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ABSTRACT
In practically all fields of science the measurements are affected by noise, which can sometimes be modelled with an appropriate probability distribution function. The results of the measurements are therefore known only within a certain interval of confidence. In many cases the noise sources is independent from the system to be studied and the quantities to be measured. In this paper, a simple approach to handle statistical uncertainties, due to an independent noise source, in a Fuzzy Logic System is developed. The theoretical analysis and various tests with a benchmark show how the statistical error bars can be interpreted as an independent "axis of complexity" with respect to the fuzzy boundaries of the membership functions. The statistical confidence intervals in the inputs can be transferred to the output and handled separately from the system intrinsic fuzzyness. The main advantages of this independent treatment of the measurement errors are shown in the case of a binary classification task: the regime confinement identification in high temperature Tokamak plasmas. Significant improvements in the correct prediction rate have been achieved with respect to the classification performed without considering the error bars in the measurements.

1. INTRODUCTION
Fuzzy logic is a very powerful conceptual framework [1] to model complex systems, which present a level of uncertainty too high to be managed with traditional boolean logic. After proving very successful in a series of engineering applications [2], in recent years this form of multivariate logic has become increasingly accepted also in the scientific domain. Its potential has become quite clear even in Magnetic Confinement Nuclear Fusion, where it has provided very competitive results in various fields, ranging from prediction to identification and even physical modelling and interpretation. With regard to the first point, disruption predictors based on Fuzzy Logic have been developed using a wide database of JET discharges [3], improving both the rate of success and the understanding of the phenomenon. In terms of identification, the determination of the plasma confinement regime, the fact whether the plasma is in the L or H mode of confinement [4] has been a particularly interesting application; in this case fuzzy logic identifiers have been compared with all the other major alternatives on a Joint European Torus (JET) database and have provided very competitive results [5]. The L-H transition has also been analysed by generating a Fuzzy Logic System automatically on the basis of Classification Trees built with the CART software package [6] and the resulting rules give significant intuitive understanding of this auto-organisation process [7].

On the other hand in Nuclear Fusion, and more generally in natural sciences, the data available is often affected by significant error bars, which have to be properly taken into account to provide accurate confidence intervals in the derived results. These measurement uncertainties are typically expressed in probabilistic terms and are therefore not naturally handled by the Fuzzy Logic intellectual framework. Indeed Fuzzy Logic is a sort of sophisticated multivariate logic based on sets with fuzzy boundaries and is therefore meant to be used to model systems, which are inherently characterised by these fuzzy boundaries between various states or configurations. This type of complexity is conceptually different from the errors induced by the measurement process, which introduces probabilistic uncertainties in the inputs and are usually quantified in statistical terms. Since the sources of noise can be very often assumed to be independent from the phenomena to be investigated, in this paper it is shown that the statistical uncertainties can be handled separately from the inherent fuzzy sets used to model the system under study. In this way the effects of the measurement errors can be isolated and given a specific treatment, while the power and richness of the fuzzy treatment are fully preserved. In the next section 2, the effects of the uncertainties in the measurements on the fuzzy inference process are analysed in the case of some simple but basic cases using a typical benchmark system found in the literature. A brute force approach, a sort of Monte Carlo statistical calculation, is compared with some approximate theoretical treatments, which allow reducing significantly the computational efforts but unfortunately do no provide completely satisfactory performance. In the following section 3, the same theoretical methods to quantify the effects of the measurement errors are applied to a classification problem in Nuclear Fusion, the confinement regime identification in Tokamak plasmas using signals of JET database [8]. Further developments of the approach and potential interesting new applications are then reviewed the last section.

2. THE STATISTICAL UNCERTAINTIES DUE TO THE MEASUREMENTS AS AN INDEPENDENT AXIS OF COMPLEXITY

In all the scientific domains the performed measurements are not perfect and they are always known within an interval of confidence. Therefore any form of inference process must be capable of handling this problem and produce results with associated probabilities. The main cause of uncertainty is normally some sort of noise, considered additive to the otherwise ideal output of the measuring devices. The real nature of this noise is not relevant for the discussions of this paper, provided it is uncorrelated with the quantities of the system to be measured.

In the case of complex physical phenomena modelled with FLSs, the fuzzy logic inference process must be adequately upgraded to cope with the statistical uncertainties due to the measurement noise. To clarify this aspect, let's take as an example the height of people. In some contexts, like for example for the definition of a suitable diet, it can be inappropriate to divide people in the two simple categories of tall and small (see figure 1a) and a more sophisticated classification may be required.

