flux surface; a displacement in the major radius of the midpoint of each flux surface. In this work, the shape of the plasma boundary is split up simply into only two components: the elongation and the HOT component. *OFIT+* has been kept intentionally simple, with a very limited set of shape parameters that determine the propagation of the boundary shape to the magnetic axis. This simplifies fitting procedures and limits the required processing power to stay within real-time constraints. Should the errors introduced by this simplification of the flux surface shapes prove to be prohibitively large, the algorithm could be expanded to provide more detail, for example by allowing to tune individually the profiles of not only elongation but also triangularity, squareness, etc.

To decompose the elongation and higher order shape component of the plasma boundary, the edge coordinates (R_{edge}, Z_{edge}) are transformed into anelongation-corrected polar form:

$$r_{edge} = \quad (R_{edge} - R_{mid})^2 + (\frac{Z_{edge}}{\kappa_{edge}} - Z_{mid})^2 , \tag{1}$$

$$\theta_{edge} = atan \left(\frac{\frac{Z_{edge}}{\kappa_{edge}} - Z_{mid}}{R_{edge} - R_{mid}} \right), \tag{2}$$

with θ the poloidal angle and where R_{mid}=(R_{max}+R_{min})/2, Z_{mid}=Z_m and κ_{edge} is the elongation at the plasma boundary, defined as

$$\kappa_{edge} = \frac{Z_{edge,\,top} - Z_{edge,\,bottom}}{R_{edge,\,max} - R_{edge,\,min}}. \tag{3}$$

Now, the scaling functions introduced for the HOT of the shape $\boldsymbol{r}_{\mathrm{HOT}}$ and elongation κ are:

$$r_{HOT}(\rho,\theta) = \rho \left(a + \rho^{p_{shape}} \left(r_{edge}(\theta) - a \right) \right) \tag{4}$$

$$\kappa(\rho) = \kappa_0 + (\kappa_{edge} - \kappa_0)\, \rho^{p\kappa}, \tag{5}$$

where p_{shape}, κ_0 and p_κ are shape parameters that must be fitted to the experiment and a is the plasma minor radius. Note that other propagation functions could be used for the elongation and shape components, the functions used here are practical examples. The flux surface shape, in terms of coordinates (R,Z) corresponding to a normalized minor radius ρ is now given by

$$R(\rho, \theta) = R_{mid} + cos(\theta)\, r_{HOT}(\rho, \theta) + \Delta(\rho) \tag{6}$$

$$Z(\rho, \theta) = Z_{mid} + \kappa(\rho) sin(\theta)\, r_{HOT}(\rho,\theta) \tag{7}$$

The term $\Delta(\rho)$ in the expression for R represents the Shafranov-shift of the individual flux surfaces. The Shafranov shift profile that is used here has the shape of the first quadrant of a scaled cosine function:

$$\Delta(\rho) = \Delta_0 cos\left(\left(\frac{\pi}{2}\rho\right)^{p\Delta} \right) \tag{8}$$

Applying OFIT+ to an optically reconstructed plasma boundary and MSE-derived magnetic axis from MAST discharge #20301, the flux surfaces as well as their elongation-corrected minor radius $r_{HOT}(\theta)$ are plotted in Figure 1. The results of the flux surface estimation algorithm are analyzed in the following chapters.

2.2 FLUX SURFACE ESTIMATION RESULTS

To test the flux surface estimation routine, flux surfaces are estimated on the basis of plasma boundaries prescribed in CRONOS and magnetic axis locations from the HELENA solver for simulations of MAST, JET and ITER plasmas. This dataset also provides flux surface layouts, serving as a comparison to the OFIT+ flux surface estimates.

To estimate the flux surface shapes, a Shafranov shift profile, the center elongation, the elongation propagation exponent and the shape propagation exponent must be chosen. In the comparison with the three CRONOS simulations shown here, the corresponding parameters were roughly tuned for each machine. The parameters used are presented in Table 1.

To quantify the error between two sets of flux surfaces, the average perpendicular distance between the flux surfaces normalized to the minor radius is calculated:

$$e_{av}(\rho) = \frac{\int \frac{\left| f_{OFIT+}(\rho,s) - f_{HELENA}(\rho,s) \right|}{f_{flux\ exp}(\rho,s)} ds}{(R_{max} - R_m) \int ds}, \tag{9}$$

where $e_{av}(\rho)$ is a measure of the average perpendicular distance between two flux surfaces $f_1(\rho)$ and $f_2(\rho)$, normalized to ρ, s is the coordinate along a flux surface during a poloidal revolution and $f_{flux,exp}$ is the flux expansion factor, defined as the local perpendicular distance between two flux surfaces divided by the perpendicular distance between these flux surfaces on the mid-plane

Figure 2 shows, for a hybrid ITER discharge treated in Citrin et al. [12], the flux surfaces solved by HELENA and estimated by OFIT.

In Figure 3 the average and maximum errors of the OFIT+ estimated flux surfaces compared to the HELENA flux surfaces are shown, for a discharge in each of the three tokamaks treated here. The largest errors occur during the plasma ramp-up phase (not covered in the MAST data), where the flux surface shapes evolve quickly due to Ohmic current diffusion. After ramp-up, average errors in ρ are between 0.5% and 1% while maximum errors are between 1 and 4%

2.3 ADAPTIVE SHAPE PARAMETERS FOR JET HYBRID DISCHARGES

The optimum shape parameters, which constrain for a given plasma boundary and magnetic axis location the remaining degrees of freedom in the layout of the flux surfaces, depend on the current in the poloidal field coils, the plasma current and the regime of the discharge (i.e. Hmode, hybrid, reversed shear). Plasma parameters such as the internal inductance l_i and normalized pressure β are therefore expected to be useful signals to adapt the flux surface shaping to the state of the discharge. For a set of simulations of JET hybrid discharges, a statistical analysis was made of the optimum shape parameters. A correlation was found between the plasma's internal inductance li and the optimum shape parameters, as shown for the core elongation κ_0 in Figure 4. On the basis of this analysis, the shape parameters were made to adapt to the value of l_i during the discharge, to improve the accuracy of the estimates during changes in the plasma. Figure 5 shows the error in the flux surface estimated using adaptive shape parameters, compared to the static parameters as given in Table 1. The most significant improvement in the estimation is seen in the early part of the discharge, during which the current is ramped up and diffusing to the core.

