EFDA–JET–PR(07)58

F.P. Orsitto, A. Boboc, C. Mazzotta, E. Giovannozzi, L. Zabeo
and JET-EFDA Contributors

Modelling of Polarimetry Measurements at JET

Modelling of Polarimetry Measurements at JET

F.P. Orsitto[1], A. Boboc[2], C. Mazzotta[1], E. Giovannozzi[1], L. Zabeo[2]
and JET-EFDA Contributors*

JET-EFDA, Culham Science Centre, OX14 3DB, Abingdon, UK

[1]*Associazione EURATOM-ENEA C R Frascati 00044 Frascati, Italy*
[2]*EURATOM-UKAEA Fusion Association, Culham Science Centre, OX14 3DB, Abingdon, OXON, UK*
** See annex of M.L. Watkins et al, "Overview of JET Results ",*
(Proc. 21 st IAEA Fusion Energy Conference, Chengdu, China (2006)).

Preprint of Paper to be submitted for publication in
Plasma Physics and Controlled Fusion

ABSTRACT.
The paper presents a study aimed to validate the ability of the presently available models to predict the *Cotton-Mouton effect*. The Faraday rotation and the Cotton-Mouton phase shift angle can be calculated by means of the rigorous numerical solution of Stokes equations. Numerical and approximated solutions are presented and compared with experimental data. A detailed comparison is done with the complete time traces of measurements, inside a limited dataset representative of JET regimes. A statistical analysis is then carried out on a dataset including data from 300 discharged. In general the Cotton-Mouton measurements are in agreement with the numerical model, and the line integral of plasma density deduced by the Cotton-Mouton measurements is in agreement (well inside the experimental error, which is close to two fringes for the polarimetry measurements, 1 fringe = $1.14 \cdot 10^{19}$ m^{-2}) with that measured by interferometer and LIDAR Thomson scattering.

1. INTRODUCTION

The JET polarimeter system [1] measures the Faraday rotation and the Cotton-Mouton phase shift angle proportional respectively to the line integral of vertical magnetic field times the electron density and to the line integrated plasma density. It is important to assess the interval of plasma parameters [2] where the proportionality is verified. In this context it is useful to have a tool to predict the polarimetry measurements using data produced by other diagnostics, to compare the theory and polarimetry measurements. This paper presents a study aimed to validate the ability of models to predict the *Cotton-Mouton effect* at high plasma density, plasma current and temperature, i.e. at ITER relevant plasma parameters. The Faraday rotation and the Cotton-Mouton phase shift angle can be calculated by means of the rigorous solution of Stokes equations [3], which define the spatial evolution of the polarisation of the laser beam inside the plasma. A simplified analytical solution (hereafter named as Type II solution [2]) could be found using an ordering between the components of the vector appearing in the Stokes equations that is typical of Tokamak plasma. The Type II solution could be used: (i) for understanding the mutual effect between the Faraday and Cotton-Mouton at high density and current; (ii) to predict the range of plasma parameters where there is a linear dependence between Cotton-Mouton phase shift angle and line integral of plasma density. Other approximated solutions to the Stokes equations are available: I) analytical approximated solution valid for small Faraday and Cotton-Mouton effects, hereafter denoted as Type I solution (see ref [4]), and II) a solution empirically obtained in ref [5] and discussed in ref [3], hereafter named as Guenther Model A solution.

Having the tools (approximate and numerical solutions) to predict the measurement of polarimetry, a complete comparison between the models and data can be carried out in the following way. First, a limited set of discharges (dataset†I) representative of the parameter space available at JET is selected (see Table I), where a detailed comparison can be made between data and models: for example *comparing the time traces of the measurements with the models*. Second, a more extensive data set (dataset II), *suitable for a statistical analysis*, is prepared including 300 discharges, where 11 time points are selected, in each discharge including the high power phase; the data are belonging to the channel 3 of the JET interferometer-polarimeter (i.e. the vertical chord at R = 3.04m), and the polarimetric measurements, as well as plasma density and temperature are validated. It has been demonstrated [6] that to calculate correctly the polarimetric signals it is important to include the effect of the plasma temperature for T_e > 5keV, as consequence of the dependence of dielectric tensor upon T_e. This effect could be important for ITER (T_e ~ 30keV); the JET data are consistent with this evaluation, but more data are necessary to assess the dependence of polarimeter signals upon T_e.

