EFDA–JET–PR(07)02

I.T. Chapman, S.D. Pinches, L.C. Appel, R.J. Hastie, T.C. Hender, S. Saarelma,
S.E. Sharapov, I. Voitsekhovitch, J.P. Graves and JET EFDA contributors

Modelling Sawtooth Stabilisation by Energetic Ions from Neutral Beam Injection

Modelling Sawtooth Stabilisation by Energetic Ions from Neutral Beam Injection

I.T. Chapman[1], S.D. Pinches[1], L.C. Appel[1], R.J. Hastie[1], T.C. Hender[1],
S. Saarelma[1], S.E. Sharapov[1], I. Voitsekhovitch[1], J.P. Graves[2]
and JET EFDA contributors*

[1]EURATOM/UKAEA Fusion Association, Culham Science Centre, OX14 3DB, UK.
[2]CRPP, Association EURATOM-Conf ed eration Suisse, EPFL, 1015 Lausanne, Switzerland
** See annex of M.L. Watkins et al, "Overview of JET Results ",*
(Proc. 21st IAEA Fusion Energy Conference, Chengdu, China (2006)).

Preprint of Paper to be submitted for publication in
Physical Review Letters

ABSTRACT.

Recent advances in modelling the effects of anisotropic energetic ion distributions have enabled the development of a complete coherent physics explanation of sawtooth stabilisation in both conventional and spherical tokamaks. As an example, a complete model has been developed to explain the asymmetric stabilisation of sawteeth with respect to neutral beam injection (NBI) direction in the Joint European Torus (JET). This asymmetric sawtooth stabilisation arises because of both the destabilising contribution from the counter-passing ions and the strong modification of the non-adiabatic stabilising contribution of the trapped ions due to flow shear. The fast particle effects including pressure anisotropy, sheared flows and the adiabatic response to the internal kink mode have been modelled in general toroidal geometry for the first time.

1. MODELLING SAWTOOTH STABILISATION BY ENERGETIC IONS FROM NEUTRAL BEAM INJECTION

The MagnetoHydroDynamic (MHD) stability of burning plasmas is a critical issue for operation of the International Thermonuclear Experimental Reactor (ITER). The reaction $D+T \rightarrow {}^4\text{He}(3.5\text{MeV})+ n(14\text{MeV})$ produces fusion-born α particles that can affect the stability of the plasma. One key MHD instability which will be affected by these α particles is the sawtooth oscillation – a periodic relaxation of the core plasma density and temperature. Based on experimental evidence, it is thought that the ® particles will lead to large amplitude sawteeth [1, 2], which have been shown to result in the triggering of other instabilities called neoclassical tearing modes [3], which can have deleterious ramifications for plasma confinement. As such, recent experiments have identified various methods for the control of sawteeth in order to confine the perturbation to the plasma core whilst retaining the benefits of small, frequent sawtooth crashes, such as the prevention of core impurity accumulation [4]. One such experimental technique is to apply neutral beam injection (NBI) heating in the opposite direction to the plasma current. This has been shown to result in shorter sawtooth periods than those in Ohmically heated plasmas in the Joint European Torus (JET) [5], the Mega Amp‘ere Spherical Tokamak (MAST) [6] and the Tokamak Experiment for Technology Oriented Research (TEXTOR) [7]. Furthermore, each experiment exhibits an asymmetry of sawtooth period with respect to neutral beam injection (NBI) direction. This Letter reports on modelling of the stability of the plasma with respect to the $n/m = 1/1$ internal kink mode – which is generally accepted to be related to sawtooth oscillations [8] – in the presence of NBI fast ions.