2.4 REVERSE SHEAR DISCHARGES

The hollow current profiles typical of reverse shear discharges result in different flux surface shapes. Figure 6 shows an interpretive CRONOS run of the reverse-shear JET discharge #53521 described in Crisanti et. al.[13]. The corresponding profiles of elongation and shift from the HELENA equilibrium are also shown, as well as the OFIT profiles of the form described in chapter 2.1. To accurately estimate the flux surfaces in this regime, the shape-functions of elongation and Shafranov shift used to estimate the flux surface shapes must be adapted. The significant error in these profiles result in a maximum and average error in the flux surface of 7.1% and 1.6% of the minor radius. However, when using the profiles of elongation and Shafranov shift extracted from the HELENA flux map in the flux surface estimation algorithm, the average and maximum error of the flux surfaces drop to 1.6% and 0.35% respectively. This illustrates that given a good estimate of the low order shape profiles, the algorithm proposed here is also suited to reverse shear discharges.

3. Q-PROFILE RECONSTRUCTION USING ONLY FLUX SURFACES AND MSE MEASURED PITCH ANGLES

The availability of the estimated flux surfaces significantly simplifies the calculation of the q-profile of the plasma by integrating over the flux surface the pitch angle of the magnetic field. This pitch angle can be measured directly by the Motional Stark Effect diagnostic. The method to determine the q-profile from the flux surface geometry and the magnetic pitch angle will be derived first. Consequently, q-profiles will be calculated on the basis of the estimated flux surfaces and compared to q-profiles calculated by CRONOS, using the HELENA equilibria.

3.1 Q-PROFILE CALCULATION

The safety factor q of a tokamak plasma is defined on a flux surface, as the number of toroidal rotations of a magnetic field line per poloidal rotation. The safety factor is thus a function of the ratio B_φ/B_θ, the toroidal and poloidal magnetic fields. The Motional Stark Effect diagnostic provides a measure of this ratio in terms of the pitch angle γ, where:

$$\frac{B_\theta}{B_\varphi} = tan\,(\gamma) \tag{10}$$

MSE is a diagnostic that provides pitch angle data along the trajectory of injected neutral beam particles, by measuring the polarization of the light emitted by these particles. The geometry of the beam therefore defines the localization of the measurement. In the case treated here, it will be assumed that the beam is injected such that it crosses the magnetic axis of the plasma, where the poloidal magnetic field B_θ and thus the pitch angle are zero.

The safety factor q will now be calculated by integrating over the contour s of a flux surface in the poloidal plane, the ratio B_φ/B_θ, proportional to the inverse pitch angle. Using the geometry of the estimated flux surfaces and the measurement of the pitch angle in the mid-plane, this ratio can defined on all flux surfaces.

Given this data, q is calculated using the contour integral:

$$q = \frac{1}{2\pi} \oint \frac{1}{R} \frac{B_\varphi}{B_\theta}\, ds. \tag{11}$$

To find the ratio B_φ/B_θ around the contour of a flux surface, the value of this ratio on the equatorial mid-plane from MSE is used as a starting point:

$$\frac{B_{\varphi,\,eq}}{B_{\theta,\,eq}} = \frac{1}{tan(\gamma_{eq})}. \tag{12}$$

By scaling of the magnetic fields around the flux surfaces, the value of the ratio is found in all poloidal locations, and thus around the contour *s*. When assuming a vacuum toroidal magnetic field that is a function only of major radius R, and assuming paramagnetic effects to be flux functions, the value of the toroidal magnetic field B_φ on a flux surface scales with the major radius *R*:

$$B_\varphi(R, R_{eq}) = B_{\varphi,eq}\,\frac{R}{R_{eq}}\,. \tag{13}$$

The value of the poloidal magnetic field B_θ on a flux surface scales with the local value of major radius R and the local flux expansion:

$$B_\theta(R, R_{eq}) = B_{\theta,eq}\,\frac{R}{R_{eq}}\,\frac{1}{f_{flux\ exp}(R,R_{eq})}. \tag{14}$$

The flux expansion factor $f_{flux\ exp}$ is defined as the local perpendicular distance between two flux surfaces divided by the perpendicular distance between these flux surfaces on the mid-plane, and effectively provides the scaling of the poloidal magnetic field B_p to the equatorialmidplane value. Now, the ratio B_φ/B_θ is given by

$$\frac{B_\varphi}{B_\theta} = \frac{B_{\varphi,eq}}{B_{\theta,eq}} f_{flux\ expasion} = \frac{f_{flux\ exp}}{tan\,(\gamma_{eq})}\,, \tag{15}$$

and q can be calculated as:

$$q = \frac{1}{2\pi} \oint \frac{f_{flux\ exp}}{R \cdot tan(\gamma_{eq})} ds. \tag{16}$$

This expression contains no explicit magnetic fields, but uses only the pitch angle of the magnetic field measured by MSE directly. The benefit of this simple expression is that the magnitude of the toroidal and poloidal magnetic fields need not be known, and therefore the resulting q-profile is unaffected by paramagnetic or diamagnetic effects. Provided a map of the flux surfaces of arbitrary resolution and detail and the magnetic pitch angle in the equatorial plane, the q-profile can now be directly numerically evaluated.

Figure 7 shows the q-profile of an ITER Hybrid discharge, calculated using only flux surfaces and synthetic pitch angles from a CRONOS simulation. By using the flux surface map from HELENA, the accuracy of equation (16) is tested, isolated from the effect of the flux surface estimation. For comparison, the q-profile calculated by CRONOS is also shown.