The paper is organised as follows: in sec II the method of rigorous numerical solution to Stokes equations is introduced, in sec IIa the Type I approximate solution is introduced, in sec IIb the Type II approximate solution is obtained and an example of comparison of approximate solutions (Type 1 and 2) with the rigorous solution and polarimetric measurements is presented; in sec III the Guenther Model A is discussed and compared with measurements and numerical solutions; in sec 4 examples of comparison between modelling and time traces of polarimetric measurements for shots belonging to dataset I (Table I reports the plasma parameters) are presented; in sec V the result of a statistical analysis carried out on dataset II is detailed; in sec Conclusions are given. Hereafter a plasma discharge is named also using ‘Pulse No:’.

2. SOLUTIONS OF STOKES EQUATIONS.

The geometry considered includes the propagation of a laser beam along a (vertical) chord (taken as z-axis) in a poloidal plane of a Tokamak. The toroidal magnetic field ($\vec{B}_t$) is perpendicular to this plane and the angle of the electric field vector of the input wave with $\vec{B}_t$ is 45^0. The polarisation of a beam can be described using Stokes vector $\vec{s}$, whose components are expressed in terms of the ellipticity (χ) (which is linked to the Cotton-Mouton phase shift angle ϕ) and Faraday angle (ψ). The spatial evolution along the z-axis of the polarisation of a beam is given by the Stokes equation [4]:

$$\frac{d\vec{s}}{dZ} = \vec{\Omega} \times \vec{s} \tag{2.1}$$

where

$\vec{\Omega} = ka(\Omega_1, \Omega_2, \Omega_3)$, and $\Omega_1 = C_1 nB_t^2$; $\Omega_2 = 2C_1 nB_x B_t$; $\Omega_3 = C_3 nB_z$

B_t is the toroidal magnetic field (Tesla), B_z the component of the poloidal magnetic field along the propagation axis, B_x the component of poloidal magnetic field orthogonal to the propagation axis, n_e the electron density (m^{-3}), $C_1 = 1.794 \cdot 10^{-22}$ and $C_3 = 2 \cdot 10^{-20}$, calculated for the laser wavelength of λ = 195μm [4], and Z = z/ka is the normalized coordinate along a vertical chord, where k is the elongation and a the minor radius. The relation between the measured Faraday rotation ψ and the Cotton-Mouton phase shift ϕ angles and the components of Stokes vector is:

$$\frac{s_2}{s_1} = \tan 2\psi \tag{2.2}$$

$$\frac{s_3}{s_2} = \tan \varphi \tag{2.3}$$

2.1. TYPE I APPROXIMATE SOLUTION

A simple approximate solution [4] (hereafter named Type I solution) is found to Stokes Equation (2.1) if the quantities $W_3 = \int \Omega_3 dz = C_3 \int n_e B_z dz$ and $W_1 = \int \Omega_1 dz = C_1 \int B_t^2 n_e dz$ satisfy to the conditions $W_i^2 << 1$. In this condition the Stokes vector and the Faraday and Cotton-Mouton phase shift angle are given by:

$$s_1 = W_3 = C_3 \int n_e B_z dz = 1/\tan 2\psi \tag{2.4}$$

$$s_2 = 1(W_1^2 + W_3^2)/2 \approx 1 \tag{2.5}$$

$$s_3 = W_1 = C_1 \int B_t^2 n_e dz = \tan \varphi \tag{2.6}$$

The evaluation of W_1 and W_3 and vector $\vec{\Omega} = ka(\Omega_1, \Omega_2, \Omega_3)$ is made using the electron density profile measured by the LIDAR Thomson Scattering projected on the vertical chord using the equilibrium reconstruction, and the magnetic field components are taken from the equilibrium reconstruction, using magnetic measurements.

3. APPROXIMATE ANALYTICAL SOLUTION (TYPE II)

The conditions $W_i^2 << 1$ are restrictive and more general approximate solutions to the Equation (2.1) can be found, noting that the following inequalities between the components of the vector $\vec{\Omega}$ hold for Tokamak plasma:

$$\Omega_3 > \Omega_1 >> \Omega_2. \tag{3.1}$$

As consequence of condition [3.1], the terms with component Ω_2 can be neglected in the equations [2.1]. The expressions (Type II solutions) for the Faraday angle and Cotton-Mouton phase shift can now be obtained by analytical integration from the simplified Stokes equations:

$$\frac{s_2}{s_1}(z) = \tan 2\psi = -\frac{1}{\tan(W_3)} \; ; \; \frac{s_3}{s_2}(z) = \tan \phi = \frac{\int_{-z}^{+z} dy \Omega_1(y) \cos(W_3(y))}{\cos(W_3(z))} \tag{3.2}$$

Approximate solutions similar to equations [3.2] were discussed in [7].