A possible membership function with fuzzy boundaries could be the one represented in figure 1b. This more involved membership is adopted to take into account the inherent complexity of the problem at hand and it does not have anything to do with the uncertainties in the measurements. Even in the case of ideal, perfect measurements of people height, a modelling with fuzzy sets could be required. In the treatment adopted in this paper, the errors in the measurements introduce another dimension of complexity. Not only any input belongs to various sets with a different degree of membership, but also that degree of membership is not absolute but must be expressed by a probability. This additional dimension in the complexity of the problem is shown pictorially in figure 2, where two axes represent the traditional fuzzy boundaries between sets and the third one the probabilistic distribution of the membership values due to the measurement errors.

The error bars in the measurements can affect the operation of a FLS in different ways depending on the form of the membership functions and the actual values of the measurements. The three fundamental possibilities are reported in figure 3. In certain eventualities, the uncertainty due to the measurement noise can be such that the input remains in a flat region of the membership function and therefore the output of the FLS is the same as if the noise was not present (case 1 of figure 3). A second situation is encountered when the noise can bring the input in a region where the same membership function is involved but with a different value (case 2 in figure 3).

This of course would change the output of the FLS in a way which depends on the details of mfs and the fuzzy rules. A last alternative arises when the noise combines with the form of the mfs in such a way as to even involve other membership functions (case 3 in figure 3), which also affects the output of the FLS.

To illustrate the impact of the measurement errors on FLSs, a simple fuzzy system well known in the literature has been used as a benchmark [9]. This FLS was developed to simulate the tipping habits of customers in a restaurant. Only two quantities, the quality of food and service, are supposed to have an influence on the customer tip, which is the output variable. The mfs of the two inputs and the single output are shown in figure 4.

The fuzzy rules implemented are:

1. If service is poor then tip is cheap
2. If service is excellent then tip is generous
3. If service is good and food is rancid then tip is cheap
4. If service is good and food is delicious then tip is generous
5. If service is good and food is excellent then tip is generous

To confirm the effects of the input uncertainty intervals summarised in figure 3, a systematic investigation of this simple model has been carried out. Various confidence intervals of different amplitude have been considered first for the single inputs and then for both of them. The expected results have been confirmed and an illustrative example is reported in figure 5. In this case an uncertainty interval of 15 %, simulating the measurement errors, has been assumed for both inputs, food and service. In figure 5a) the difference between the outputs of the FLS with and without the errors on the inputs is reported. For the inputs affected by noise, the central point and the two extremes of the error bars have been calculated with the FLS; the derived outputs have then be averaged and the difference between this value and the one obtained by the FLS without added noise has been calculated (and this difference is what is plotted in figure 5a). In figure 5b), the output variances due to the errors in the input are summarised. Inspection of figure 5 confirms that the output of the FLS is affected only when the uncertainties in the inputs induce their values to span regions covered by different mfs or where the mfs have different values.

In many cases typical of the exact sciences, even if the noise cannot be reduced at will, it is very often possible to derive detailed information about its probability distribution function. A typical realistic assumption, for example, is for the noise to obey Gaussian statistics. In any case, this additional information in general can help to handle the impact of the noise on the operation of a FLS. To this end it must be remembered that, in the interpretation proposed in this paper, the statistic of the noise remains completely independent from the fuzzy mfs and the fuzzy rules. Therefore, if the probability of the actual inputs in their uncertainty intervals can be calculated, the corresponding distribution of the outputs can be derived numerically. Since the FLS can be interpreted as a nonlinear transfer function, the output calculated assuming perfect measurements in general does not coincide with the peak of the output probability distribution function. When this is the case, the value calculated without noise does not represent the most likely value of the output and can therefore be misleading.

