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Analyses of Substantially Different Plasma Current Densities and Safety Factors Reconstructed from Magnetic Diagnostics Data

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F.S. Zaitsev[1], D.P. Kostomarov[1], E.P. Suchkov[1], R.J. Akers[2], V.V. Drozdov[2], E.R. Solano[3], A. Murari[4], S. Matejcik[5], M. De Bock[2], N.C. Hawkes[2], L.C. Appel[2] and JET EFDA contributors*

JET-EFDA, Culham Science Centre, OX14 3DB, Abingdon, UK

[1]Moscow State University, Moscow 119991, Russia
[2]EURATOM-CCFE Fusion Association, Culham Science Centre, OX14 3DB, Abingdon, OXON, UK
[3]Laboratorio Nacional de Fusion, Asociacion EURATOM-CIEMAT, 28040, Madrid, Spain
[4]Consorzio RFX-Associazione EURATOM ENEA per la Fusione, I-35127 Padova, Italy.
[5] Department of Experimental Physics, Comenius University, Slovakia
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ABSTRACT
The problem of plasma current density and safety factor reconstruction using magnetic field measurements is considered. In the traditional formulation, the problem is strongly ill-posed. In particular, substantially different current densities and safety factors can be equally well attributed to the same set of measurements. The paper presents an accurate mathematical formulation of the inverse problem and its variants. A numerical algorithm is given, which allows to find all substantially different solutions or to prove the absence of multiple solutions. Examples of very different current density and safety factor reconstructions for measurements with finite accuracy are presented. Cases of MAST, JET and ITER-like plasmas are considered. It is shown that including the Motional Stark Effect (MSE) measurements as constraints, provided the accuracy of MSE measurements is sufficient, allows identifying one solution. The approach of the paper can be used for a wide range of inverse problems in physics and help in selecting additional conditions, which can identify the most likely solution among several.

1. INTRODUCTION

The development of methods for the reconstruction of plasma characteristics inaccessible for direct measurements is an important direction of research in controlled fusion. Such methods allow obtaining valuable information about processes inside the plasma, comparing theoretical forecasts with real experiment, understanding plasma behavior and producing reliable control techniques.

In this paper the problem of toroidal plasma current density reconstruction is considered. It is known that this problem is strongly ill-posed [1-4]. For example, ref. [4] shows that substantially different current densities can be compatible with measurements and proposes a technique for finding these different densities. Nevertheless several methods and codes have been developed for equilibrium and current density reconstruction, such as EFIT [5] or SCoPE [2,3], which are commonly used in practice.

Typically, methods for current density reconstruction search for some solution of the inverse problem and do not address the question about the existence of other solutions. However, for the correct interpretation of a plasma discharge it is very important to find all substantially different solutions and then, using additional information, select the one appropriate to the real physical process under study. This paper is devoted namely to the analysis of this problem: the identification of all possible solutions and the selection of the most likely one.

With regard to the organization of the paper, section 2 presents an accurate mathematical formulation of the inverse problem for the reconstruction of the poloidal flux and the components of the toroidal current density. Some different alternatives are discussed. In the traditional formulation the problem reduces to finding three functions, which satisfy the elliptic equation with non-linear right-hand-side in the plasma and some additional conditions at the plasma boundary. The traditional formulation can be modified with different constraints. One of them is based on the Motional Stark Effect (MSE) measurements. Account of MSE gives additional information about the magnetic field inside the plasma and helps reducing the freedom in constructing a solution.

Section 3 describes a numerical method for determining all substantially different solutions of the inverse problem. The method is based on solving the set of direct problems constructed with the theory of ε-nets. The numerical solution is complicated by the non-circular form of the plasma cross-section, specific additional conditions and it is typically very heavy in terms of computational resources.

