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ABSTRACT

A new mechanism whereby Neoclassical Tearing Modes (NTMs) can be triggered through toroidal mode coupling to a frequency sweeping fishbone instability driven by energetic particles is proposed. The novel physical picture is the presence, prior to the seeding process, of a “pre-NTM” magnetic island of relatively small width, rotating at a frequency within the diamagnetic drift band, so that the polarization current is stabilizing. The main effect of the fishbone is to exert an electromagnetic torque that changes this frequency so that the polarization current becomes destabilizing, driving the island above the critical seed island width, in which case NTM growth ensues.

INTRODUCTION

Neoclassical Tearing Modes (NTMs) are MagnetoHydroDynamic (MHD) instabilities that modify the magnetic topology in toroidal fusion devices such as tokamaks, leading to the formation of helically winding structures called “magnetic islands” that may reach large sizes and significantly affect plasma performance. NTMs are usually triggered by another MHD instability, most often a sawtooth reconnection event (see [1] and references therein). Empirically, it has been found that the sawtooth period is the main physical parameter determining the triggering, and, consequently, significant effort is spent on controlling sawteeth in present-day tokamaks so as to prevent the appearance of NTMs [2]. It was also observed experimentally that the energetic particle-driven fishbone instability can trigger NTMs [3]. However, this is not observed for every fishbone burst, and the problem of identifying the conditions for the fishbone triggering of NTMs remains open.

Next-step burning plasmas, e.g. on ITER, will substantially differ from plasmas in present-day machines because they will generate a large population of fusion-born alpha particles. With energetic ions present in abundance, energetic particle-driven instabilities, such as fishbones, may become an inevitable feature, and the fishbone triggering of NTMs could turn out to be the dominant effect even if the sawtooth behaviour is well controlled. Therefore, it is important to have a better understanding of this process in order to be able to devise ways of controlling it. In this Letter, we describe a mechanism for this type of triggering, whereby the fishbone is considered as an external frequency sweeping perturbation that affects the NTM through toroidal mode coupling. The model is comprised of only two equations for the magnetic island’s width and phase, and the various parameters involved are identified by considering a typical instance of a fishbone triggered NTM on JET.

That toroidal mode coupling may trigger NTMs was first suggested in [4]. In the latter work, a forced reconnection was considered, namely, the MHD perturbation was acting as a direct drive for the initial growth of the magnetic island, much as in the so-called Taylor problem [5]. The main drawback of such an explanation is that the NTM should then be born in phase with the triggering event, which is seldom observed, e.g. on JET [6].

The mechanism of NTM triggering by the fishbone essentially uses the frequency sweeping character of the perturbation. This Letter shows that NTMs can be triggered by the fishbone under the following conditions: (i) the tearing mode is classically unstable, i.e. the stability parameter, Δ', is positive [7], (ii) the Δ'-driven island is stabilized at small amplitude by the polarization current [8-10] to produce what we henceforth refer to as a "pre-NTM" magnetic island, and (iii) the main effect of the fishbone is to exert an electromagnetic torque that modifies the frequency of the pre-NTM island so that the polarization current becomes destabilizing.

The basic formalism of the toroidal mode coupling problem has already been described in [4], and we refer the reader to this reference for more details. The main result is that the classical stability parameter of a (*m, n*) NTM, where *m* and *n* are the poloidal and toroidal mode numbers respectively, is changed by the n^{th} harmonic of a (1, 1) fishbone into :

$$\tilde{\Delta}' = \Delta' + \Delta'_c \frac{A(t)}{\Psi} e^{i(\phi - n\phi_{fb})} \tag{1}$$

where Δ' is the intrinsic stability parameter of the mode, Δ'_{c} is a toroidal coupling coefficient, $A(t)$ is the amplitude of the n^{th} harmonic of the fishbone, ϕ and ϕ_{fb} refer to the phase of the NTM and of the fishbone in the plasma rest frame, and Ψ is the reconnected flux. The latter is related to the normalized island width $\tilde{w} = 4\sqrt{\Psi L_s/(\mathrm{B_0 r_s}^2)}$, where $L_s = qR/s$ is the local shear length, $s = q'r_s/q$ the local shear, $q(r)$ the safety factor, R the major radius, B_0 the toroidal magnetic field, and r_s the radial location of the rational surface (i.e. $q(r_s) = m/n$).