As an illustrative example, this aspect has been investigated for the case of the benchmark previously described. For simplicity sake, the noise has been assumed to have a uniform probability distribution within the error bars but this does not affect the quality of the results in any significant way. For the cases discussed in the following, the two inputs, food and service, have considered known with an uncertainty of 40%. The probability of the FLS outputs has been tested numerically. The inputs have been scanned and for each individual value one hundred points have been selected, within the confidence interval, and then randomly chosen with equal probability. Therefore for each couple of inputs a statistics of ten thousand combinations has been analysed. The output values have been determined with the FLS and compared with the single value obtained neglecting the noise and assuming a unique, exact value for each input. In figure 6 the outputs for two different input combinations are shown. It is clear that in general the output of the FLS without noise can be very different from the most likely one, once the perturbation due to the noise is taken into account. In particular, the output of the top plot in figure 6 is generated for a service value of 2 and a food value of 1; in the case in the bottom plot of figure 6 the service and food values are 3 and 4 respectively. Analyzing figure 4, for the last set of inputs (shown in the bottom plot) the noise produces a significant difference in the output. This discrepancy is general and has been confirmed using different distribution functions of the noise.

The impact of the noise in the inputs is easy to quantify in the case of binary classification problems. To this end, it has been assumed that the output value of 0.76 indicates the propensity of the customer to give a tip; for values below this threshold the costumers do not tip and above it they do. Again it has been assumed that the inputs are affected by an uncertainty of 40% and the same number of one hundred test points has been chosen as in the previous case. Both inputs have integer values in the range [1,7] and all possible combinations have been taken in account, producing a total of 49 outputs. Now for each of the input combinations, the average of the ten thousand outputs has been calculated to determine whether it is below or above the threshold of 0.76. This average value has then been compared with the one obtained assuming the input to be perfectly known. The difference between the decisions to tip or not for the the cases with and without noise is plotted in figure 7.

The main problem with the statistical, brute force approach of calculating the outputs for a large number of inputs, representing the statistical distribution of the inputs affected by noise, is linked to the "curse of dimensionality". The number of combinations to be calculated increases exponentially with the complexity of the FLS and becomes unmanageable very quickly in practical applications. An approximate but less computationally intensive treatment would be therefore very beneficial. After investigation of various alternatives, two types of calculations have been performed. The activation of all the rules is calculated first for three values of the inputs, the nominal one and the maximum and minimum possible values given the error bars. Then the outputs of the FLS is calculated in two different ways: a) by calculating the average activation of the rules first and then determining the single output or b) by calculating the three outputs (corresponding to the three activations) and then averaging over the three outputs. The results of these types of calculation for the benchmark example have been compared with the previous exhaustive approach of simulating the noise statistics with one hundred cases. The quality of these two approximations is shown in figure 8. It must be noted that the approach of averaging the outputs seem to provide the best approximation. On the other hand both alternatives are not completely satisfactory even for this very simple FLS. The performance of the approximate methods is expected to degrade significantly with the complexity of the FLS and this is confirmed by the analysis reported in next section.

The main added value of the treatment described in this section is that a probability can be attributed to each output of the FLS. The interpretative powers of both fuzzy logic and probability can therefore be combined. The outputs are calculated according to the rules of fuzzy logic but their probability can also be determined on the basis of the statistics of the noise. The additional representational power of this combined approach can be very useful in the exact sciences as illustrated by the application to nuclear fusion described in the next section. The approach of computing only a limited number of strategically chosen activation values of the rules must be adopted with extreme caution because it can provide significantly distorted outputs. The relevant result of the treatment described in this section is that a significant improvement of the classification power of FLS can be obtained by taking into account the noise statistics as shown in the next section for a practical case.

3. A CASE STUDY: CONFINEMENT REGIME IDENTIFICATION IN JET

The methodology described in the last section has been applied to the case of confinement regime identification at JET. The main objective of the analysis is a typical classification problem: to determine whether a discharge is in the L or H mode of confinement for every point in time. To this end, a database of 53 discharges has been verified by experts at JET, who have determined specifically the times of the L to H and H to L transitions for each discharge. The most relevant signals to be used have been derived with exploratory analysis techniques and in particular with the CART algorithm [5] within a wider class indicated by the experts. In the end, the six signals summarised in table I have been retained for the purpose of studying the LH transition. They are the most relevant ones for the database, whose results are presented in this paper and their information content is estimated to be about two orders of magnitude higher than the next most important one (the first one discarded). In order to investigate the potential of the proposed approach to treat the error bars, a previously automatic method to generate FLSs from CART trees has been used [7]. A series of Fuzzy classifiers of different level of complexity have been derived and their success rate verified. A first classification has been performed without considering the measurement errors. The FLSs have been trained with a random set of 38 discharges and then tested on the remaining ones (not used for the training). For each FLS, the best threshold has then been determined in an empirical way by choosing the value witch provided the best performances.