Examples of substantially different solutions for MAST, JET and ITER parameters are given in section 4. It is shown that the inclusion of the MSE constraints in the problem, provided the accuracy of the MSE measurements is sufficient, can allow identifying one solution among several.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The problem of toroidal axially symmetric plasma equilibrium reconstruction can be expressed in terms of the following equations

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right)+\frac{\partial^2\psi}{\partial Z^2}=-\mu_0 Rj_\eta,\quad (R,Z)\in S, \qquad (1)$$

$$j_\eta = R\frac{dp(\psi)}{d\psi}+\frac{1}{2\mu_0 R}\frac{dF^2(\psi)}{d\psi}, \qquad (2)$$

$$\psi\big|_{(R,Z)\in\Gamma}=0, \qquad (3)$$

$$\int_S j_\eta ds = I \neq 0, \qquad (4)$$

$$\left\|\frac{\partial\psi}{\partial\vec{n}}\bigg|_{(R,Z)\in\Gamma}-\Phi\right\|/\|\Phi\|\le\delta\text{, where, for example, }\|\Phi\|\equiv\max_{(R,Z)\in\Gamma}|\Phi|. \qquad (5)$$

Equation (1), (2) constitute the well known Grad-Shafranov equation, which is a two-dimensional elliptic equation with non-linear right-hand side. Here the notation of ref. [6] is used, where various symbols indicate: (R,η,Z) - the usual system of cylindrical coordinates with the Z-axis oriented along the tokamak axial symmetry axis; $\psi(R,Z)$ - the poloidal flux function, equal to the covariant component A_η of the vector potential of the magnetic field $\vec{B}=\vec{\nabla}\times\vec{A}$; Γ - the known, e.g. from optic measurements [2] or the solution of other problems, plasma boundary in the meridian section of the tokamak; S - the area bounded by Γ; I - the toroidal plasma current; j_η - the toroidal current density; $p(\psi)$ - the kinetic pressure; $F(\psi)$ - the poloidal current function, related to the toroidal magnetic field $B_{\text{tor}}=F/R$; $\vec{n}$ - the external normal unit vector with respect to S; Φ - the derivative at the plasma boundary Γ in the direction of vector, μ_0 - the vacuum magnetic permeability coefficient. $\vec{n}$. Φ can be expressed through the components of the poloidal magnetic field $\vec{B}_{\text{pol}}$

$$\frac{\partial\psi}{\partial\vec{n}}\bigg|_{(R,Z)\in\Gamma}=\pm\sqrt{\left(\frac{\partial\psi}{\partial R}\right)^2+\left(\frac{\partial\psi}{\partial Z}\right)^2}=\pm\frac{\sqrt{B_R^2+B_Z^2}}{R}=\frac{B_{\text{pol}}}{R},$$

and contains information about the external magnetic measurements. δ is related to the inaccuracy of the magnetic measurements and it is introduced in the problem because values of $\partial\psi/\partial\vec{n}$ can be found from experiments only with limited accuracy.

In a typical, though not the most general, direct problem, one function ψ is to be determined from known functions p and F using equations (1)-(4), preset plasma boundary Γ and total current I. In the case of the inverse problem, considered in this paper, a triplet (ψ,p,F) is to be found from conditions (1)-(5) subject to a given Γ, I and the normal derivative Φ with inaccuracy δ.

The normal derivative Φ can be determined either from the solution of the Grad-Shafranov equation outside the plasma using measured values of ψ a and $\vec{B}$ at some specific points or from the solution of the direct problem (1)-(4) for given p and F. The total toroidal current I can be expressed through the normal derivative Φ with a curvilinear integral using the Green formula

$$I = \int_S j_\eta ds = -\frac{1}{\mu_0}\int_\Gamma \frac{\Phi}{R} dl.$$

So equation (4) also gives a normalization for Φ.

Here we consider a relatively novel inverse problem, which consists of finding all substantially different solutions (ψ_k, p_k, F_k) satisfying conditions (1)-(5), given the inaccuracy of the available measurements δ. The meaning of "substantially different" depends on the particular plasma characteristics of interest. Usually noticeably different qualitative and quantitative behaviors of the toroidal current density j_η or safety factor q, especially near the magnetic axis, are considered.

One of the most interesting applications of the developed method is a somewhat simplified variant of the considered inverse problem, when one of its solutions (ψ^*, p^*, F^*) is known together with $\partial\psi^*/\partial\vec{n}$, for example from an equilibrium reconstruction code, such as SCoPE or EFIT. In this case the problem reduces to finding all substantially different to (ψ^*, p^*, F^*) solutions (ψ_k, p_k, F_k), which satisfy conditions (1)-(5).