The real part of $\tilde{\Delta}'$ given by Eq. (1) influences the island width evolution, whereas its imaginary part has an effect on its phase, i.e. it accounts for the electromagnetic torque exerted by the fishbone on the magnetic island [11]. In order to delineate clearly the main effect of the fishbone, we neglect its contribution to the reconnection itself (although it may have a small stabilizing effect on the NTM when the latter is not locked to it [12,13]), i.e. take $\mathrm{Re}(\tilde{\Delta}') = \Delta'$, so that the only effect of the fishbone will be on the island frequency. Therefore, the island width is governed by the Rutherford equation [14]:

$$\tau_R \frac{d\tilde{w}}{dt} = r_s \Lambda(\tilde{w}) + r_s \beta_p \left(\frac{a_{bs}\tilde{w}}{\tilde{w}^2 + \tilde{w}_d^2} - \frac{a_{GGJ}}{\sqrt{\tilde{w}^2 + 0.2\tilde{w}_d^2}} \right) + \varepsilon^{3/2} \frac{g(\dot{\phi}) T_A^2 \tilde{w}}{\tilde{w}^4 + \tilde{w}_{pol}^4} \,, \tag{2}$$

where $\tau_R \simeq \mu_0 r_s^2/(1.22\eta)$ is proportional to the resistive diffusion time, μ_0 and η being the permeability of free space and plasma resistivity respectively, β_p is the plasma's poloidal beta, $\varepsilon = r_s/R$, and the other coefficients will be defined shortly. As to the phase equation of the island, it is assumed to satisfy the following relation (compare to [15]):

$$\tau_M \frac{d}{dt}\left[\tilde{w}\,(\dot{\phi} - \omega) \right] = \frac{T_M}{T_A^2}\, r_s \Delta'_c\, (\tilde{w}\,\tilde{w}_{fb})^2\, \sin(\mathrm{n}\phi_{fb} - \phi) + (\omega - \dot{\phi})/\tilde{w}, \tag{3}$$

where $\tau_M = G_W r_s^2/(6G_V \mu_a)$ is proportional to the viscous diffusion time, $\tau_M = G_3 \tau_M/(\sqrt{2}G_w)$, $\tilde{w}_{fb} = 4\sqrt{A(t)L_s/(B_0 r_s^2)}$, μ_a is a phenomenological perpendicular viscosity, and $G_{3,\,V,\,W}$ are coefficients of order unity [15]. Note that the first term on the right hand side of Eq. (3) is proportional to $\mathrm{Im}(\tilde{\Delta}')$. This equation differs from the one given in [15] insofar as we have introduced a new parameter, ω, that refers to the island's "natural" frequency, i.e. to the frequency of the island without any external perturbation. Since, from a theoretical standpoint, it is still debated what this frequency exactly is, its value has to be inferred from experimental data.

We now give the definition of the different terms on the right hand side of Eq. (2). The first one is the classical tearing mode driving term that determines the width of saturated Δ'-driven islands. The functional dependence of this classical term can be inferred from the theory rigorously derived recently in cylindrical and slab geometry [16-19]. It is of the form $\Lambda(\tilde{w}) = \Delta' + a^2\tilde{w}[\ln(\tilde{w}/\tilde{w}_0) - b]$, where a^2, b, and $\tilde{w}_0$ are coefficients whose precise definition can be found in the aforementioned references. The term proportional to $a_{bs} \simeq 0.6\, L_q/L_p$, where $L_q = q/q'$ and $L_p = -p/p'$, is the contribution of the bootstrap current [14], taking into account the perpendicular diffusion length $\tilde{w}_d \simeq 5.1\sqrt{R/(r_s n s)}\,(\chi_\perp/\chi_\parallel)^{1/4}$ [20]. Finally, the stabilizing effect of toroidal curvature leads to the Glasser-Greene-Johnson term [21] that is proportional to $a_{GGJ} \simeq 6\varepsilon^2 L_q (q^2 - 1)/(q^2 s L_p)$ and that has also been shown to depend on $\tilde{w}_d$ [22].

