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Observation and Explanation of the JET $n = 0$ Chirping Mode

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* See annex of J. Pamela et al, "Overview of JET Results",
(Proc.  IAEA Fusion Energy Conference, Vilamoura, Portugal (2004)).

ABSTRACT

Persistent rapid up and down frequency chirping modes with a toroidal mode number of zero ($n = 0$) have been observed in the JET tokamak when energetic ions, with a mean energy ~ 500 keV, were created by high field side ion cyclotron resonance frequency heating. This heating method enables the formation of an energetically inverted ion distribution function that allows ions to spontaneously excite the observed instability, identified as a global geodesic acoustic mode. It is interpreted that phase space structures form and interact with the fluid zonal flow to produce the pronounced frequency chirping.

INTRODUCTION

Future burning plasma experiments, such as ITER, will deal with plasmas which consist of a core component characterized by some base temperature and an energetic particle component with a much higher mean energy. Such plasmas often give rise to nonlinear phenomena that are based on both MHD fluid properties and intrinsic kinetic particle properties. In this Letter we investigate an unusual set of JET data [1, 2] which exhibits nonlinear oscillations based on these dual properties. Specifically, high clarity, up and down frequency chirping modes are observed at frequencies that are initiated at about 1/3 the TAE frequency. R. Heeter [1] was the first to realize that these were $n = 0$ modes (n is the toroidal mode number) but the physics of the mode was undetermined. Here we infer that they are due to the combined effects of MHD fluid behaviour that is related to the geodesic acoustic mode (GAM) [3] and zonal flow (see refs. [4, 5, 6] and references therein) and the kinetic response arising from the kinetic resonant interaction of the energetic particle component.

In Fig. 1 we show a specific case where $n = 0$ chirping oscillations were observed in JET. Frequency chirping is seen that persists for nearly a second (in some cases chirping persists for the entire time the ion cyclotron resonance heating (ICRH) is applied). These oscillations are initiated only when high field side (HFS) ICRH of minority protons is applied. The heating power varies, but is typically ~ 5 MW. It is found, through numerical modelling with the SELFO code [7, 8], that the protons form an energetic tail with a mean energy of ~ 500 keV. The chirping is quenched by the application of comparable neutral beam injection (NBI) heating power or the termination of the ICRH. These modes are picked up by Mirnov coils, which measure the magnetic fluctuations at the plasma's edge. These signals determine the instantaneous mode frequency and toroidal mode number. The initiating frequencies typically occur between 20 and 60 kHz depending on equilibrium plasma conditions. Most of these oscillations have been observed in discharges with a reversed q profile deduced from the existence of Alfvén cascades [9] but they have also been observed in discharges with steady frequency TAEs and no Alfvén cascades, suggesting that these modes can be excited in either flat or monotonically increasing q profiles. Additional evidence for this mode arises from reflectometer signals [10] and from soft X-ray detectors, both showing

strong frequency correlation with the Mirnov coil signals.

The initiating frequency of the $n = 0$ mode is plotted in the white curve in Fig. 1. This initiation frequency is found to scale as $T_e^{1/2}(r/a = 0.15)$, as measured by the multi-channel ECE diagnostic. Note that $T_e \gg T_i$ as energetic protons (~ 500 keV) primarily heat electrons rather than deuterium ions in the bulk plasma. When the mode is excited the effect of T_i is observed by suddenly applying neutral beams (the beam energy is a mix of 80 and 120 keV components which directly heat core deuterium ions and electrons at comparable rates) with a combined power that exceeds 5 MW. The initiation frequency of the mode first increases substantially over a short time interval of ~ 0.1 s and then the chirping mode terminates. This is interpreted as being due to the sudden increase of T_i in accord with theoretical considerations discussed below.

The Linear Eigenmode. The experimental data shown in Fig. 1, where $T_e \gg T_i$, demonstrates that the basic mode frequency scales as $T_e^{1/2}$. This scaling suggests that the $n = 0$ continuum geodesic acoustic mode (GAM) [3] is the appropriate linear mode as its dispersion relation is $\omega^2 = \frac{\gamma p}{\rho R^2} \left(2 + \frac{1}{q^2}\right)$ where γ is the adiabatic constant, ρ is the mass density, $p = \rho T_e(1 + T_i/T_e)$ is the plasma pressure and R is the major radius of the tokamak. As $T_i/T_e \ll 1$, we obtain the observed scaling of frequency with electron temperature. However, as the GAM mode is a continuum mode there is no definite frequency associated with it. In past literature the global property of the GAM has been attributed to the additional physics mechanisms in the plasma such as the anisotropy of particle sources and turbulent fluxes. [4, 5, 6] In contrast we have found that a global geodesic curvature mode (GGAM) emerges from first principle MHD equations. The eigenfunction structure of the GGAM is found to have similar structure as the GAM, with the addition of a significant magnetic component.

The properties of the GAM are obtained by examining the following equations that have been derived in MHD theory for a continuum mode [11] on a magnetic flux surface,

$$\rho \frac{\omega^2 |\nabla\psi|^2}{B^2} Y + \frac{\partial}{J\partial\theta} \left(\frac{|\nabla\psi|^2}{B^2 J} \frac{\partial Y}{\partial\theta} \right) + \gamma p \kappa_s Z = 0 \quad (1)$$

$$\kappa_s Y + \left(\frac{\gamma p + B^2}{B^2} \right) Z + \frac{\gamma p}{\rho \omega^2 J} \frac{\partial}{\partial\theta} \left(\frac{1}{B^2 J} \frac{\partial Z}{\partial\theta} \right) = 0 \quad (2)$$

We refer to reference [11] for the definition of the quantities represented here in their standard notation. Note that the geodesic curvature is defined as $\kappa_s = 2\vec{\kappa} \cdot (\vec{B} \times \nabla\psi/B^2)$ with $\vec{\kappa}$ the magnetic curvature and the dependent variables defined as, $Y(\theta) = \vec{\xi} \cdot (\vec{B} \times \nabla\psi/|\nabla\psi|^2)$ and $Z(\theta) = \nabla \cdot \vec{\xi}$, which are surface quantities.