For the extraction of the unique triplet (ψ_k, p_k, F_k) in case of the existence of substantially different solutions, it is necessary to add to equations (1)-(5) other constraints provided by some additional data. A number of papers, for example [7,8], indicate that important additional information about the magnetic field inside the plasma can be obtained from Motional Stark Effect (MSE) measurements. Such measurements allow determining the angle χ_{MSE}, relative to the toroidal B_{tor} and the vertical B_{Z} components of the magnetic field, at N_{MSE} points (R_i, Z_i) inside the plasma with accuracy δ_{MSE}. Appropriate constraints to the problem (1)-(5) have the form

$$\left\| \chi(R_i, Z_i) - \chi_{\mathrm{MSE}}(R_i, Z_i) \right\| \le \delta_{\mathrm{MSE}}, \quad i = 1, ..., N_{\mathrm{MSE}}, \text{ where} \tag{6}$$

$$\chi(R,Z) = \arctan\left(\frac{B_{\mathrm{Z}}(R,Z)}{B_{\mathrm{tor}}(R,Z)}\right), \quad B_{\mathrm{tor}} = \frac{F}{R}, \quad B_{\mathrm{Z}} = \frac{1}{R}\frac{\partial\psi}{\partial R}.$$

An important element in the formulation of the problem is choosing the class of functions p and F, from which a solution (ψ, p, F) is searched. It is desirable to narrow the class of functions sought for as much as possible. We apply the approach normally used for solution of the direct problem (1)-(3). In the direct problem functions p and F are considered as input parameters and should be preset. The main interest in the direct problem is to find ψ appropriate to the fixed ranges of $p(\psi)$ and $F(\psi)$ values. In this case it is necessary to consider functions $p(\rho(\psi))$ and $F(\rho(\psi))$ *, in which ρ always runs over all given values, e.g. the segment [0,1], for any bounded ψ. Otherwise p and F may not fall in the required range, since the values of ψ become known only after the solution of the direct problem (1)-(3). Assuming ψ to be bounded and non-negative $\psi \ge 0$ the simplest form of ρ convenient for differentiation is

* Here the new functions $p(\rho(\psi))$ and $F(\rho(\psi))$ are denoted with the same letters p and F, already used for functions $p(\psi)$ and $F(\psi)$, since further it does not lead to collisions of notations.

$$\rho(R,Z)=\frac{\max\limits_{(R,Z)\in S}\psi-\psi}{\max\limits_{(R,Z)\in S}\psi},\quad \rho\in[0,1] \text{ in S.} \tag{7}$$

Thus usually, in the direct problem (1)-(3), the functions $p(\rho(\psi))$ and $F(\rho(\psi))$ are considered, because their range is known beforehand, since $\rho\in[0,1]$. The geometrical interpretation of the reformulation of $p(\psi)$ and $F(\psi)$ in terms of $p(\rho(\psi))$ and $F(\rho(\psi))$ is that it consists of a transition from setting p and F as functions of ψ level lines with unknown numeration to functions of ψ with the known in advance level lines numeration $\rho(\psi)\in[0,1]$. The substitution represented by (7) does not change the differential properties of the problem, but the considered class of functions p and F is restricted, in particular functions growing to infinity with $|\psi|$ are discarded.

Functions p and F in case of $p(\rho(\psi))$ and $F(\rho(\psi))$ become invariant with respect to the ψ normalization, i.e. for any constant $C>0$ we have $p(\rho(\psi))=p(\rho(C\psi))$ and $F(\rho(\psi))=F(\rho(C\psi))$. Therefore the free parameter $\lambda\equiv\left(\max_{(R,Z)\in S}\psi-\min_{(R,Z)\in S}\psi\right)^{-2}$ appears in equation (1) of the direct problem (1)-(3), which is usually chosen from the condition (4) of given total current I.

In the inverse problem, by analogy with the direct one, we will search functions p and F from the class $p(\rho(\psi))$ and $F(\rho(\psi))$ with $\rho\in[0,1]$ (7), and function ψ from the class of bounded non-negative functions. More precisely, we consider the problem (1)-(5) or (1)-(6) with additional equation (7), used only for functions p and F in the right hand side (2) of equation (1).

Up to now, the question about the uniqueness of the solution of the inverse problem (1)-(5) with $\delta=0$ is studied analytically only in some particular cases [9]. The answer depends on the form of the area S and the form of the right-hand side (2) in (1). It is shown that the problem can have either one or several solutions. The areas S and functions p, F, Φ and $\delta\neq 0$ met in practice, require numerical solution of the problem (1)-(5).