3.2 CALCULATING Q-PROFILES USING ESTIMATED FLUX SURFACE SHAPES

In this section, q-profiles are calculated using flux surfaces estimated with *OFIT+*, mimicking the situation of real-time q-profile estimation using only the plasma boundary and MSE measurements. The data used consists of a plasma boundary and the magnetic field pitch angle on the equatorial mid-plane from CRONOS. For the flux surface estimates, the magnetic axis is taken as the zerocrossing of the measured pitch angle profile.

In Figure 8, the estimated flux surfaces as shown in Figure 2 for the ITER Hybrid discharge are used to calculate the q-profile using the routine described in section 3.1. In this example, the minor-radius averaged absolute error in q is 2.7% while the maximum absolute error at the plasma edge is 7.3%. Similarly to Figure 3, the average and maximum error time traces of discharges in each of the four tokamaks treated here are shown in Figure 9. In the results shown here, the maximum error in the q-profile occurs always very near the plasma centre (ρ=0) or plasma edge (ρ=1). The minor radius averaged error in q is below 10% for all the data shown here.

Figure 10 shows the flux surfaces and q-profiles or JET discharge #79630 during ramp-up, where the average error in q is 8.5%. Because the flux surface shape parameters are tuned to the flattop part of the discharge, in this early phase of the discharge the errors in the resulting q-profile are significantly larger. Similarly, for the JET reverse shear discharge shown in Figure 6 the corresponding q-profile is plotted in Figure 11.

This shows the sensitivity of the q-profile calculation to the layout of the flux surface, as the 'hybrid' q-profile, calculated using the flux surfaces from HELENA does not show this increased error. Adapting the shape parameters to the state of the discharge will therefore not only improve the accuracy of the flux surfaces, but also of the resulting q-profiles.

To obtain a qualitative insight into the effect on the safety factor of different profiles of elongation and Shafranov shift, it is once more observed that the safetyfactor is proportional to the ratio of the toroidal magnetic flux, as applied by the toroidal field coils, to the poloidal Safety magnetic flux, as caused by the toroidal plasma current. This ratio is measured in the equatorial midplane through the magnetic pitch angle. The pitch angle at other poloidal locations on a flux surface is determined by the local flux expansion: the normalized perpendicular distance between two adjacent flux surfaces, and the major radius. The profiles of elongation and Shafranov shift have a large influence on the flux expansion between the flux surfaces, and through this insight, their effect on the q-profile can be understood.

The plasma elongation profile is generally a monotonically increasing function of minor radius. For a given plasma boundary and magnetic axis, a high gradient in the elongation profile results in more local flux expansion and therefore a higher local safety factor q. The resulting lower core elongation however, causes a lower central safety factor. In Figure 10 this effect can be clearly identified: the low centre elongation results in an underestimate of the core safety factor, while the increased gradient in the elongation profile in the plasma edge results in an overestimate of the safety factor.

The Shafranov shift profile is generally a monotonically decreasing function of minor radius. The effect of a higher Shafranov shift is to lower the safety factor, by moving flux surfaces to a larger major radius where the confining toroidal field is smaller. However, the effect of gradients in the Shafranov shift is to increase the local value of q due to the increased flux expansion in the high-field-side of the plasma, effectively increasing the average toroidal flux density at the flux surface.

The results presented so far have been obtained using plasma boundaries and pitch angles obtained from CRONOS reconstructions or simulations. For a single equilibrium in a MAST discharge actual measured signals of the plasma edge and pitch angles were available, as well as an EFIT reconstruction to serve as comparison. In this case, a plasma boundary reconstructed optically by OFIT [5] and pitch angles measured by the MSE [4] diagnostic were used as input for *OFIT+* and the resulting flux surfaces and q-profile were compared to the corresponding EFIT reconstruction. The resulting qprofile, as obtained using actual measured signals, is shown in Figure 12. Both q-profiles are drawn in the normalized minor radius coordinate, although this scale differs slightly between the two results. This discrepancy is caused by a small difference in the location of the plasma edge between EFIT and OFIT. The average and maximum error in q is 4.9% and 10% respectively, showing no significant increase in error over the results obtained using CRONOS data.

The estimation of the flux surface shapes and calculation of the q-profile takes approximately 1.5ms on a desktop computer with an Intel Core 2 6600 central processing unit, when executed as a Matlab script, when a set of 100 flux surfaces is estimated and used to calculate the value of the safety factor q. This shows the efficiency algorithm: in an unoptimized environment and with a high spatial resolution the processing time is already near the typical feedback control sampling time for current tokamaks of 1ms.

3.3 SENSITIVITY OF Q-PROFILE TO ERRORS IN MAGNETIC AXIS AND PLASMA BOUNDARY

So far, all shown results and comparisons have assumed exact measurements of the plasma boundary and the magnetic axis, and only the discrepancy of the flux surface shapes was analyzed. In a realistic application however, there may be errors in the real-time determination of the plasma boundary and magnetic axis. To analyze the effect of these errors on the calculated q-profile, a perturbation analysis is carried out.

Errors in the q-profile resulting from errors in the measured magnetic pitch angles have three sources. First of all, the inherent measurement uncertainty in the MSE diagnostic results in an inherent error bar on the calculation of the q-profile. From equation 16 it can be shown that this uncertainty is equal to:

$$\frac{\delta q}{\delta \gamma_{eq}} = \frac{-\left(1+tan^2(\gamma)\right)}{2\pi} \oint \frac{f_{flux\ exp}}{R \cdot tan^2(\gamma_{eq})} ds \qquad (17)$$

where $\delta\gamma$ is the uncertainty in the measured pitch angle.

Firstly, a radial offset of the magnetic axis is treated. The adopted approach is to displace the magnetic axis R_m by a distance Δ_{rm}. The displacement in major radius of magnitude Δ_{rm} is applied to the core flux surfaces and decays linearly to zero at the plasma edge, while maintaining for each flux surface the pitch angles corresponding to the unperturbed magnetic axis. This represents the situation where there is an error in the pitch angle measurement, resulting in an error in the location of the magnetic axis. For the ITER hybrid shot at t=890s shown in Figure 2 and Figure 7, the magnetic axis R_m is displaced outwards by 2%, corresponding to approximately 13cm. The resulting flux surfaces and q-profile are shown in Figure 13. An outwards shift of the magnetic axis results in a lower core safety factor and a higher edge safety factor. The average safety factor error introduced by the 2% shift in magnetic axis is 7.8%, the maximum error in the plasma core is 18%.