The same training procedure has then been performed taking into account the uncertainties in the measurements. The assumed error bars are also reported in table I. Two different estimates of the error bars have been considered to cover realistically the entire set of discharges in the database. Within the error bars, four different values of the signals have been selected and then the combinations of the various inputs have been randomly chosen. Given the complexity of the FLSs under test and the number of inputs, handling more alternatives is prohibitive in terms of computational time required. For each combination of these inputs the value of the output has been calculated. The plasma is then assumed to be in the state of the higher probability, i.e. in the state (L or H) which is more likely on the basis of the statistical distribution of the outputs. The results of the two categories of FLSs, with and without the error bars, are compared in table II for the time interval [-100 ms, 100 ms] around the transition. A significant improvement in the success rate is always achieved by taking into account the most likely output instead of the simple value calculated assuming perfect measurements. In some cases the success rate of the classifier increases of more than 12 %. Moreover the positive effect of taking into account the statistics of the noise is significant also for the most performing FLs. It is important to note that this interval so close to the transition is the most challenging from the point of view of the classification and therefore it is the most vulnerable to noise.

This is confirmed by the analysis of time intervals further away from the transition time, for which the improvement in the performance is lower because the influence of the noise is expected to be less critical. It is also worth noticing that the improvement is in general more evident for higher level of the assumed noise on the input signals as shown by the results of the case two. On the other hand it has been verified that the increase in the success classification rate tends to saturate if the assumed noise becomes unrealistically high. This suggest a possible way to determine whether the error bars assumed are correct by increasing the noise level up to the point where the performance of the FLS does not improve.

For some of the most performing FLs the same test has also been carried out for a higher number of input combinations corresponding to different noise values, in order to increase the statistical basis of the results. In general increasing the number of input values of the signals has a positive effect on the classification capability of the FLs, because of course the statistical basis for the decision becomes more solid. On the other hand, already for less than ten different values of the inputs the computational time becomes prohibitive (several days). Therefore the same alternatives used for the benchmark, of estimating the rule activation for the nominal values of the inputs and the points corresponding to the limits taking into account the error bars, have also been tested on the JET database. Unfortunately even the most performing alternative of averaging the outputs of the three values does not provide reliable estimates, as illustrated in table III. Even if the results are in general improved, there is a significant decrease in the success rate for the most performing FLSs. This indicates that in general the proposed shortcuts to the full statistical analysis distort the results probably because they do not reflect accurately enough the statistical distribution of the noise.

As a conclusion to this section, it is also worth mentioning that the proposed type of analysis, calculating the average of the FLS outputs to take into account the confidence interval of the measurements, is still computationally relatively manageable.

On the contrary it would be practically very difficult or even impossible to use the same approach starting form the inputs to the CART algorithms. The CART algorithm has indeed significant difficulties in coping efficiently with trees of the dimension required by treating statistically the confidence intervals of the measurements already for problems with of the order of 10 inputs. Moreover, it has been shown that applying the FLS classifier on trees generated by CART improves the performance than using only the CART system [10]. Therefore the proposed method to derive the statistical distribution of the outputs within the error bars of the measurements, in addition to improving the performance of the FLS classifier, is also the only computationally practical and simple way to attack the problem.

5. PROSPECTS OF FURTHER DEVELOPMENTS

A specific strategy has been tested to address the issue of taking into account the error bars in the measurements used as inputs to Fuzzy Logic Systems. The approach of considering the uncertainty of the measurements as an independent axis of complexity is conceptually sound for many real system and provides a clear improvement in the performance of the FL classifiers when the statistic of the noise is properly taken into account. Indeed the advantages of the adopted approach have been tested using a JET database to classify plasma depending on their mode of confinement. These results confirm the importance of the noise for the time interval close to the confinement regime transition. Moreover the propose analyis can provide an independent confirmation of the error bars estimated for the various measurements.

With regard to future developments, it would be nice to confirm the potential of the strategy for a concrete regression problem. Also the case of a FLS with more than one output could be analysed. Practical problems with wider error bars would probably also deserve some specific attention.