3. Numerical method for construction of substantially different solutions

We rewrite relation (2), the right-hand side of (1), in terms of the normalized flux (7). Equations (1)-(3) become

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right)+\frac{\partial^2\psi}{\partial Z^2}=\frac{d\rho}{d\psi}\left(-\mu_0R^2\frac{dp}{d\rho}-\frac{1}{2}\frac{dF^2}{d\rho}\right), \tag{8}$$

$$\psi|_{\Gamma}=0, \tag{9}$$

$$\rho(R,Z)=\frac{\max\limits_{(R,Z)\in S}\psi-\psi}{\max\limits_{(R,Z)\in S}\psi-\min\limits_{(R,Z)\in S}\psi}. \tag{10}$$

Due to absence of the restriction in the sign of ψ in the algorithm, the definition (10) contains $\min\limits_{(R,Z)\in S}\psi$.

Equation (5) gives

$$\frac{d\rho}{d\psi} = \frac{I}{\int_S \left(R\frac{dp}{d\rho} + \frac{1}{2\mu_0 R}\frac{dF^2}{d\rho} \right) ds}. \tag{11}$$

By substituting (11) in (8) we get an equation for ψ, given the total current I.

Note that, assuming convexity and non-negativity of ψ (in this case $\rho = 1$ is satisfied at the plasma boundary) the direct problem (8)-(10) in terms of $\overline{\rho} = 1 - \rho$ becomes an eigenvalue problem for $\lambda \equiv \left(\max_{(R,Z)\in S}\psi - \min_{(R,Z)\in S}\psi \right)^{-2}$

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \overline{\rho}}{\partial R} \right) + \frac{\partial^2 \overline{\rho}}{\partial Z^2} = \lambda \left(-\mu_0 R^2 \frac{dp}{d\overline{\rho}} - \frac{1}{2}\frac{dF^2}{d\overline{\rho}} \right), \tag{12}$$

$$\overline{\rho}\big|_\Gamma = 0. \tag{13}$$

Substituting $d\rho / d\psi = -\sqrt{\lambda}$ in equation (11), one can conclude that the set of possible λ determines the set of allowable total currents I for given $p(\rho)$ and $F(\rho)$. However, for typical practical direct problems, not all but only one value of λ is required, which provides the condition of the given total current (11).

The numerical method proposed below does not require neither convexity nor non-negativity of ψ, since at each iteration the operator at the left-hand-side of equation (8) is inverted with the boundary condition (9) and the condition $\rho = 1$ at the plasma boundary is not used. Function ρ is needed for setting up the right-hand-side of equation (8) through ψ obtained in the previous iteration.

Assume that functions $dp / d\rho$ and $dF^2 / d\rho$ can be presented as polynomials in ρ

$$-\mu_0 \frac{dp}{d\rho} = \sum_{i=0}^{m} \alpha_i \rho^i, \quad -\frac{1}{2}\frac{dF^2}{d\rho} = \sum_{i=0}^{m} \beta_i \rho^i . \tag{14}$$

The method for determining all substantially different solutions is based on special enumerative technique for values of coefficients α_i and β_i in equation (14).

The right-hand side of equation (8) with fixed coefficients α_i and β_i becomes known and provides the possibility of solving the direct problem (8)-(11). We search for the numerical solution by iterations over $s = 1, 2, ...$ analogously to [10]

$$\Lambda \psi^{(s)} = \alpha^{(s-1)} (R^2 \sum_{i=0}^{m} \alpha_i (\rho^{(s-1)})^i + \sum_{i=0}^{n} \beta_i (\rho^{(s-1)})^i),$$

$$\alpha^{(s-1)} = -\mu_0 I / \int_S (R \sum_{i=0}^{m} \alpha_i (\rho^{(s-1)})^i + \frac{1}{R}\sum_{i=0}^{n} \beta_i (\rho^{(s-1)})^i) ds.$$

Here Λ is the difference operator, which approximates the left hand side of (8) taking account of the boundary condition (9). The coefficient α ensures validity of (4) for $\psi = \psi^{(s-1)}$.