Vertical displacements of the plasma do not in the first order affect the q-profile, as the magnitude of poloidal and toroidal flux are unaffected. However, it can occur that, due to vertical displacements, the MSE diagnostic measures a pitch angle that does not coincide with the magnetic axis of the plasma. Vertical displacements of the magnetic axis will thus affect the reconstructed q-profile. Figure 14 shows how a vertical error of 2% of the plasma height between the MSE line of sight and magnetic axis strongly affects the core measurement of the pitch angle. The effect of the incorrect core-pitch angle directly translates into a strongly perturbed safety factor q in the plasma center, shown in Figure 13. The core safety factor error is 84%, while for ρ>0.2 the safety factor is unaffected. The q-profile reconstruction very close to the plasma center is thus sensitive to vertical displacements of the magnetic axis. Additional diagnostics that provide information on the vertical location of the magnetic axis may be applied to correct for this effect.

Finally, an error in the plasma edge in the form of a rigid, radial displacement is analyzed. In this case, the magnetic axis is not displaced and the pitch angle measurements are also unaffected, although they will now correspond to a different . The plasma edge is displaced inward by 2% of the magnetic axis' major radius. An outward shift would limit the analysis, as there are no pitch angle measurements available outside the unperturbed plasma edge. The resulting flux surface and q-profile are shown in Figure 15. The core safety factor is unaffected, going towards the plasma edge the error gradually increases. The 2% displacement of the plasma boundary in this case results in an average safety factor error of 8.7% and a maximum error of 20% at the plasma edge.

CONCLUSIONS

A fast and simple algorithm was developed to estimate the shape of the flux surfaces in a tokamak plasma by propagating the shape of the plasma boundary to the magnetic axis. These flux surfaces form an input/output map for 2d-spatially distributed actuators and sensors required for 1d-profile control experiments.

The flux surface estimation algorithm is tested using simulation results from the CRONOS suite of codes, providing the plasma boundary, the location of the magnetic axis and, to serve as a reference, a flux map as the solution of the HELENA Grad-Shafranov solver, part of the CRONOS suite of codes. Plasma discharges from MAST, JET and ITER were analyzed, where four static shape parameters are manually tuned for each machine. The average errors in the estimated flux surfaces for each machine, when compared to the flux map from HELENA, are within 2% of the normalized minor radius. For the tested data, maximum errors during ramp-up are within 10% of the normalized minor radius and within 5% during the later parts of the discharge. For the class of hybrid JET discharges, a regression analysis was applied to find a correlation between the plasma shape parameters and the state of the discharge. A clear correlation with l_i was found, and applied to adapt the flux surface estimation. This improved the accuracy of the flux surface estimates during ramp-up. The optimum shape parameters are expected to relate to plasma parameters differently for each class of plasma discharges. This effect is observed for a reverse shear discharge in JET, which shows different flux surface shape propagation. Concluding, different classes of plasma discharge will require different shape parameters and profiles, and could incorporate also the effect of other quantities such as the normalized pressure β or the Mach number.

By using CRONOS data as input to the flux surface estimation algorithm, the effects of errors in the determination of the plasma boundary and the magnetic axis were effectively eliminated. In a real-time environment however, uncertainties in the measured plasma boundary and magnetic axis will also propagate into the flux surface layout.

Using the estimated flux surfaces and the magnetic pitch angle measured on the plasma midplane, the profile of the safety factor q can easily be computed by evolving and integrating the pitch angle around each flux surface. The scaling of the pitch angle along a flux surface depends only on the local major radius and flux expansion, which can be derived directly from the set of flux surfaces. Using the estimated flux surfaces and the magnetic pitch angles from CRONOS simulations, qprofiles are calculated for the plasma discharges treated here and compared to the q-profiles provided by

CRONOS. The resulting minor radius averaged error in q is below 13% for all tested data, and below 3% for the ITER and JET discharges after ramp-up. Peak errors are up to 40%, occurring always very near the plasma edge or plasma centre.

For a single MAST equilibrium measurements of plasma boundary and magnetic pitch angle were available, which allowed to test *OFIT+* on a real dataset. These results show that the accuracy of the reconstruction is maintained when using measured signals, as opposed to the synthetic measurements obtained from CRONOS. For the estimation of the flux surfaces in real-time, the plasma boundary and the location of the magnetic axis must be available in real-time. Additionally, to obtain the q-profile, the magnetic pitch angle must be known on a chord that coincides with the magnetic axis. The estimation of the flux surface and calculation of the q-profile then takes approximately 1.5ms when executed on a computer with an Intel Core 2 6600 central processing unit, executed as Matlab code. Optimizing code and operating under a real-time environment should further speed up the processing time, making this a highly suitable tool for real-time plasma control.

OFIT+ provides accurate, fast and robust estimates of the flux surface layout and the profile of the safety factor q for most parameter domains presented here. Because of the numerical simplicity, stability of the results and the small number of required inputs, the method is ideally suited for use in profile control experiments.

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Machine Parameter	MAST	JET	ITER
p_Δ (shift exponent)	1.7	1.5	1.5
κ_0 (center elongation)	1.45	1.3	1.4
p_κ (elongation exponent)	10	6	5
p (shape exponent)	5	4	5

Table 1: Shape parameters per machine

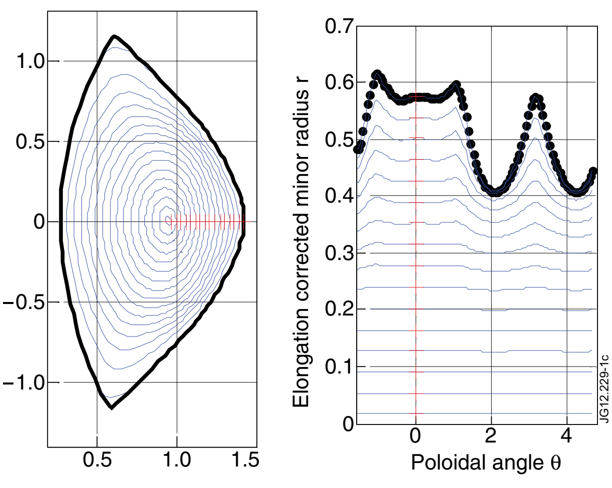

Figure 1: (left) Flux surfaces estimated using OFIT[5] plasma boundary and magnetic axis from MSE[4]. The shape parameters used in this case are p_Δ=1, κ_0=1.6, p_κ=10 and p_{shape}=3. (right) The propagation of the HOT component of the plasma boundary shape to the magnetic axis, as described by equation (4), provides accurate flux surface estimates near the plasma boundary.