From a methodological point of view, mixtures of continuous and categorical inputs are expected to present some additional challenges to the proposed approach. On the other hand, the prospects of finding a simplified technique to avoid the statistical scan of the inputs plus noise do not look very bright. FLSs can be very nonlinear computational processes and therefore any approximate treatment which distorts the statistics of the noise even if lightly can have very detrimental effects on the classification accuracy. General alternatives to the exhaustive calculation over the noisy inputs are therefore not viable even if they can provide satisfactory results for some specific, simple cases.

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Signal Name	Acronym	Unit	Uncert. Case 1	Uncert. Case 2
Magneto-hydrodynamic energy	Whmd	(J)	± 15%	± 20%
Toroidal magnetic field	BT80	(T)	± 2%	± 2%
Electron temperature	Te	(eV)	± 10%	± 20%
Beta normalised	Bndiam		± 15%	± 20%
X-point radial position	Rxpl	(m)	± 0.01%	± 0.01%
X-point vertical position	Zxpl	(m)	± 0.01%	± 0.01%

Table 1: List of the signals used as predictors for the classification trees

Term Node Number	Thr	Performance without noise (%)	Incert. Case 1		Incert. Case 2	
			Performance (%)	Improvement (%)	Performance (%)	Improvement (%)
20	0.45	55.17	60.9	5.73	65.2	10.03
20	0.5	57.43	60.5	3.07	61.67	4.24
18	0.45	55.57	59.73	4.16	65.37	9.8
18	0.5	57.93	65.73	7.8	68.7	10.77
16	0.45	56.27	59.97	3.7	63.7	7.43
16	0.5	59.6	69.1	9.5	71.83	12.23
14	0.45	56.2	58.43	2.23	61.5	5.3
14	0.5	73.43	76.83	3.4	77.67	4.24
12	0.45	68.7	72.4	3.7	73.77	5.07
12	0.5	73.2	76.4	3.2	77.5	4.3
4	0.45	60.93	72	11.07	76.33	15.4
4	0.5	69.23	74.53	5.3	75.6	6.37
2	0.45	61.67	67.43	5.76	69.63	7.96
2	0.5	67.2	70.1	2.9	69	1.8

Table 2: The performances of the various FLSs, with and without the noise on the inputs.

Term Node Number	Thr	Performance without noise (%)	Incert. Case 1		Incert. Case 2	
			Performance (%)	Improvement (%)	Performance (%)	Improvement (%)
20	0.45	55.17	62.9	7.73	68.13	12.96
20	0.5	57.43	63.63	6.2	72.3	14.87
18	0.45	55.57	57.57	2	58.67	3.1
18	0.5	57.93	64.47	6.54	65.33	7.4
16	0.45	56.27	57.57	1.3	59.97	3.7
16	0.5	59.6	64.47	4.87	61.13	1.53
14	0.45	56.2	57.87	1.67	57.73	1.53
14	0.5	73.43	64.4	-9.03	60.7	-12.73
12	0.45	68.7	70.43	1.73	65.93	-2.77
12	0.5	73.2	70.83	-2.37	71.07	-2.13
4	0.45	60.93	71.43	10.5	70.23	9.3
4	0.5	69.23	74.4	5.17	66.93	-2.3
2	0.45	61.67	67.43	5.76	68.6	6.93
2	0.5	67.2	69.13	1.93	77.53	10.33

Table 3: The performances of the various FLSs for the method of calculating the activation of the rules for the nominal inputs and the maximum and minimum of their possible values given the error bars.

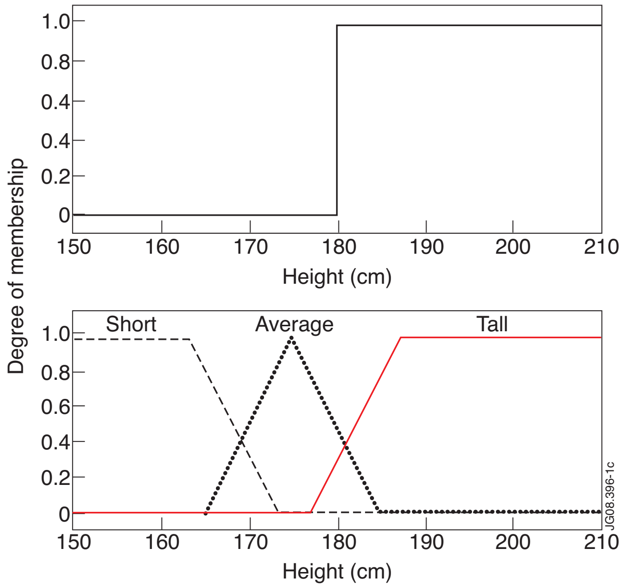


Figure 1: a) modelling people height with crisp sets b) modelling people height with fuzzy sets without any provision for uncertainties in the measurements.