The values of ρ are calculated with $\psi^{(s-1)}$ according to (10). However, when determining $\rho^{(s-1)}$, one should keep in mind that unfortunate initial approximations, peculiarities of the iteration process, finite accuracy of computer arithmetic operations and etc, can change $\psi^{(s-1)}$ during the calculations. In order to always have $\rho^{(s-1)}=0$ in equations (14) appropriate to the magnetic axis, i.e. to ψ extremum in plasma (excluding the boundary), we set $\rho^{(s-1)}=\rho$ using (10) when ψ extremum in plasma is non-negative and $\rho^{(s-1)}=1-\rho$ in the opposite case.

A modern effective method for solution of this discrete problem is presented in the book [2], for example. Iterations continue until a steady state or achievement of some maximum number. If the steady state solution exists, we denote it as ψ^{∞}. Inequality (5) is checked for ψ^{∞}. If it is valid then $\psi=\psi^{\infty}$ is taken as the solution of the problem (1)-(5), (10), (14). While selecting solutions, it is possible to check additional constraints, such as $\psi\geq 0$, or (6) or fit to some required pressure p range, etc. with the aim to narrow the set of different solutions (ψ, p, F).

Thus finding all substantially different solutions of the problem (1)-(5), (7) or (1)-(7) reduces to an accurate overhaul of the values of coefficients in polynomials (14). This can be done by different methods. One is based on usage of the ε-net of the finite number of polynomials [11,9], which cover a priori defined sufficiently broad class of functions $p(\rho(\psi))$ and $F(\rho(\psi))$ with given absolute accuracy ε. If the number of polynomials in the ε-net is large then only a subset from a given layer can be considered. The details of the ε-net construction are out of the scope of this paper. Solutions (ψ_k, p_k, F_k) appropriate to the elements of the ε-net give different solutions of the inverse problem (1)-(7). Using one or other criteria one can select substantially different ones from (ψ_k, p_k, F_k).

The numerical algorithm is implemented in the code SDSS (Substantially Different Solutions Searcher) in Fortran 95. The code has special graphic interface, written in JAVA, to help setting up the input data, constructing ε-nets, visualizing and analyzing the results.

4. Examples of substantially different solutions

The solution of the direct problem (1)-(4) with MAST-like plasma parameters has been considered. MAST parameters close to pulse number 9037 have been chosen: elongation 1.7, minor radius 0.5 m, magnetic axis at $R=$ 0.7 m, total toroidal current I = 560 kA, $B_{\text{mag.ax.}}=0.52$ T. The functions $dp^*/d\rho$ and $d(F^*)^2/d\rho$ have been calculated with code SCoPE [2,3,6] and presented with polynomials of the 2-d order for $dp^*/d\rho$ and 3-d order for $d(F^*)^2/d\rho$ with ≈ 1% accuracy, see dashed curves in figure 1, right. The normal derivative Φ was calculated using ψ^*. The inverse problem (1)-(5), (7) has been considered initially. We searched for solutions substantially different from (ψ^*, p^*, F^*) in the sense of noticeable deviations in toroidal current density (2) and safety factor q.

Two ε-nets with about 10^7 polynomials in each have been constructed. Selecting polynomials from the ± 30 % belts around $dp^*/d\rho$ and $d(F^*)^2/d\rho$, we are left with 6000 variants of the right hand sides (2). The method, described in the previous section, gives several hundred solutions (ψ_k, p_k, F_k) of the inverse problem (1)-(5), (7) satisfying inequality (5) with inaccuracy δ < 2.5%. Figure 1 illustrates the given solution (ψ^*, p^*, F^*) (dashed) and an obtained one (solid). It is clear that the various solutions present not just quantitative (up to 40%) but also qualitative differences. In addition to the SCoPE solution with a hollow current density and non-monotonic safety factor profile, the inverse problem has another solution with a non-hollow current density and monotonic q profile. The fluxes ψ in the two alternatives differ by less than 5%. However, the two reconstructions have completely different interpretations of the pulse and therefore should be identified and analyzed.

It is important to note that a very similar result has been obtained for the inverse problem with fixed plasma pressure $p = p^*$.