The trigger condition for a sawtooth collapse is believed to be associated with the linear stability threshold for a reconnecting 1/1 mode. Although this differs from the threshold for ideal MHD instability, it nevertheless depends strongly on the magnitude of the ideal potential energy, $\delta W = \delta W_{thermal} + \delta W_{hot}$. It has been proposed [1] that – provided the magnetic shear at $q = 1$, s_1, exceeds a critical value – reconnection is triggered when $c\rho r_1 > \delta W$, where c is a normalisation coefficient, r_1 is the radial position at which $q = 1$ and ρ is the ion Larmor radius. In Reference [1], a $1^1\!/_2$D transport code was used to study the time evolution of $s_1(t)$, $r_1(t)$ and $\delta W(t)$ in order to determine when the trigger inequality is satisfied. Typically, the ideal potential energy $\delta W(t)$ is positive (ideal stable) and reaches a quasi-stationary value on a relatively short time scale (the reheating timescale for δW_{th}, and the energy deposition timescale for δW_h), while $s_1(t)$ and $r_1(t)$ evolve on a slower timescale, determined by core resistive diffusion. Assuming that the trigger criterion can be represented by a linear time dependence for $r_1(t) \approx \hat{r}_1 t/\tau_\eta$, with τ_η a measure of the resistive diffusion time, and a quasi-stationary value of $\delta W(t)$, then the sawtooth period, τ_s, can be represented in the form $\tau_s/\tau_\eta \sim (\delta W_{th} + \delta W_h)/c\hat{r}_1$ with longer ramp times predicted when δW_h is larger, and shorter ramp times predicted when δW_h is smaller. In what follows we assume that this applies so that the sawtooth period is roughly proportional to δW.

In MAST, the asymmetric stabilisation of sawteeth has been explained in terms of the direction of strong toroidal flows relative to the ion diamagnetic drift [6]. Whilst fast particles do have a stabilising influence upon sawteeth in spherical tokamaks, they cannot explain the experimentally observed minimum in sawtooth period. However, in larger aspect ratio devices, where the toroidal rotation is smaller, the sawtooth behaviour can only be explained by the effects of the fast ions. An appropriate tool for studying the effects of anisotropic fast particles on the internal kink mode is the HAGIS code [9]. HAGIS solves the Hamiltonian equations describing the guiding centre motion of ions in realistic toroidal geometry. The code has been extended to calculate the contribution of the fast beam ions to the potential energy of the internal kink mode, δW_h. The code now also includes the effects of the equilibrium flow shear, which modifies the toroidal precession frequency of the particle orbits as well as the electric potential experienced by the beam ions, $\phi_\mathrm{f} = rB_0 v_\zeta/q$ where ϕ_f is the electric potential due to the plasma flow, v_ζ is the toroidal rotation velocity, r is the minor radius and B_0 is the equilibrium magnetic field. The eigenmode structure is computed by the MHD stability code, MISHKA-F [10], which includes the effects of flows and ion diamagnetic drifts in toroidal geometry. HAGIS subsequently determines the stability of the mode including kinetic effects, where since $\beta_{hot} \sim 0.2\beta_{bulk}$ ($\beta = 2\mu_0 p/B^2$) it is appropriate to assume that the kinetic modification of δW does not alter the form of the perturbation, $\vec{\xi}$. HAGIS treats the effects of finite orbits on the adiabatic response to the internal kink mode, as well as the effect of pressure anisotropy on δW_h, which have been neglected in other hybrid kinetic-MHD codes [11]. The kinetic effects of thermal ions at finite rotation have not been considered since this was examined in Reference [12].

The fast ion distribution function is separated into an equilibrium component, $f_0(\varepsilon^0, P^0{}_\zeta, \mu)$, and two perturbed components, $\delta f_h = \delta f_{hk} + \delta f_{hf}$, a non-adiabatic (kinetic) and an adiabatic (fluid) part respectively. Here, the particle energy ($\varepsilon^0 = mv^2/2$), the canonical momentum ($P^0{}_\zeta = mB_\zeta v_{||}/B - e\psi_p$) and the magnetic moment ($\mu = mv_\perp{}^2/2B$) are the unperturbed constants of motion, where m is the particle mass, ζ is the toroidal angle, v is the particle velocity, ψ_p is the poloidal flux, e is the fast ion charge and "$||$" and "$_\perp$" mean components parallel and perpendicular to the magnetic field respectively. Analytic theory developed for large aspect ratio circular plasmas [13] implies that these contributions to the perturbed distribution function can be expressed as

$$\delta f_{hk} \quad \sum_{l=-\infty}^{\infty} \frac{\omega - \Delta\Omega - n\omega_{*h}}{\omega - \Delta\Omega - n\langle\zeta\rangle + l\omega_b} \; \frac{\delta f_h}{\delta\varepsilon^0} \; e^{-i(\omega + l\omega_b + \langle\dot{\zeta}\rangle)t} \tag{1}$$