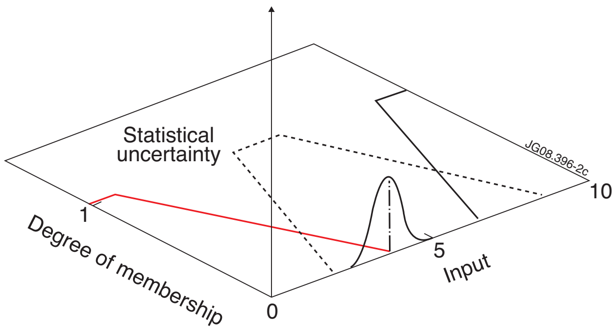


Figure 2: Statistical errors in the measurements of the inputs to the FLS create a second axis of uncertainty. The x axis represents the input value, the y axis the degree of membership, the vertical dimension the statistical uncertainties in the inputs and therefore the consequent probability distribution of the membership values.

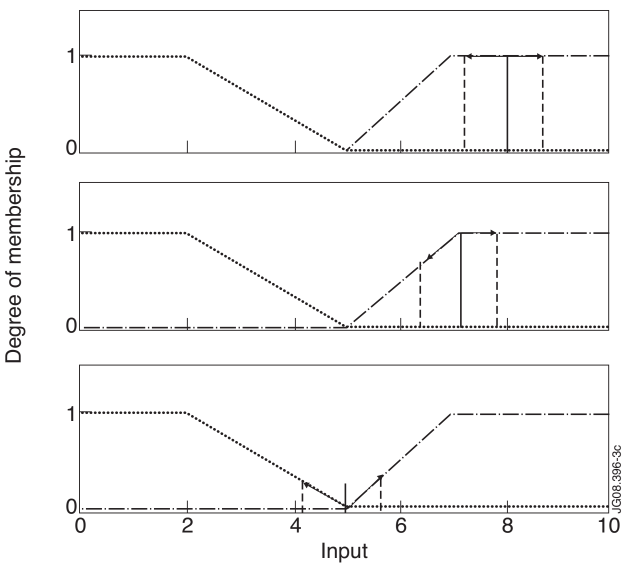


Figure 3: Three main alternatives encountered when adding noise to the inputs of a FLS.

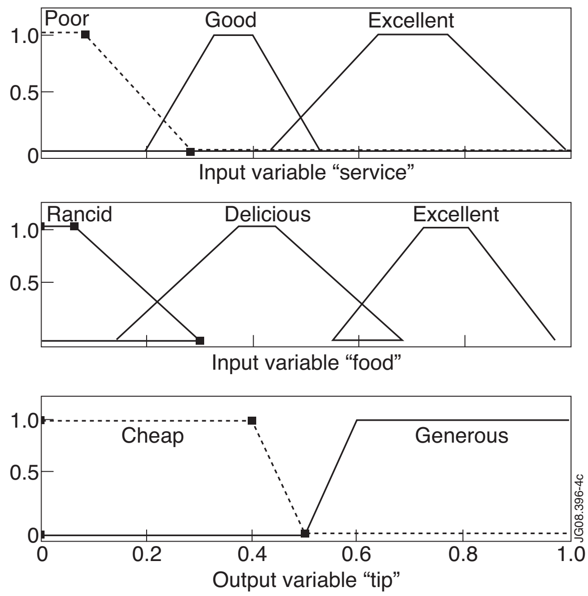
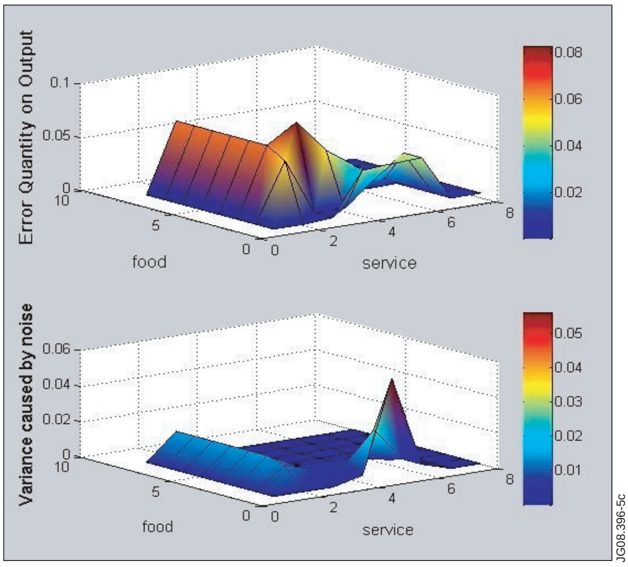