Figure 1 shows a comparison between the polarimeter data (solid line) with models (numerical solution, Type I and Type II) for Pulse No: 60980 (B_T = 2.4T, I_P = 2MA, $n_e \leq 3.9.10^{19}$ m^{-3}, T_e = 3.1keV), channel 3. A good agreement is found between polarimeter data, and Type I, Type II and the numerical solution. The experimental error bar on the phase shift ϕ is $\Delta\phi \sim 2^o$, corresponding to a $\Delta\tan\phi \leq 0.036$ for the data shown in fig.1, while the difference between measurements and calculations is less than 0.003. Since 1 fringe = 1.143 10^{19} m^{-2} the difference of 0.003 corresponds to 0.001 of a fringe, using [2.6].

4. GUENTHER MODEL A SOLUTION VERSUS EXPERIMENTAL DATA.

An approximate solution to the Stokes equations [2.1] can be obtained [3,5] if an hypothesis (Guenther model A) is introduced (see ref. [3,5] for a detailed discussion on this approximation):

$$\frac{\Omega_1(z)}{\Omega_2(z)} = K(z) = const \tag{4.1}$$

This hypothesis leads immediately to the solution:

$$s_1 = Ks_3 + q \tag{4.2}$$

where

$$K = \frac{s_{1f} - s_{10}}{s_{3f} - s_{30}} \quad \text{and} \quad q = s_{10} + Ks_{30} \tag{4.3}$$

and s_{1f}, s_{3f} (s_{10} and s_{30}) are the values of the Stokes components at plasma exit (entrance). The values of W_{3A} and W_{1A} are given by the following equations [3,4]:

$$W_{3A} = K\, W_{1A} \tag{4.4}$$

While

taking $\sigma = \frac{s_{20}}{|s_{20}|} - \frac{s_{2f}}{|s_{2f}|}$; $\nu = \frac{s_{20}}{|s_{20}|}$

$$W_{1A} = \frac{\sigma}{\sqrt{1+K^2}} + (-1)^{\sigma}\nu\frac{F(s_{3f}) - \sigma F(s_{30})}{\sqrt{1+K^2}} \tag{4.5}$$

where

$$F(u) = \arcsin\frac{(1+K^2)u - Kq}{\sqrt{1+K^2-q^2}} \tag{4.6}$$

The values of s_{1f} and s_{2f} are obtained from a numerical solution of the Stokes equations: in practice the values of W_{1A} and W_{3A} are intended to give an estimation of the Cotton-Mouton and Faraday, in any conditions.

As it can be noted for Pulse No: 60980 there is agreement between Model A and the numerical solution (see fig.2). It has been checked that the condition [4.2] is well verified for Pulse No: 60980.

5. COMPARISON BETWEEN MODELLING AND DATASET 1.

A comparison between the time traces, i.e. the time evolution of the measurements in a discharge, of the polarimeter, and model calculations is done for a limited number of shots, named dataset I. Table I gives the plasma parameters of shots included in dataset I: particular care has been given to the validation of data used for the comparison with models. The shots are representative of: i) low density (n_e ~ 2-4•$10^{19}m^{-3}$) and electron temperature (Pulse No's: 60980 and 61092); ii) high density (max density n_e~$10^{20}m^{-3}$) and low T_e (T_e~2-4keV, Pulse No's: 67782 and 67777); iii) low density and high T_e (max T_e~10keV, Pulse No's: 66002 and 66015); iv) intermediate density and T_e (Pulse No: 66016). Example of comparison of data with models are given in fig.3 and 4 related with Pulse No: 66002 at high T_e and Pulse No: 67777 at high n_e.