Inclusion of MSE constraint (6) appropriate to (ψ^*, p^*, F^*) in the problem (1)-(5), (7) with inaccuracy $\delta_{\text{MSE}} < 1.5^0$ of measurements in MAST allows rejecting all substantially different solutions found for the inverse problem (1)-(5), (7). By increasing the inaccuracy δ_{MSE}, noticeably differing current densities and safety factors appear as solutions of the problem (1)-(7). The results also depend on the number of MSE measurements and the positions of these measurements.

One should note, that the inaccuracy δ_{MSE} of MAST MSE diagnostics can be less than 0.5^0 [13]. Measurements of high quality naturally help a lot in filtering out “false” solutions.

A similar study has been done for JET-like plasmas. Pulse 58837 has been considered: elongation 1.7, minor radius 0.85 m, magnetic axis at $R =$ 3.05 m, total toroidal current I = 2.37 MA, $B_{\text{mag.ax.}} = 2.4$ T. Over one hundred solutions of the inverse problem (1)-(5), (7) have been found using ε-net technique. Figure 2 shows the solution (ψ^*, p^*, F^*) (dashed) produced by the code EFIT and an obtained substantially different one with δ < 8 % (solid). Again large qualitative differences in current density and safety factor are present. Moreover, a solution with $q > 1$ at the magnetic axis exists along with EFIT reconstructed one with $q < 1$. However, taking into account the MSE constraints with $\delta_{\text{MSE}} < 0.3^0$, achieved in JET measurements, eliminates the solutions different from the original one (ψ^*, p^*, F^*), which was produced with EFIT using MSE and polarimetry constraints.

Finally ITER scenario 4 - like [12] has been considered with elongation 2, minor radius 1.8m, magnetic axis at $R =$ 6.4 m, total toroidal current I = 9 MA, $B_{\text{mag.ax.}} = 5.3$. About three hundred differing solutions of the inverse problem (1)-(5), (7) have been found using ε-net technique for a ± 20 % belts around $dp^*/d\rho$ and $d(F^*)^2/d\rho$ of the target solution (ψ^*, p^*, F^*) and δ < 8 %. Figure 3 shows the original solution (ψ^*, p^*, F^*) (dashed) and a substantially different one (solid) obtained from the inverse problem. One can see over 20 % difference in current density and safety factor. A more striking difference in j_η and q with δ < 5 % was found by increasing the width of the belts around $dp^*/d\rho$ and $d(F^*)^2/d\rho$ (thin curves). However, the inverse problem with the MSE constraint (6) and $\delta_{\text{MSE}} = 0.3^0$ leave no solutions substantially different from (ψ^*, p^*, F^*).

Conclusions

The results of this work confirm the fundamental statement that finding one solution is not sufficient for some ill-posed inverse problems and that it is essential to explore the existence of other substantially different solutions, which also satisfy the relations and constraints of the problem. The presented approach, based on the theory of ε-nets, can be used in practice for the determination of the existence or absence of substantially different solutions. The approach can be applied also to other inverse problems of controlled fusion. It can also be used for theoretical purposes to verify numerically the existence of at least one solution of the inverse problem (1)-(7).

The formulation of the constraints, required for obtaining the most realistic solutions of the inverse problem for current density and safety factor reconstruction, is an unresolved theoretical issue. The possible existence of multiple, substantially different solutions, as shown in this paper, indicates the importance of being able to identify all of them. To avoid faulty interpretation of experiments, caused by possibly very different reconstructions, it is advisable to supplement equilibrium reconstruction codes with a module for searching for all substantially different solutions. Some previous results may require reexamination for studying the appropriateness of reconstructed current density and safety factor.

The presented study suggests the following strategy for equilibrium reconstruction. Initially a standard code, such as EFIT or SCoPE, can be used for finding a solution (ψ^*, p^*, F^*) of the inverse problem. Then the problem (1)-(5), (7) should be solved, with for example the technique of section 3, to verify of the existence of reconstructions (ψ_k, p_k, F_k) substantially different from (ψ^*, p^*, F^*). If these do exist then additional constraints, such as MSE (6), should be applied for reducing the number of solutions of equations (1)-(5), (7). If the additional constraints leave only one solution (ψ, p, F), which may not necessarily coincide with (ψ^*, p^*, F^*), then it is possible to state, that with the given inaccuracy ε in the given class of functions, reconstructions substantially different from (ψ, p, F) do not exist.