The last term is the contribution of the polarization current [9], with $T_A \simeq \sqrt{9.3q}\,\tau_A/(ns\varepsilon)$, $\tau_A = R/v_A$ being the Alfvén time and $v_A = B_0/\sqrt{\mu_0 \rho}$ the Alfvén speed, where ρ is the plasma mass density. Its exact form is taken with the same island width dependency as in [23] but, in line with the recent work of Fitzpatrick et al. [24], we assume that $\tilde{w}_{pol} \sim \rho_i$, where ρ_i is the ion Larmor radius. Contrary to the other terms, this one depends on the island frequency, which is a crucial point as far as NTM triggering is concerned, for it allows the fishbone to act on the island width in an indirect way through the phase, Eq. (3). Therefore, it is important to keep track of this frequency dependence, which is why we have introduced the factor $g(\dot{\phi})$ that is modelled here in the following way:

$$g\,(\dot{\phi}) = (\dot{\phi} - \omega_E)\,(\dot{\phi} - \omega_E - \omega_{*pi}) \text{ if } \dot{\phi} > \omega_E \tag{4a}$$

$$g\,(\dot{\phi}) = (\dot{\phi} - \omega_E)\,(\dot{\phi} - \omega_E - \omega_{*e}) \text{ if } \dot{\phi} < \omega_E \tag{4b}$$

where $\omega_E = mE_r/(r_s B_0)$ is the drift frequency due to the radial electric field E_r, $\omega_{*pi} = \omega_{*i}(1 + \eta_i)$, $\omega_{*i} = -mT_i\, n_i'/(e r_s B_0 n_i)$ and $\omega_{*e} = -mT_e\, n_e'/(e r_s B_0 n_e)$ are the ion and electron drift frequencies respectively, n_e (resp. n_i) is the electron (resp. ion) density, T_i (resp. T_e) is the ion (resp. electron) temperature, $\eta_i = d\log T_i/d\log n_e$, and is the elementary charge. Equation (4) is in agreement with the fact that the polarization current is stabilizing in the electron or ion drift branch and destabilizing otherwise [25-27]. It also complies with the expression derived in [9].

In order to illustrate this model and show that it can indeed describe the triggering of NTMs, we apply it to a specific event that occurred during pulse 61151 on the JET tokamak (see Fig.1). This example is quite relevant in that no other major perturbation is present at the onset of the (3,2) NTM, which leads one to conclude that the triggering is due to the fishbone only. The experimental values of the model's main parameters are listed in Table I. Concerning the natural frequency, it can be seen on Fig.1 that the NTM frequency, in the laboratory frame, rises from a value that is approximately 17–18kHz to about 20kHz at t ~ 7.15s. Since the main physical profiles do not change significantly over that period of time, as we have checked, it is reasonable to assume that this value is the Doppler-shifted natural frequency to which the NTM relaxes after it has been triggered (the low amplitude $n = 1$ mode is assumed to have a negligible effect on the NTM's dynamics). We thus take $\omega_{lab}/(2\pi) = 20$kHz, where $\omega_{lab} = \omega + 2\Omega_{3/2}$, $\Omega_{3/2}$ being the plasma's toroidal rotation frequency at the $q = 3/2$ surface.

There remain a few unknown parameters in our model, and we now address them in turn. First, there are Δ' and Δ'_c that both depend on the equilibrium profiles in an intricate way. We shall not attempt to compute them precisely but rather give them a typical value, e.g. $r_s\Delta'_c = 2$ and $r_s\Delta' = 8$. Note that the sign of Δ'_c is chosen arbitrarily, since it can be changed by a mere redefinition of the island phase, whereas we have assumed Δ' to be positive so as to ensure the presence of the pre-NTM island. As to the a, b, and $\tilde{w}_0$ coefficients, they essentially affect the large saturated island width, which is not the main concern of this Letter, so they are assigned values of, e.g. $a^2 = 20$, $b = 1$ and $\tilde{w}_0 = 1$. Following the estimates derived in [5] for typical JET plasmas, we take $\tilde{w}_d = 0.04$ and assume that the equilibrium radial electric field is small so that the **E**×**B** drift frequency is negligible. The last two parameters have to be chosen arbitrarily, but within the reasonable assumptions that $\tau_M \sim \tau_R$ and $\tilde{w}_{pol} \sim \rho_i/r_s$. Therefore, we take $\tau_M = 3\tau_R$ and $\tilde{w}_{pol} = 1.6\,\rho_i/r_s$.