Finite temperature effect is modelled in the Stokes equations adding to the vector $\vec{\Omega}$ the corrections $\delta\vec{\Omega} = (\Omega_1,\Omega_2,\Omega_3)$. They can be expressed in terms of the small parameter $\tau = \frac{T_e}{m_e c^2}$. The theory is reported in [6]. It turns out that $\delta\Omega_1 = (12\tau\Omega_1, 12\tau\Omega_2, 6\tau\Omega_3)$. Assuming $T_e = 5$keV and following the model, it results $\tau = 0.01$, and the corrections to the Faraday term is $\delta\Omega_1/\Omega_1 \approx 12\%$, while that to the Cotton-Mouton term $\delta\Omega_1/\Omega_1 \approx 12\%$.

Figure 3 shows the comparison between the numerical solution of Stokes equation with and without T_e corrections for the Pulse No: 66002 where the maximum T_e is 11keV, in the time interval between 15 and 20s. The correction for high Te tends to overestimate the measurements.

Figure 4 shows a comparison between the results of numerical solution and measurements, showing also how the Model A slightly underestimates the measurements and numerical results for the high-density Pulse No: 67777.

6. STATISTICAL ANALYSIS.

To perform a statistical analysis of the comparison between data and models, 305 JET shots taken in two periods, March-September 2003 and April-September 2006, are included in the dataset II: 11 temporal points for each shot are included, starting by 45s until 65s to include stationary phase and to check the influence of field rise.

Data for B_t (toroidal magnetic field), plasma current, temperature and density by LIDAR Thomson Scattering and by the FIR interferometer, and measurements of FIR polarimeter (channel 3) were checked. The range of values are: toroidal magnetic field $1 < B_t < 3.5$T; plasma current $1 < I_p < 3.5$MA; line integrated density on channel 3, measured by the interferometer, $2 < n_{eL(ch3)} < 30 \cdot 10^{19} m^{-2}$; electron temperature $1 < T_e < 11$keV.

The data of the models are calculated using EFIT reconstructions and LIDAR temperatures and densities; they are compared with polarimetry measurements. The statistical analysis of JET data is presented in more detail in [8]. An extensive comparison including a detailed study of data at high density, i.e. for densities $n_{eL} = 20 - 30 \cdot 10^{19} m^{-2}$ is presented in [9].

Figure 5 shows a comparison between the line integral of plasma density deduced by the measurements of polarimetry using the equation [II.6] and the same quantity measured by the Thomson scattering: the agreement is clear, the result of a linear regression is also shown. It is worth noting that the constant of proportionality deduced by the linear fit is $C_{1exp} = 0.00194$, while the theoretical evaluation is 0.00179 (see sec. 2).

The comparison between the measurements and Type I solution is given in fig.6, while the comparison between the numerical solution and measurement is presented in fig.7. In both cases the linearity is confirmed within the experimental errors.

CONCLUSIONS

The paper is dedicated to a comparison of modelling with polarimetry measurements in particular for

Cotton-Mouton phase shift angle. The presently available models are compared with data. The numerical solutions of Stokes equations, obtained from first principles and without any free parameter, are in general good agreement with data.

It has been shown that, for the plasma parameters considered in the present paper, the difference between the measurements of phase shift and the calculated one using the numerical solution of Stokes equation is well within the experimental error bar.

Approximate solutions can be used to evaluate Cotton-Mouton phase shift angle: I) it has been shown that the Type II is in agreement with data and with numerical solution of Stokes equations, at the same level of accuracy of the latter; ii) while the Model A is less accurate than the Type II and Type I approximate solutions.

With reference to the dataset used for this study (mainly densities $n_{eL}<15\cdot10^{19}m^{-2}$), in general the line integral of plasma density deduced from Cotton-Mouton measurements is in agreement with interferometer and the Thomson scattering measurements, so the Cotton-Mouton measurement can be used to evaluate the line-integral of plasma density using the formula [II.6], with C1=0.00194. The accuracy of this evaluation is well within 1 fringe for line-integrated densities $n_{eL}<15\cdot10^{19}m^{-2}$.