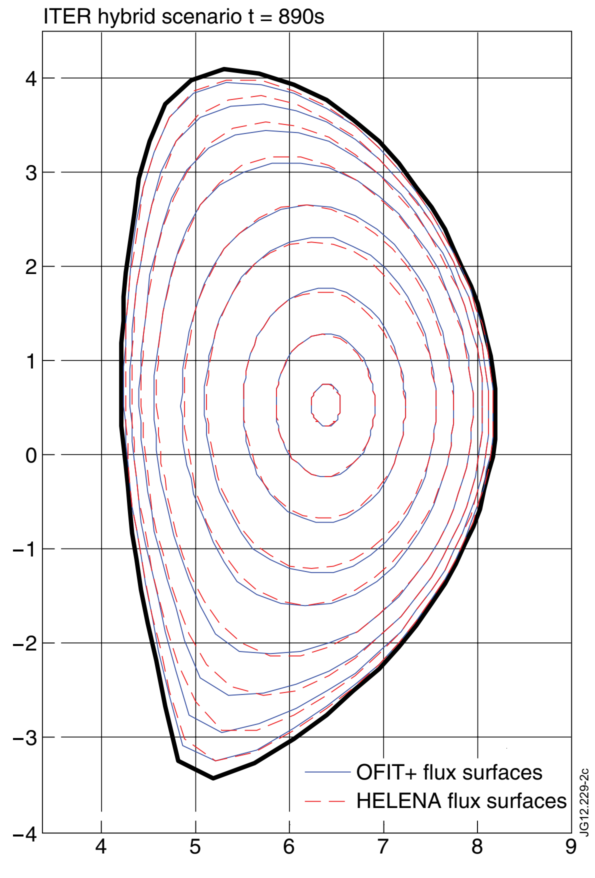

Figure 2: Comparison of flux surfaces estimated by OFIT+ and calculated by HELENA. The average displacement, expressed in the normalized minor radius is 0.59%, the maximum is 2.8%.

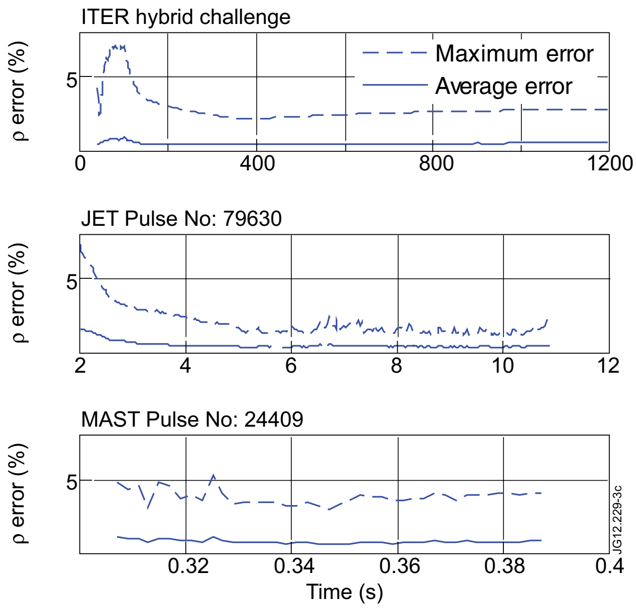

Figure 3: Average and maximum traces of error between OFIT+ and HELENA flux surfaces, for plasma discharges in ITER, JET and MAST.

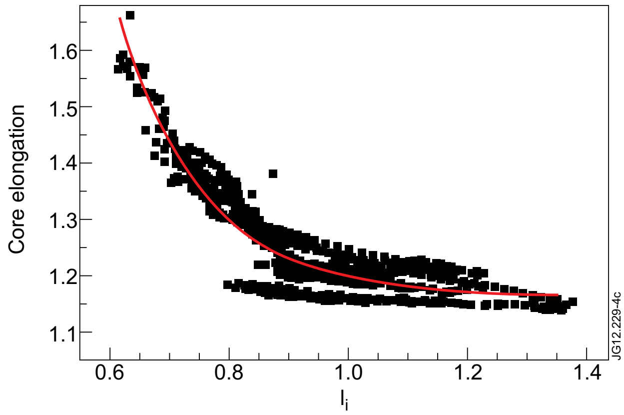

Figure 4: Correlation of core elongation with l_i for JET Hybrid regime.

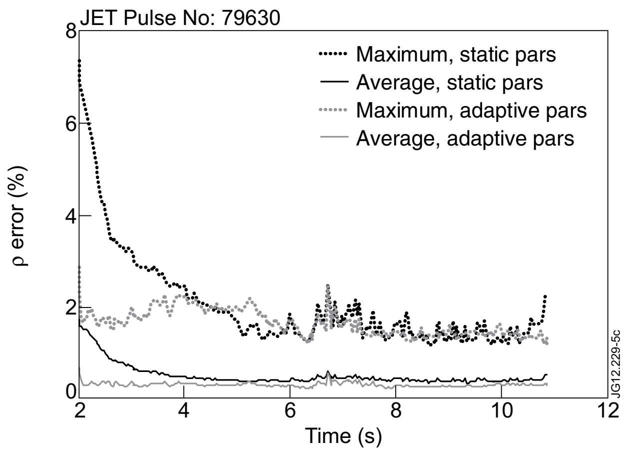

Figure 5: Error of flux surface estimates using adaptive shape parameters, compared to estimation using static parameters.