Given this set of parameters, it is interesting to have a look at the right hand side of Eq. (2) to understand the stability of the NTM without any perturbation. The fact that we have taken Δ' to be positive allows for another stable equilibrium, $\tilde{w}_p$ (see Fig.2), which is the pre-NTM island that has already been mentioned. It is too small to be observed experimentally (about 1mm), but is still larger than the linear resistive layer width so that the system is indeed in the nonlinear Rutherford regime [28]. If this pre-NTM island is ever driven above the seed island width $\tilde{w}_{sat}$, the system will naturally evolve towards the large saturated state, $\tilde{w}_{sat}$, which is the "seeding" process.

We finally need to model the fishbone instability, which amounts to giving an explicit expression for its amplitude and frequency. Obviously, deriving its (effective) amplitude at the position of the NTM from experimental data would be a difficult task. Therefore, the choice of $\tilde{w}_{fb}$ is again arbitrary. It is, however, possible to determine its frequency from Fig.1, and we see that it drops from an initial value $f_i \sim 20$kHz to a final one $f_f \sim 9$kHz in a time of order 10ms. We therefore model its evolution as an exponential decay between these two values over a time $t_{fb} = 8$ms. Solving Eqs. (2) and (3) numerically, we obtain the result shown on Fig.3 for a maximum fishbone amplitude $\tilde{w}_{fb} = 0.8$. We see (Fig.3(a)) that the fishbone has a braking effect on the pre-NTM island, so that the polarization current eventually becomes destabilizing, which results in a very rapid growth compared to the resistive diffusion time τ_R. It is noteworthy that, by the time the fishbone amplitude is negligibly small, the island has not quite overshot the seed island width (Fig.3(b)), but is nonetheless able to grow due to the fact that it has not yet relaxed to its natural frequency. The fishbone thus acts as an indirect trigger for the NTM through the electromagnetic torque it exerts on the pre-NTM island. It is the polarization current that actually drives the bifurcation from the partially-reconnected equilibrium to the fully-reconnected one, which is why this new physical picture may be coined enhanced, as opposed to forced, reconnection.

It is worthwhile to investigate the influence of the fishbone's parameters within the context of our model. Obviously, its amplitude should be the most important one. This is shown on Fig.4(a), where, along with $\tilde{w}_{fb}$, the viscous diffusion time has been varied as well. The result is qualitatively straightforward, as it is all the easier for the fishbone to trigger an NTM when its amplitude is large or the viscosity is small, and there is no NTM triggering below a certain threshold $\tilde{w}_{fb}^{crit}$(dotted line).

More interestingly, the role of the fishbone's final frequency and frequency sweeping rate (roughly given by $f_i - f_f$) is shown on Fig.4(b). The runs on this figure have been performed by making a scan of f_f with a 100kHz step and then, for each such value, changing $f_i - f_f$ continuously. It appears that the NTM is never triggered outside a certain window, and the unstable region itself is filled with gaps of varying size, so that a sub-region of greater instability can be seen (dashed line). Given the crudeness of our model, these results should by no means be taken as quantitative, yet their qualitative features exemplify the complexity of the triggering event. The interaction between a fishbone and a pre-NTM island thus seems to require a fine tuning to be efficient, possibly explaining why, in most instances, it takes more than one fishbone before an NTM is eventually triggered.

Obviously, a lot of work remains to be done in order to improve and validate this model, but we believe we have shown that the mechanism we propose is plausible, both from a theoretical and experimental point of view. This new physical picture emphasizes the importance of the polarization current and, hence, stresses the need for a more comprehensive theory concerning the natural frequency and the phase evolution of magnetic islands. We do not claim, however, that this mechanism applies whenever an NTM is triggered, since a physical phenomenon that could significantly affect any of the parameters involved in our model (e.g. a sawtooth precursor) may also lead to instability, not to speak of forced reconnection that cannot be definitely excluded in some cases. The triggering of NTM certainly is a complicated problem, but we think that this Letter is a step forward in the endeavour to understand it.