and $\delta f_h \sim \vec{\xi} \cdot \vec{\nabla}\psi_p \delta f_h / \delta P^0{}_{\zeta}$ where $\omega_{*h} = (\delta f_h / \delta\varepsilon^0)$ is the hot ion diamagnetic frequency, $\Delta\Omega = \Omega_E(r) - \Omega_E(r_1)$ is the sheared toroidal flow, $\tilde{\omega}$ is the Doppler shifted mode frequency, $\omega_b = 2\pi/\tau_b$ and τ_b is the poloidal orbit transit time. In HAGIS the adiabatic [?] and nonadiabatic components of the perturbed distribution function are: $\delta f_{hf} = \alpha g \delta f_0 / \delta P^0{}_{\zeta} + \phi \delta f_0 / \delta\varepsilon^0$ and $\delta f_{hk} - \dot{P}^0{}_{\zeta}\, \delta f_0 / \delta P^0{}_{\zeta} - \dot{\varepsilon} \delta f_0 / \delta\varepsilon^0$ respectively, where the covariant ζ component of the magnetic field, $B_{\zeta} = g(\psi)$, ε is the energy and the vector potential is given by $\mathbf{A} = \alpha \mathbf{B}_0$. Given δf, the hot particle contribution to the potential energy of the $n = 1$ internal kink mode is then calculated as [15–17]

$$\delta w_h = \frac{1}{2} \int d\Gamma(m v_{\|}^2 + \mu B_h)\, \delta f \sum_m \vec{\kappa} \text{ ¥ } \vec{\xi}^{(m)*} (r, t) e^{-i(n\zeta - m\theta)} \tag{2}$$

where μ is the poloidal angle, $\vec{\kappa} = b \cdot \vec{\nabla}\mathbf{b}$ is the magnetic curvature vector and $\mathbf{b} = \mathbf{B}/B$.

Previous modelling has concentrated primarily on the effects of the trapped fast particles [11]. However, the neutral beam heating in JET gives rise to a predominantly passing population. Recent analytic theory [18, 19] has suggested that the co-passing energetic particles can stabilise the 1/1 internal kink mode whereas counter-passing fast ions can have a destabilising influence. These effects are modelled in general tokamak geometry for the first time, and are found to be of paramount importance for analysing the asymmetric dependence of sawtooth stability with respect to the direction of the NBI. Wang *et al* [19] proposed that the non-adiabatic passing ion effects arose due to the gradient $\vec{\nabla} f_h$ integrated over the $q = 1$ radius. In contrast, Graves [18] suggested that the non-adiabatic passing particle effects are counter-acted by an adiabatic contribution, but that an additional adiabatic contribution survives from the fast ions which intersect the $q = 1$ flux surface. This latter mechanism depends on $\delta f_h / \delta P_{\zeta}$ at $q = 1$ only and is more sensitive to localised heating. Figure 1 shows the passing particle contribution to δW_h for a non-symmetric fast ion distribution which is Maxwellian with respect to energy and Gaussian with respect to pitch angle. The distribution function is artificially taken to be zero outside a finite radius, indicated on the x-axis. It is evident that when no gradient exists in a region around 'q'$= 1$ ($s \sim 0.3$) bounded by the orbit width, the passing ions do not contribute to the kink mode stability. The fact that unbalanced passing ions contribute only via a radial gradient in f_h close to $q = 1$ has important implications for sawtooth control in ITER using Negative NBI heating at varying deposition radii [18]. The strong contribution of the passing particles comes from ions close to the trapped-passing boundary [20] where their orbit widths, Δ_b, are large, $\delta W_h \sim \Delta b$.

In order to analyse the JET experimental results [5] concerning sawtooth stability, HAGIS has been used to calculate δW_h. Together with the contribution from MHD effects including toroidal flow, calculated using MISHKA-F, the stability of the kink mode for a range of beam powers can be evaluated. The JET discharge which has been analysed is shot 60998 (co-injected beams of 4.13MW,

I_p = 2.3MA, B_T = 2.5T, $\bar{n}_e = 3.2 \times 10^{19}\,\mathrm{m}^{-3}$ and $v_\xi \sim$ 45km/s). The neutral beam current drive has been calculated [?] and found to be broadly deposited and less than 10% of the Ohmic current, meaning it is relatively insignificant in these discharges. The equilibria are reconstructed using the HELENA code [21], which takes as input the plasma shapes and q-profiles from EFIT [22] and the pressure profile obtained from the TRANSP Monte-Carlo transport code [23]. The position of the $q = 1$ surface is constrained by the inversion radius found from the Soft X-ray diagnostic. The pressure profile includes a contribution from the neutral beam fast particles, which are treated as isotropic at this stage.