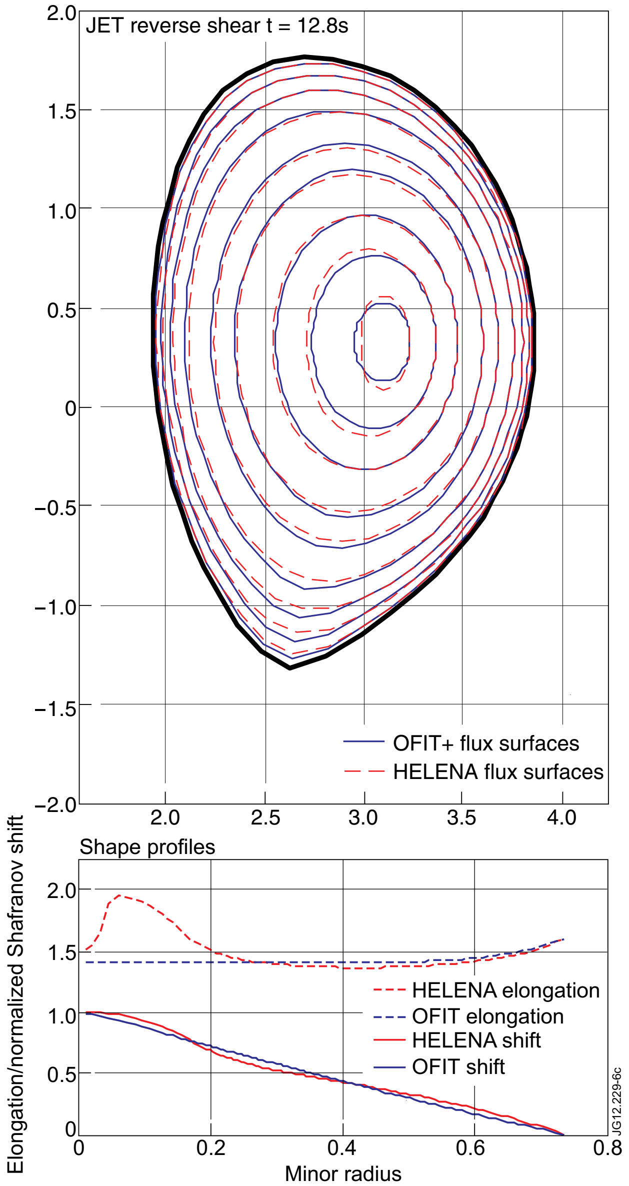

Figure 6: (top) Flux surfaces and (bottom) shape profiles in a JET reverse shear discharge, showing maximum and average errors in the OFIT+estimated flux surfaces of 7.1% and 1.6% of the minor radius.

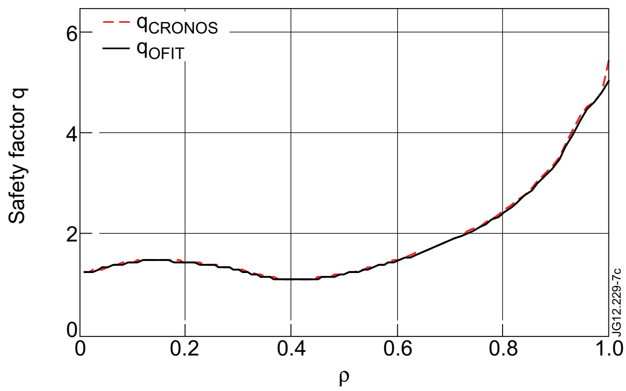

Figure 7: q-profile in a hybrid ITER discharge, calculated using flux surfaces and pitch angles from a CRONOS simulation. The safety factor profile from CRONOS is shown for comparison. The average error in q is 1.2%, the maximum error at the plasma edge is 8.3%. The maximum error for <0.95 is 1.9%.

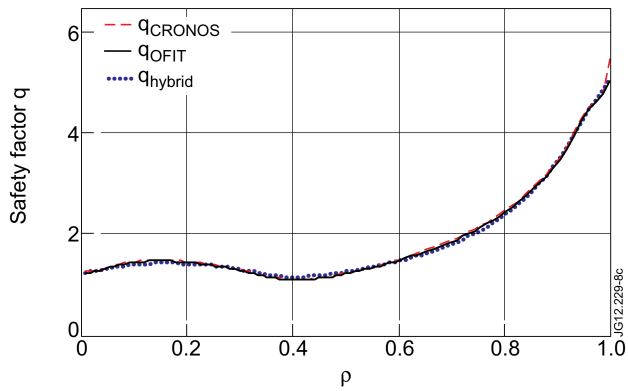

Figure 8: q-profiles from CRONOS (red), calculated using HELENA flux surfaces (black) and calculated using OFIT+ estimated flux surfaces (blue). The error in q obtained using the OFIT+ estimated flux surfaces is 2.7% average. The maximum error at the plasma edge is 7.3%. The maximum error for <0.95 is 4.7%.

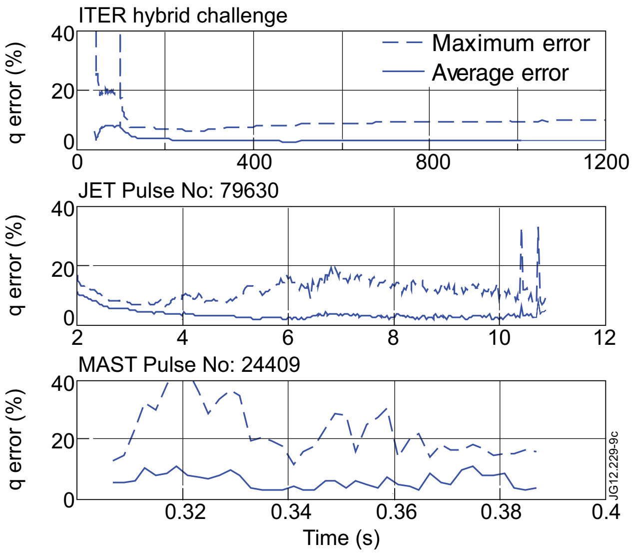

Figure 9: Average and maximum traces of error between OFIT+ and CRONOS q-profiles, for plasma discharges in ITER, JET, and MAST.