Such studies on existence of substantially different solutions should be done for every plasma pulse under consideration, since no general theorems on the subject are available yet. However the results of the paper indicate that MSE constraint (6) can help much in extracting one solution among several different ones. A more accurate formula for χ can be used in (6) for the enhanced analyses of real plasma pulses [13]

It is important to note that the technique of section 3 allows evaluation of the maximum possible error in the MSE measurements for choosing one solution among substantially different candidates.

The presented study reveals, that the inverse problem solvers, which allow using high order polynomial expansions for $dp/d\rho$ and $dF^2/d\rho$ or do not use polynomial expansions at all, such as code SCoPE [2,14], are advantageous for current density and safety factor reconstruction in complicated realistic scenarios, as in ITER for example, where $dp/d\rho$ and $dF^2/d\rho$ cannot be accurately approximated with low order polynomials.

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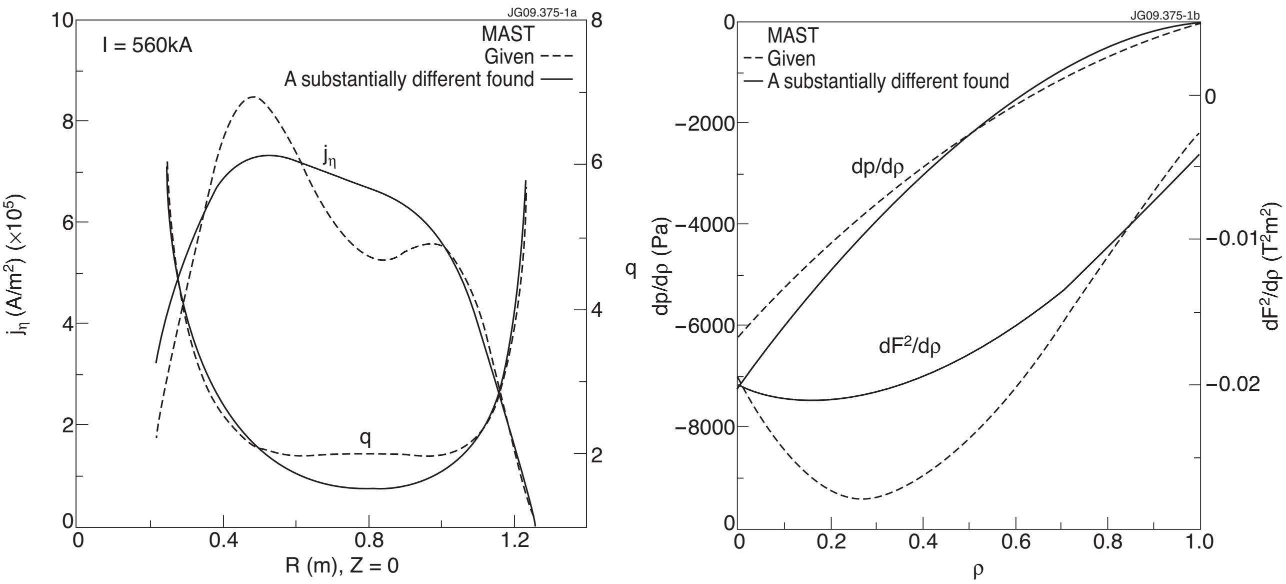

Figure 1: MAST-like plasmas. Left: current density in plane Z=0: dashed - given, solid – found. Right: components of–j_η: dashed - given, solid - found