The fast particle distribution function was obtained from TRANSP. The exact distribution function is retained, though it can be described approximately as a slowing down distribution with respect to energy and a Gaussian distribution with respect to pitch angle, centred around $\lambda = v_{||}/v = 0.5$. The dependence of the distribution function upon energy and pitch angle is illustrated in Figure 2. By retaining the complete distribution function, the complicated dependence of the pitch angle distribution width upon the normalised poloidal flux and the particle energy is treated accurately. This is important since the degree of anisotropy of the fast particle distribution can significantly change the contribution of the trapped particles to δW_h.

The contribution to the stability of the $n = 1$ internal kink mode from each class of particles for a static plasma is shown in Figure 3. The potential energy is normalised in the same way as Reference [13] such that $\delta W_h = \delta W_h \mu_0 / (6\pi^2 R_0 \vec{\xi}_0^{\,2} \varepsilon_1^4 B_0^2)$ where $\varepsilon_1 = r_1/R_0$ and ξ_0 is the displacement at the magnetic axis. It can be seen that for the realistic beam distribution function employed in these simulations, the passing particles – which are often ignored in studies concerning energetic particles – are as important as the trapped particles. In accordance with analytic theory [18], the co-passing particles are strongly stabilising whereas the counter-passing particles give a destabilising contribution which nearly balances the strong stabilisation from the trapped population. Whilst only ideal stability is considered here, it has been shown [24] that including resistivity only slightly modifies the stability boundary and the instability drive is still from the asymmetric passing ions. The non-adiabatic trapped particles are always stabilising [16], and for the level of parallel anisotropy present in this distribution function, the adiabatic terms are only weakly stabilising. The contribution from the passing particles helps to explain the different sawtooth behaviour exhibited in JET when injecting the beam in different directions relative to the toroidal field, since the $\vec{\nabla} B$-drifts of the energetic particles determine whether they are stabilising or not. However, as shown below, only the inclusion of toroidal flow shear enables an understanding of the minimum in sawtooth period.

The effect of toroidal flow shear is modelled by prescribing the experimental toroidal rotation profile measured by the Charge Exchange diagnostic. The co- and counter-NBI profiles are very similar and are approximately linearly sheared with respect to poloidal flux, ψ. Whilst the absolute values of the flows are only relatively small, there are strong toroidal flow *shears* present in JET, which can modify the stabilisation of the energetic ions [25]. The effect of sheared rotation on both $R(\delta W_h)$, which quantifies the stabilising effect of the fast ions, and $F(\delta W_h)$, agrees well with Figure

6(b) of Reference [13].

Conservation of the third adiabatic invariant, Φ – which produces strong stabilisation from trapped fast particles [16] – is only obtained [25] when $\langle \omega_d \rangle + \Delta\Omega - \tilde{\omega} >> 0$, where $\langle \omega_d \rangle$ is the bounce averaged hot particle toroidal drift precession frequency. Since this condition is more readily satisfied for $\Delta\Omega > 0$, co-rotating plasmas with velocity shear support more effective stabilisation of the kink mode. Conversely, the stabilising effect is diminished in counter-rotating plasmas ($\Delta\Omega < 0$) since Φ-conservation is inhibited, and the stabilising contribution can only come from the fewer higher energy ions.

When the non-adiabatic effect of the trapped particles is modified by the sheared flow, the contribution to the stability of the internal kink mode changes significantly, as illustrated in Figure 4. For each power, the modelled toroidal flows are as measured experimentally by the Charge Exchange diagnostic. The mode frequency, $f = \omega_{*i} \sim 0.5$kHz is of the same order as the toroidal plasma rotation frequency. The minimum in mode stability occurs at approximately 3MW of counter NBI power, in excellent agreement with the minimum in sawtooth period exhibited experimentally [?]. This minimum arises because (i) the flow shear in JET reduces the stabilising effect of the trapped ions injected counter-current, and (ii) the counter-passing ions provide a strongly destabilising contribution. In co-rotating plasmas the sheared flows amplify the stabilising contribution from the trapped ions and the co-passing ions are also strongly stabilising.