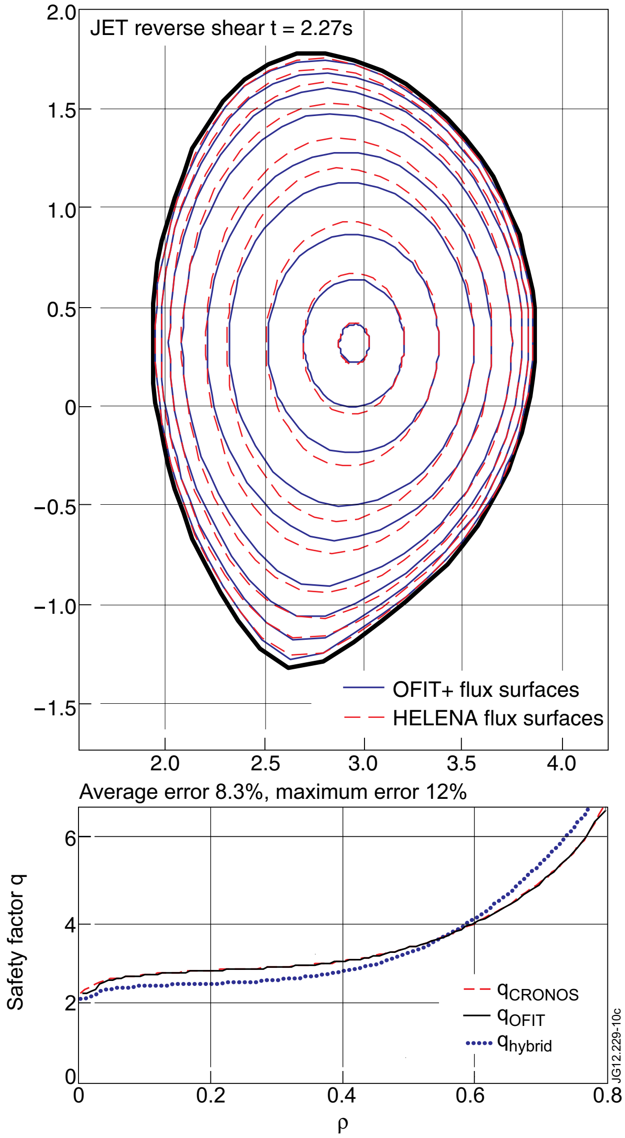

Figure 10: Flux surfaces and q-profiles during ramp-up of a JET discharge, showing larger errors due to the static flux surface shape parameters which are optimized for the flattop phase of the discharge.

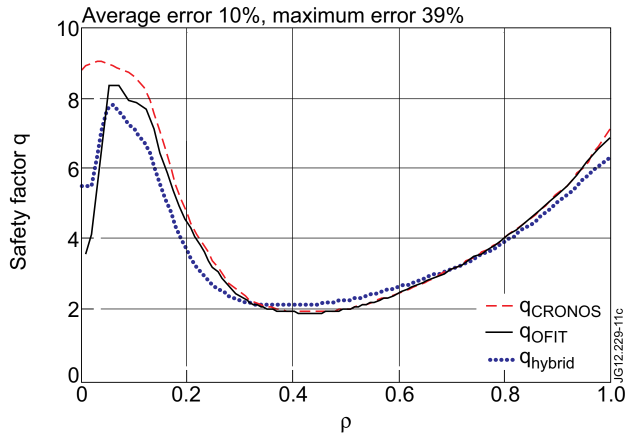

Figure 11: q-profiles of JET reverse shear discharge Pulse No: 535121, as shown in Figure 6.

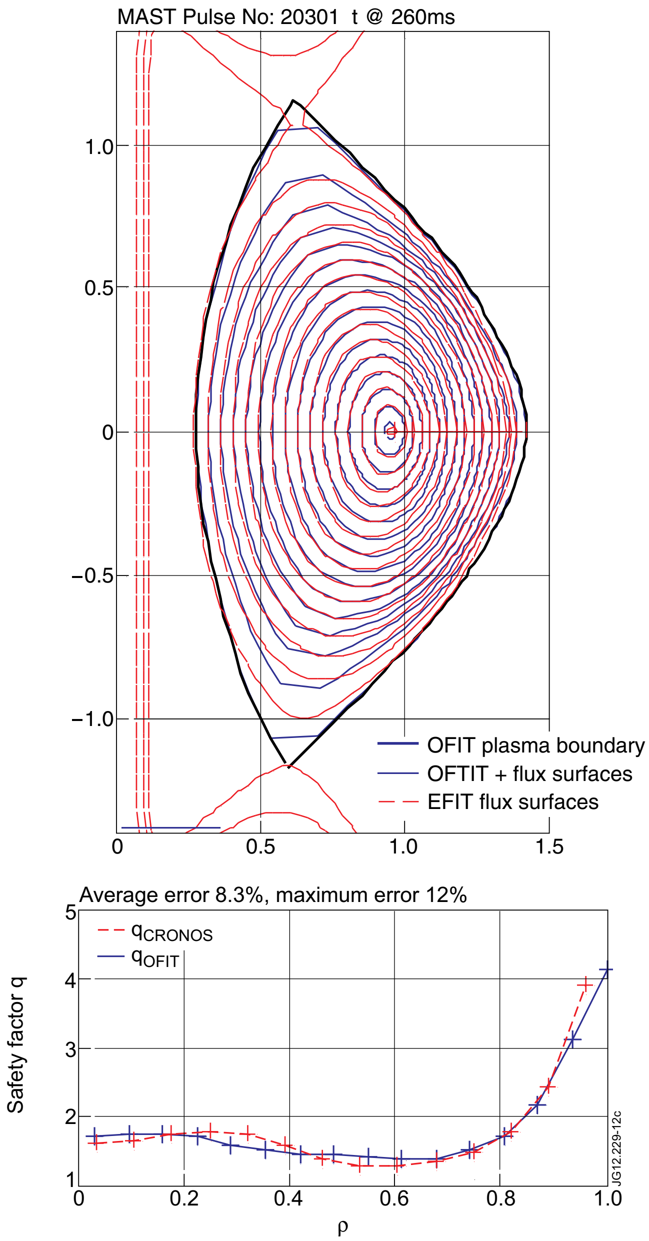

Figure 12: OFIT+ result obtained using an optically reconstructed plasma boundary and pitch angles measured using the MSE diagnostic. The flux surfaces and q-profile from the corresponding EFIT1 reconstruction is shown for comparison.