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$R = 3.1$	$B_0 = 0.5$	$r_s = 0.5$
$S = 1.2$	$L_s = 0.42$	$L_q = 0.42$
$L_p = 0.38$	$a_{bs} = 0.9$	$a_{GGJ} = 0.08$
$\beta_p = 0.9$	$\tau_R = 18.8$	$\tau_A = 4.1 \times 10^{-7}$
$\omega_{*pi} = 24.4 \times 10^3$	$\omega_{*e} = -9.4 \times 10^3$	$\Omega_{3/2} = 6 \times 10^4$

TABLE. I: Values of the model's main parameters for JET Pulse No: 61151, in SI units (except for temperatures expressed in keV), at t = 7.05s

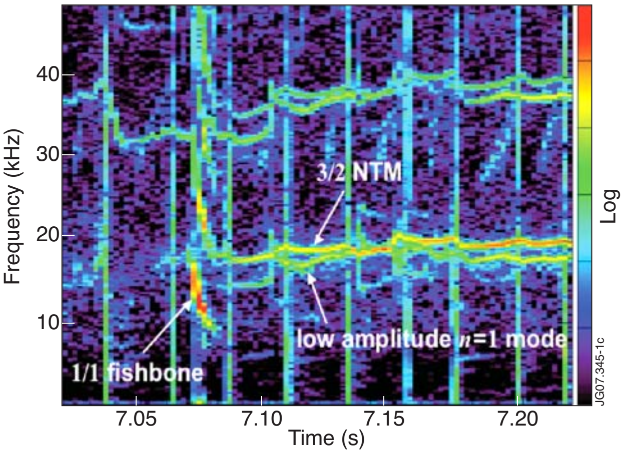

Figure 1: Magnetic spectrogram of the triggering of a 3/2 NTM by a fishbone during Pulse No: 61151 on the JET tokamak.

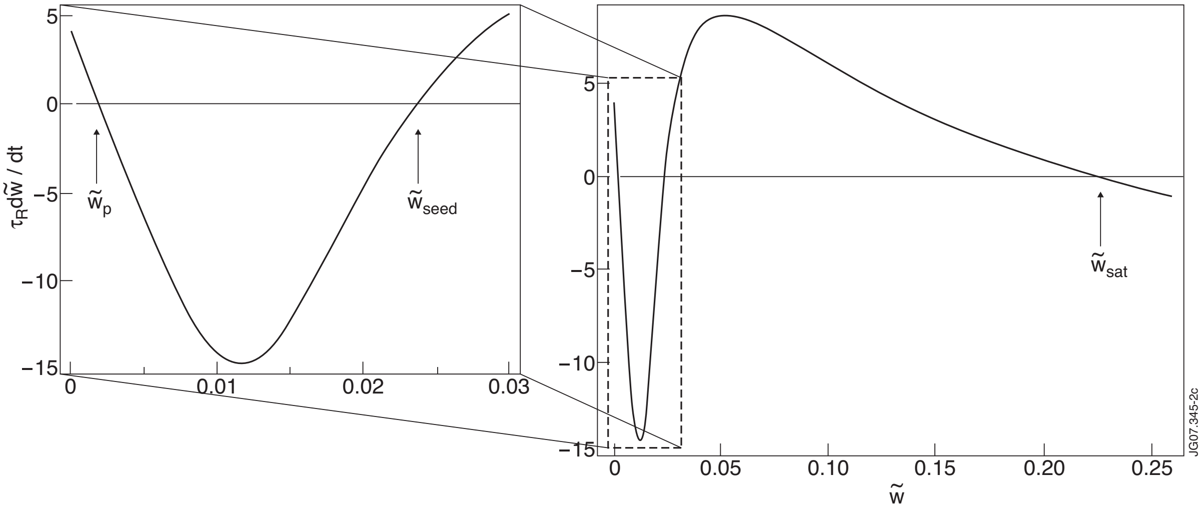

Figure 2: Right hand side of Eq. (2) for the chosen set of parameters, without any perturbation (i.e. $\dot{\phi} = \omega$). The pre-NTM, seed and saturated island widths are indicated by arrows.