We present thorough modelling of JET discharges which exhibit an asymmetric dependence of 1/1 mode stability upon the direction of neutral beam heating. The effects of anisotropy and flow shear have been included for both adiabatic and non-adiabatic trapped and passing fast particles. It is found that the passing ions, which represent the majority of energetic particles injected into JET through NBI heating, are strongly stabilising when injected in the same direction as the plasma current, but destabilising for counter-NBI. For co-NBI, the stabilising role of the non-adiabatic trapped ions is increased when flow shear is included. Conversely, for counter-NBI, the trapped particles are less stabilising. This means that the plasma is most unstable to $n = 1$ kink modes when heated with counter-current neutral beams, in excellent accordance with the experimental data. By considering the effects of toroidal flow shear and pressure anisotropy, a complete model now exists which can accurately simulate $n = m = 1$ mode stability including kinetic effects in general toroidal geometry.

ACKNOWLEDGEMENTS

Thanks to Dr D Borba and Dr R Akers for helpful discussions. This work was funded jointly by the United Kingdom Engineering and Physical Sciences Research Council and by the European Communities under the contract of Association between EURATOM and UKAEA. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

REFERENCES

[1]. F. Porcelli, D. Boucher, and M. Rosenbluth, Plasma Phys. Control. Fusion **38**, 2163 (1996).

[2]. B. Hu et al., Phys. Plasmas **13**, 112505 (2006).

[3]. O. Sauter et al., Phys. Rev. Lett. **88**, 105001 (2002).

[4]. M. Nave et al., Nucl. Fusion **43**, 1204 (2003).

[5]. M. Nave et al., Phys. Plasmas **13**, 014503 (2006).

[6]. I. Chapman et al., Nucl. Fusion **46**, 1009 (2006).

[7]. H. Koslowski, Fusion Sci. Technol. **47**, 260 (2005).

[8]. R. Hastie et al., Phys. Fluids **30**, 1756 (1987).

[9]. S. Pinches et al., Comput. Phys. Commun. **111**, 133 (1998).

[10]. I. Chapman et al., Phys. Plasmas **13**, 065211 (2006).

[11]. C. Angioni et al., Plasma Phys. Control. Fusion **44**, 205 (2002).

[12]. J. Graves et al., Plasma Phys. Control. Fusion **347**, B121 (2005).

[13]. J. Graves et al., Phys. Plasmas **10**, 1034 (2003).

[14]. J. Candy et al., Phys. Plasmas **4**, 2597 (1997).

[15]. B. Briezman et al., Phys. Plasmas **5**, 2326 (1998).

[16]. F. Porcelli, Plasma Phys. Control. Fusion **33**, 1601(1991).

[17]. J. Graves, Phys. Plasmas **12**, 090908 (2005).

[18]. J. Graves, Phys. Rev. Lett. **92**, 185003 (2004).

[19]. S. Wang et al., Phys. Rev. Lett. **88**, 105004 (2002).

[20]. J. Graves et al., in preparation (2007).

[21]. G. Huysmans et al., in Proc. CP90 Conf. on Computational Physics Proc. (World Scientific, 1991), p 371.

[22]. L. Lao et al., Nucl. Fusion **30**, 1035 (1990).

[23]. R. Budny et al., Nucl. Fusion **32**, 429 (1992).

[24]. Y. Kolesnichenko et al., Phys. Plasmas **12**, 022501 (2005).

[25]. J. Graves, R. Hastie, and K. Hopcraft, Plasma Phys. Control. Fusion **42**, 1049 (2000).

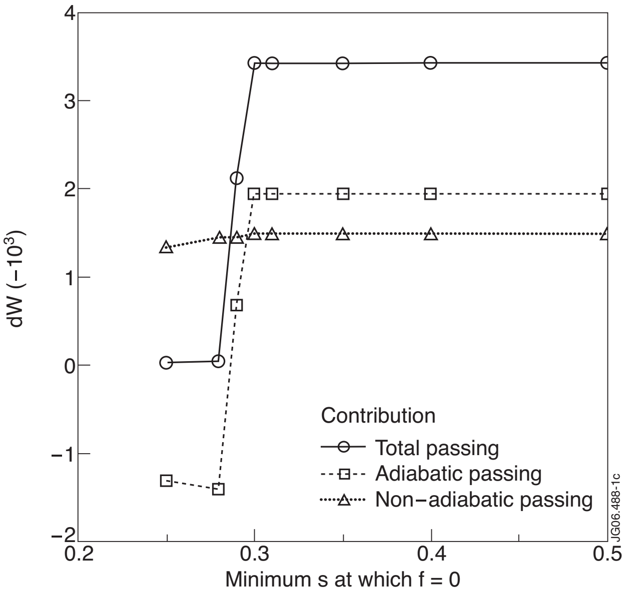

Figure 1: The contribution to δW_h from passing ions for a hot ion distribution function nullified outside $s = \sqrt{\psi}$.