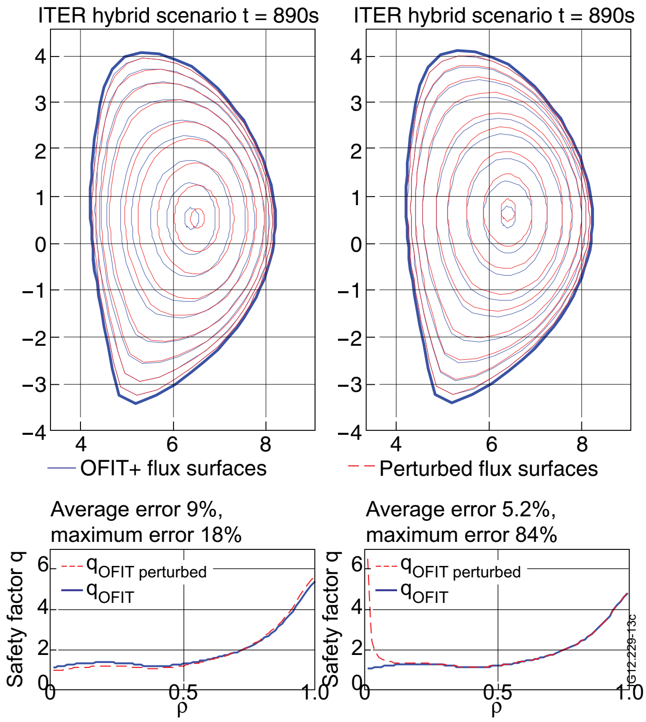

Figure 13: (left) Effect of a 2% (13cm) horizontal displacement in magnetic axis on calculated q-profile in ITER Hybrid discharge. (right) Effect of a 2% (16cm) vertical displacement in magnetic axis of the plasma height on calculated q-profile in ITER Hybrid discharge.

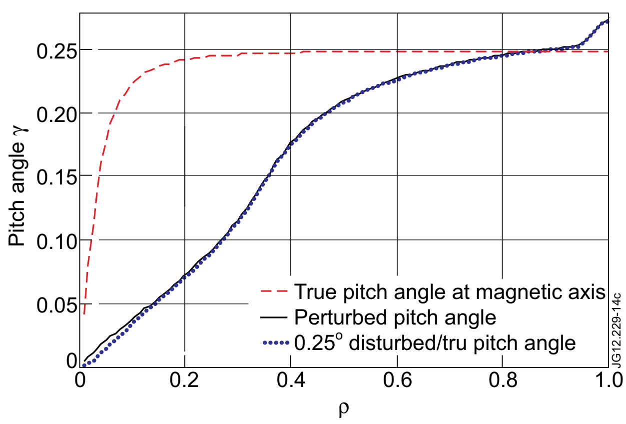

Figure 14: Effect of 2% vertically displaced magnetic axis on pitch angle measurement.

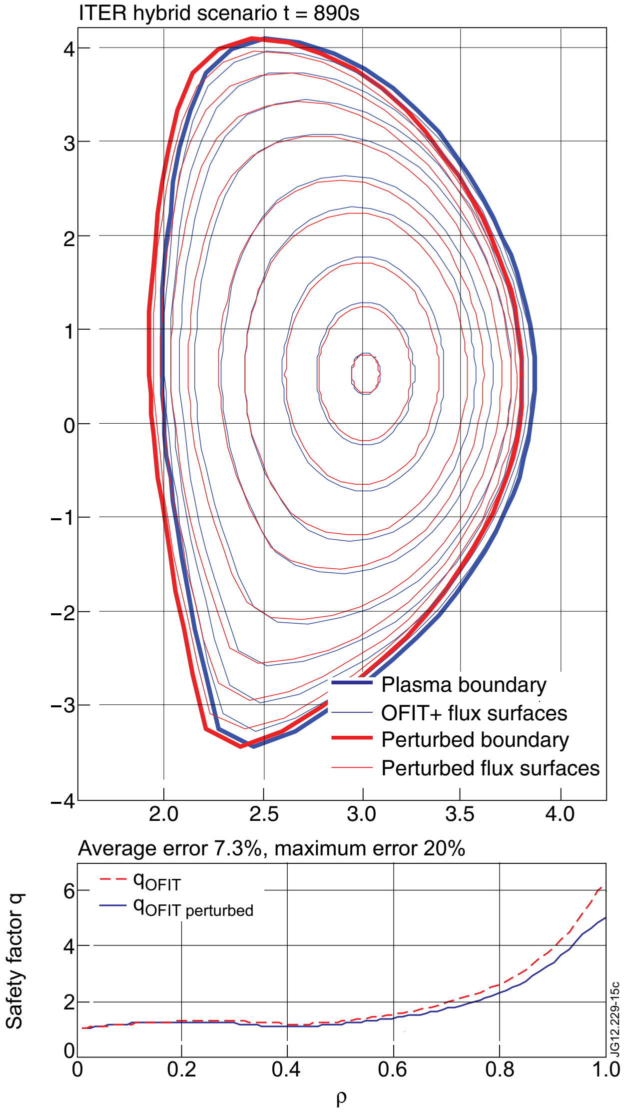

Figure 15: Effect of a 2% (13cm) displacement of the plasma boundary major radius on calculated q-profile in ITER Hybrid discharge.