ACKNOWLEDGEMENTS

The authors would like to thank M Brombin for reading the manuscript and for useful discussions, A Murari, S Segre and P Buratti for interesting observations about the analysis presented in this paper. This work, supported by the Euratom Communities under the contract of Association between EURATOM/ENEA, was carried out within the framework the European Fusion Development Agreement. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

REFERENCES

[1]. Braithwaite G, Gottardi N, Magyar G, O’Rourke J, Ryan J. and Veron D (1989) *Rev. Sci. Instrum.* **60** 2825

[2]. Orsitto F.P. et al. (2006) *Proc. European Physics Society* (Roma) paper P1.073 (Report EFDA JET CP(06)03-02)

[3]. Guenther K. et al. (2004) *Plasma Phys. Control. Fusion* **46** 1423

[4]. Segre S.E. and Zanza V. (2006) *Plasma Phys. Control. Fusion* **48** 339

[5]. Segre S.E. (1995) *Phys. Plasmas* **2** 2908

[6]. Segre S.E. and Zanza V. (2002) *Phys. Plasmas* **9** 2919

[7]. Segre S.E. (1996) *Phys Plasmas* **3** 1182

[8]. Mazzotta C. et al. ‘Models comparison for JET polarimeter data’, *Proc. Workshop on ‘Burning Plasma Diagnostics’* (Varenna) September 2007.

[9]. Brombin M. et al. ‘Systematic comparison between line integrated densities measured with interferometry and polarimetry at JET’, to be submitted to Review of Scientific Instruments.

Pulse No:	**n_{emin} (**) (10^{19} m^{-3})**	**n_{emax} (**) (10^{19} m^{-3})**	**∫nedlCh3(*) (10^{19} m^{-2})**	**T_{emin} (keV)**	**T_{emax} (keV)**	**B_T**	**I_P (MA)**	**W_1**	**W_3**
60980	2	3.9	4-9	1.5	3.1	1.6/2.4	2/1.6	0.016	0.25
61092	1.6	5.3	2-14	2.3	4	2.3/1.9	1.9/2.35	0.02	0.5
67782	3	12	6-27	2	4	2.7	2.5	0.11	1.38
67777	2.7	12	6-28	2.5	3.5	2.7	2.5	0.11	1.4
66015	3	6	3.5-11	2.5	10	3	2.5	0.07	0.7
66002	3	6	3-7	2.4	11	3	2.5	0.07	0.65
66016	3.1	6.9	7-16	2.5	8.2	3	2.5	0.08	0.8

(*) Line integrated density interval as measured by the FIR interferometer, on channel 3.
(**) Measured values of electron density by LIDAR Thomson Scattering

Table I. Plasma parameters of shots included in dataset I

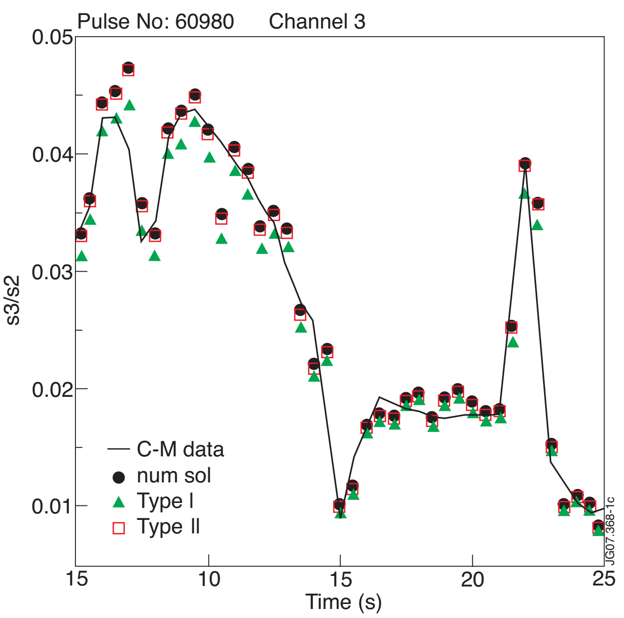

Figure 1: Tan $\phi = s_3/s_2$ versus time: Pulse No: 60980, channel 3, polarimeter data Cotton-Mouton phase shift measurement (— continuous line) compared with models (●) numerical solution, (▲)Type I and (□) Type II). Plasma parameters are given in Table I.