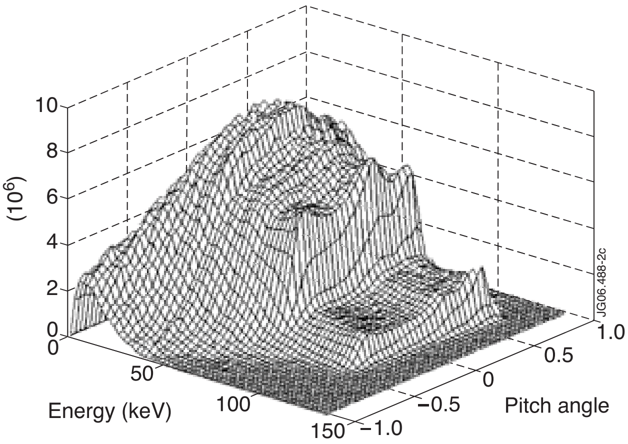

Figure 2: The fast particle distribution function as a function of particle energy and pitch angle, $\lambda = v_{||}=v$, at $r/a = 0.35$ $\theta = 0$. The distribution function is approximately Gaussian with respect to λ for high energy particles, but approaches isotropy for low energy particles. The distribution function dependence upon λ is also biased in terms of radial location.

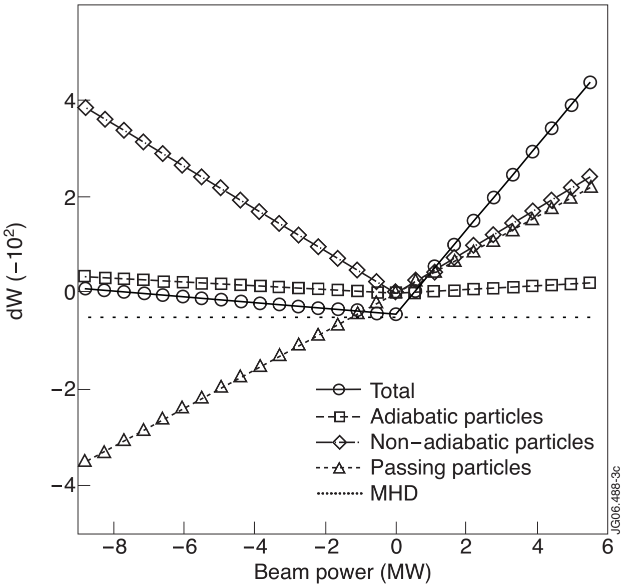

Figure 3: The contribution to δW from each energetic particle species with respect to injected beam power for a JET equilibrium (Pulse No: 60998) unstable to a 1/1 kink mode.

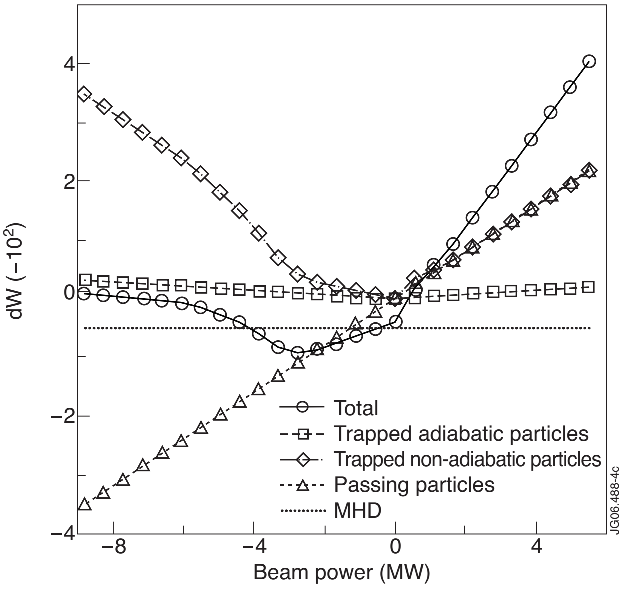

Figure 4: The contribution to δW from each energetic particle species with respect to injected beam power for a JET equilibrium (Pulse No: 60998) including flow shear effects.