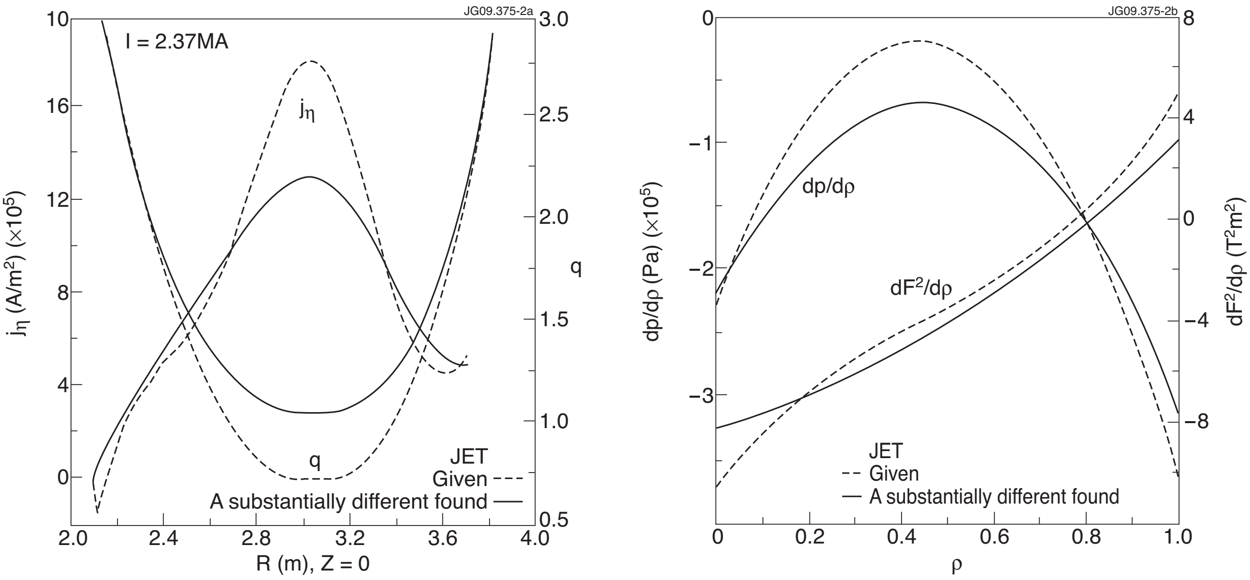

Figure 2: JET-like plasmas. Left: current density in plane Z=0: dashed - given, solid – found. Right: components of– j_η: dashed - given, solid - found.

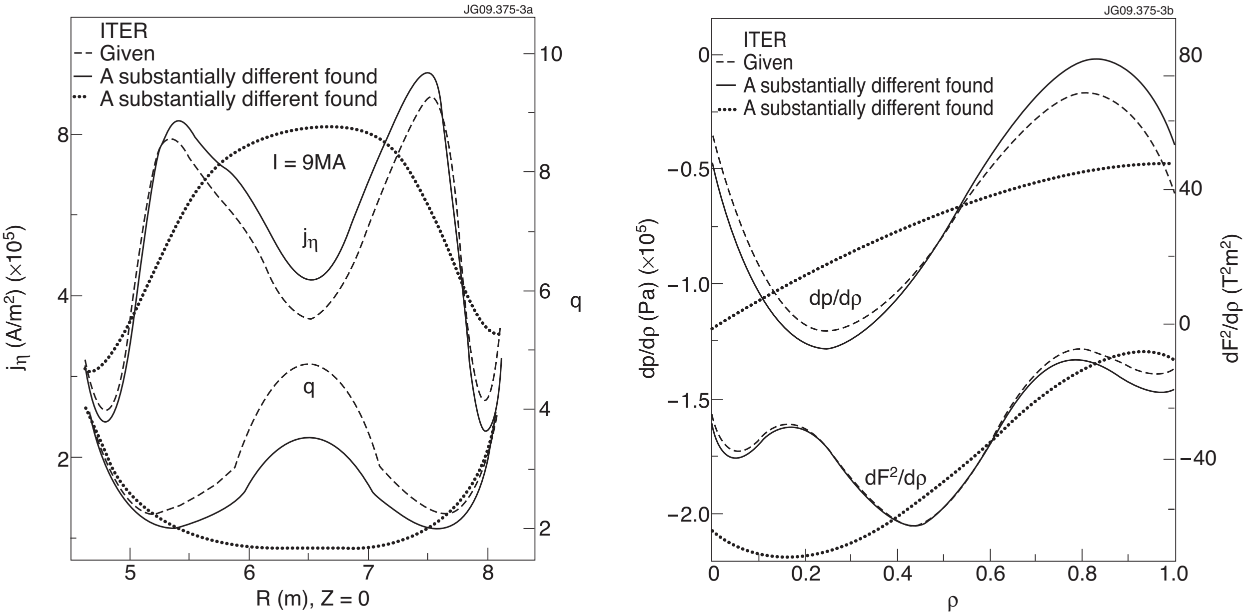

Figure 3: ITER-like plasmas. Left: current density in plane Z=0: dashed - given, solid – found. Right: components of– j_η: dashed - given, solid - found.