Figure 4: Membership functions of the inputs and the ouput of the simple model for the tipping behaviour of customers in restaurants: a) the input food b) the input service c) the output tip.



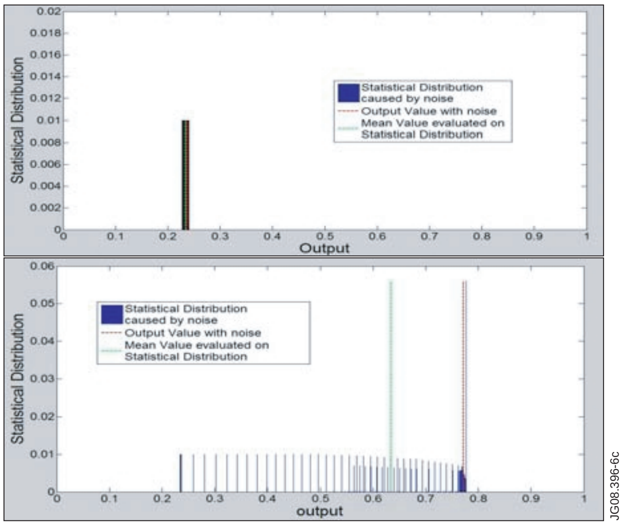
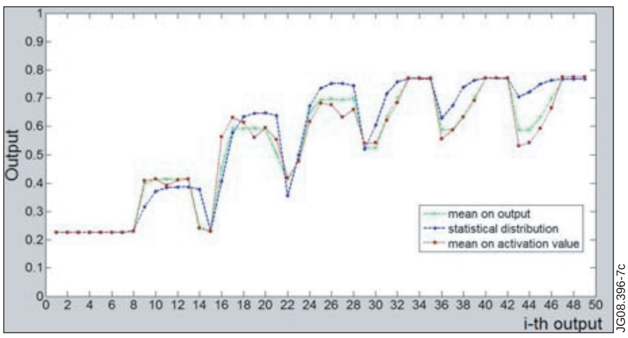


Figure 5: a) Error on the output of the FLS caused by the uncertainty on the inputs. The error has been defined as the difference between the output calculated without noise and the average of the outputs corresponding to the middle and the extremes of the confidence intervals b) Variance of the three outputs corresponding to the middle and extremes values of the inputs with error bars.

Figure 6: Output probability distribution functions for the benchmark comparing the cases with and without noise. The red dashed line indicates the value of the output considering the nominal values of the inputs (without noise) while the green dashed line indicates the mean values of the statistical distributions. The two plots show the outputs for two different sets of inputs (see text). In the case of the top graph, the difference of the outputs due to the noise is very small and the various curves blend into each other.



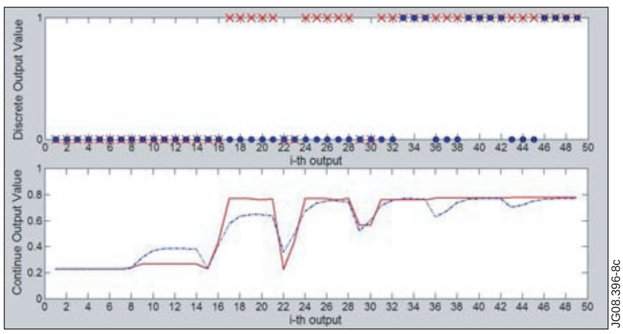


Figure 7: Top, output with binary classification; bottom, representation of the continuous output values. The difference in the classification of the tipping benchmark for the cases with and without noise can be observed. Indeed, there is a clear region of the input space where the most likely output for the noisy case is different from the one calculated without taking the noise into account.

Figure 8: Comparison of the results obtained with the theoretical approximation of calculating a) limited amount of activation values compared with the exhaustive computation of the case b).