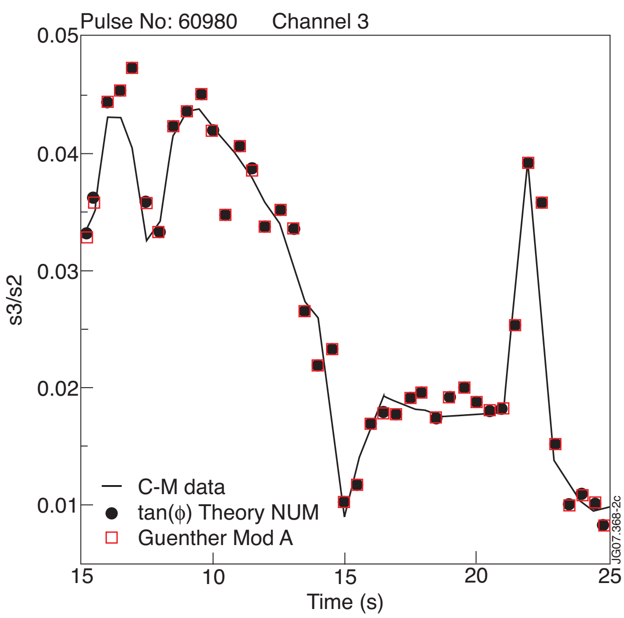

Figure 2: Tan $\phi = s_3/s_2$ versus time: Pulse No: 60980, channel 3, polarimeter data Cotton-Mouton phase shift measurement (— continuous line) ,(●) numerical solution, (□) Guenther Model A.

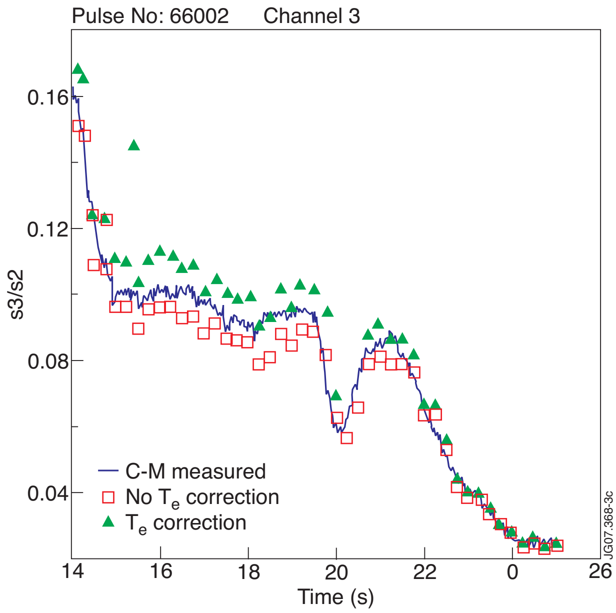

Figure 3: Tan ϕ = s_3/s_2 versus time: Pulse No: 67782, channel 3, Polarimeter Data Cotton-Mouton phase shift measurement (— continuous line), numerical solution of Stokes equations with (▲) and without (□) temperature corrections.

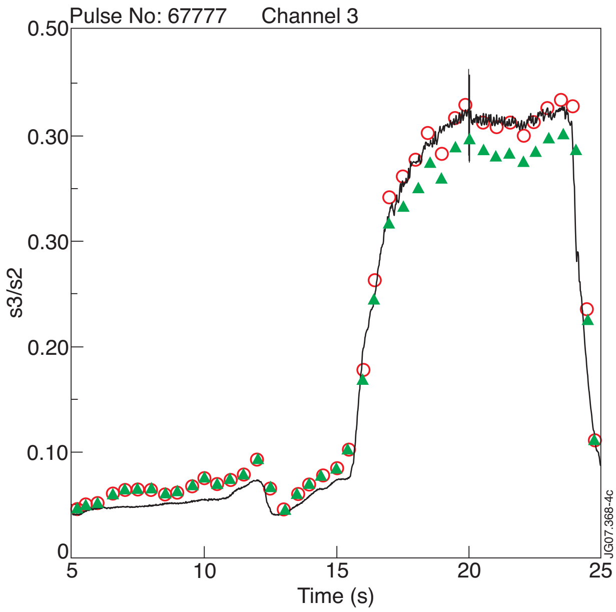

Figure 4: Tan ϕ = s_3/s_2 versus time: shot 67777, channel 3, Polarimeter Data Cotton-Mouton phase shift measurement (— continuous line), numerical solution of Stokes equations with temperature corrections (□), Guenther Model A (△).