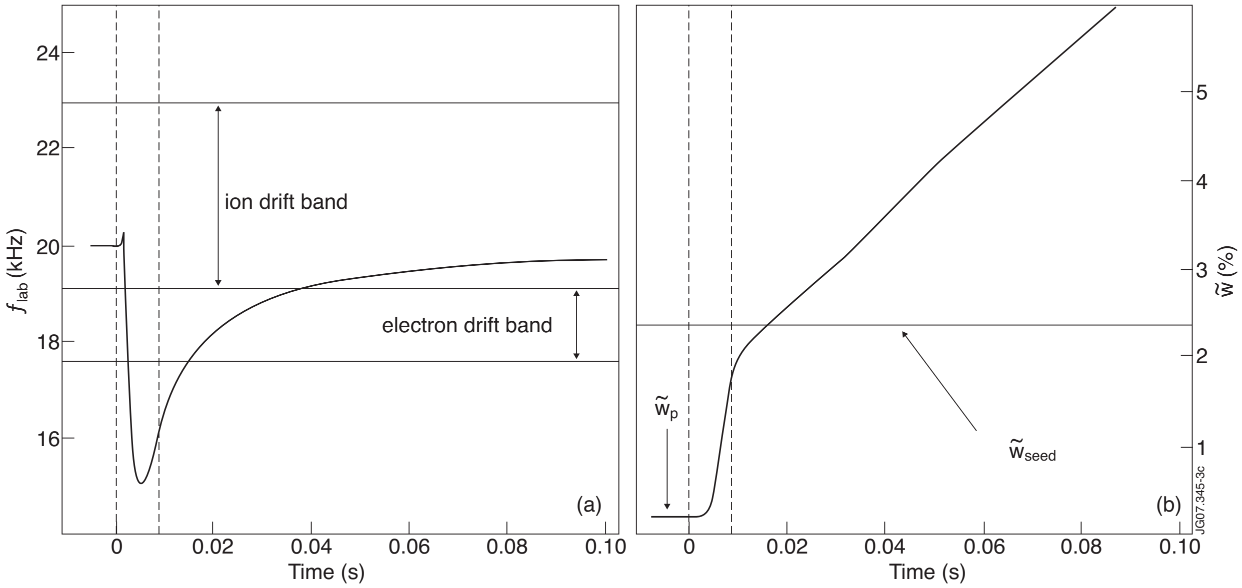

Figure 3: Evolution of the island's laboratory frequency (a) and width (b); the dashed lines indicate the beginning and end of the fishbone.

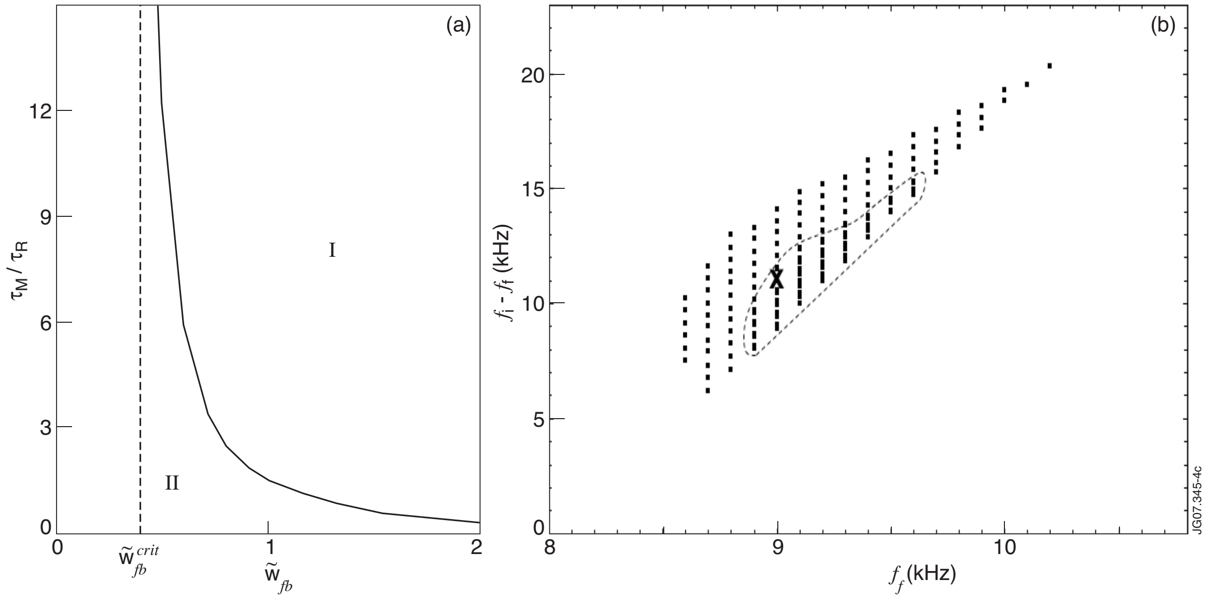

Figure 4: Influence of: (a) the viscosity and the fishbone's amplitude on the triggering (region I) or not (region II) of the NTM; and (b) the chirping rate and final frequency of the fishbone (boxes refer to cases where the NTM is triggered and the cross mark corresponds to the run of Fig.3).