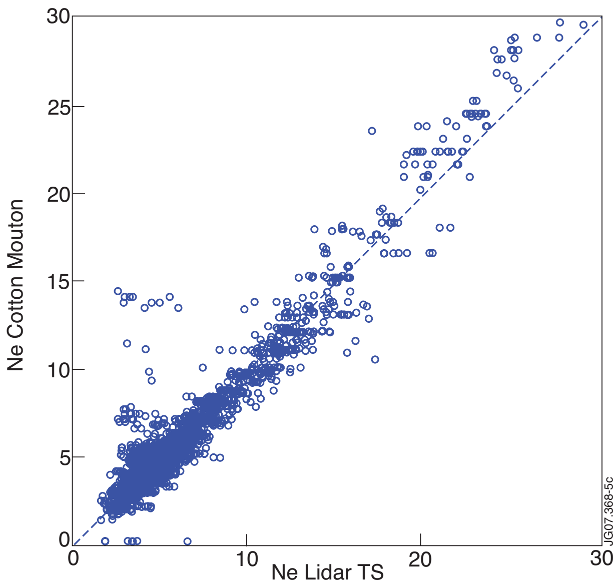

Figure 5: Line integral of plasma density deduced by Cotton-Mouton phase shift angle (y-axis) versus the line integral of plasma density on channel 3, obtained from LIDAR Thomson Scattering. The linear fit (dashed line) to the data is shown. The unity in both axis is $10^{19}m^{-2}$. The number of points corresponding to densities $\geq 15\cdot10^{19}m^{-3}$ is 4% of the total, i.e. about 130 points are shown on the figure.

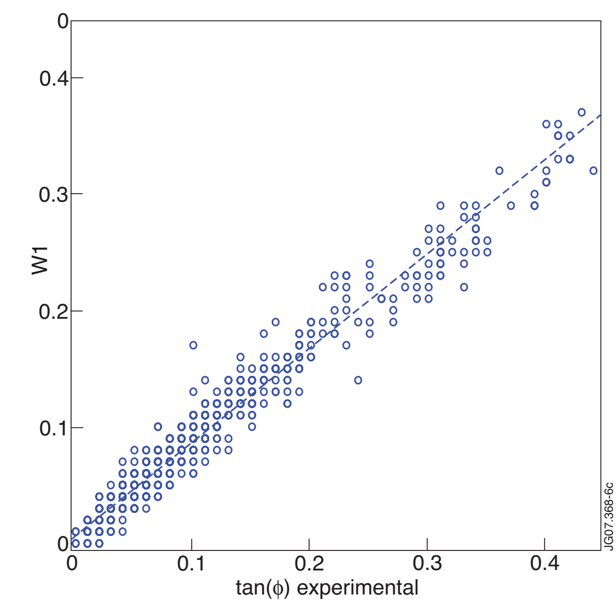

Figure 6: Type I solution (W_1) versus the measured Cotton-Mouton phase shift angle: linear fit to the data is shown. The number of points corresponding to a density higher than 15 $10^{19}m^{-2}$ is about 4% of the total.

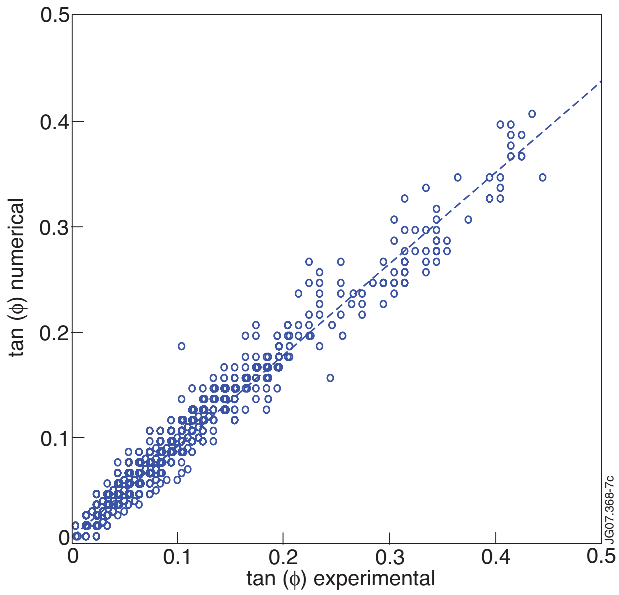

Figure 7: Cotton-Mouton phase shift angle from numerical solution of Stokes equation versus the measured phase shift angle; the result of a linear fit is shown.