To solve for a low frequency mode at low beta we take $Y(\theta) = 1 + \delta Y(\theta)$ (where $\delta Y(\theta)$ is assumed $\ll 1$) and with the periodicity conditions we find,

$$\begin{aligned} \frac{\partial}{\partial\theta} \left(\frac{|\nabla\psi|^2}{B^2 J} \frac{\partial \delta Y}{\partial\theta} \right) &= -J\gamma p \kappa_s Z - \rho \frac{\omega^2 J |\nabla\psi|^2}{B^2}, \Rightarrow \\ \int_{-\pi}^{\pi} d\theta J \left(\gamma p \kappa_s Z + \rho \frac{\omega^2 |\nabla\psi|^2}{B^2} \right) &= 0 \end{aligned} \quad (3)$$

Then using Eq. (2) to solve for $Z(\theta)$, we find

$$Z + \frac{\gamma p}{\rho \omega^2 J} \frac{\partial}{\partial \theta} \left(\frac{1}{B^2 J} \frac{\partial Z}{\partial \theta} \right) = -\kappa_s. \quad (4)$$

The solution to Eq. (4) in the large aspect-ratio, circular cross-section limit is,

$$Z(\theta) = [2(\sin \theta) B_\theta / B] \left[1 - \frac{\gamma p}{\rho \omega^2 (r B / B_\theta)^2} \right]^{-1}. \quad (5)$$

Using this expression and the relation $\kappa_s = 2(\sin \theta) B_\theta / B$, we evaluate the integral in Eq. (3) to find the desired eigenfrequency.

Using the resistive MHD normal mode analysis code CASTOR [12] for a model reversed shear equilibrium with circular cross section, a global mode was found when the GAM continuum had a maximum frequency at an off-axis radial position. Such a maximum can occur when there is shear reversal. The eigenmode structure is shown in Fig. 2 and the mode frequency is found to lie just above the maximum GAM frequency, see Fig. 3. Like the GAM, we see in Fig. 2(a) that Y , has a radially localized mode structure that is dominantly $m = 0$ while Z , which is proportional to the plasma density perturbation, is also highly localized radially with primarily an $|m| = 1$ poloidal mode component. Local GAM theory predicts $Z_{m=1}/Y_{m=0} = B_\theta/B$. For the parameters of the numerical calculation this ratio is reproduced within an inverse aspect ratio of the peak of the mode. In Fig. 2(c) we plot the poloidal m -spectrum of the geodesic component of the perturbed magnetic field, $\delta B^g \equiv (\delta \vec{B} \cdot \vec{b} \times \nabla \psi / |\nabla \psi|)_m$, as a function of radius. Though this component, dominated by $|m| = 2$, is predominantly localized in the same region as Y and Z , there is a smaller, but significant, non-localized component of the perturbed magnetic field, not present in Figs. 2(a) and 2(b), outside the core of the mode that can be detected by Mirnov coils.

The GAM's maximum frequency will be on the magnetic axis when the electron temperature monotonically decreases radially (this is always the case in our experimental studies) and the q -profile monotonically increases radially (this appears to be the case in a few of the experimental observations). In this case a GGAM has not been found in our MHD studies. However, as the GGAM eigenmode when found is very localized, we conjecture that the inclusion of additional finite Larmor radius terms into the theory will enable a global mode to be established when the GAM frequency has a maximum at the magnetic axis. Alternatively, it is possible that the GGAM is being established in these experiments via other mechanisms such as due to an anisotropic source or turbulence as is cited in the literature [4, 5, 6].

We also note that accounting for the ion temperature will improve the comparison of the data with theory. In Fig. 1(a) the white curve is the inferred frequency of the mode when one neglects the ion temperature. Accounting for finite T_i/T_e would increase the frequency of the theoretical curve, and that should make the correlation of experimental and theoretical frequencies closer to each other. Increasing ion temperature also explains the increase in the frequency when more than 5 MW of 80/120 keV neutral beams are injected. These deuterium beams cause a direct heating of ions, which then increases both T_i and T_e , as well as T_i/T_e to enable an increase of the frequency of

a mode associated with a GAM as ω is proportional to $(T_e + T_i)^{1/2}$. We also note that when neutral beam heating is imposed, additional dissipative processes arise that can prevent the GGAM from being spontaneously excited. Two additional mechanisms are apparent. These are the ion Landau damping that arises from the increased ratio of T_i/T_e that results from increased ion beam heating and the other is the resonance damping that arises from the neutral beams themselves which forms a monotonically decreasing distribution function as a function of energy for constant μ and P_ϕ . We suggest that these increased dissipative processes are the cause of the observed quenching of the spontaneous excitation of the GGAM that takes place following the imposition of neutral beams, but only after a time delay of ~ 0.1 s during which the above mentioned increase of the GGAM initiation frequency arises.

Free Energy Source. The source of instability to spontaneously excite $n = 0$ modes must come from an inverted energy distribution, $\partial F(E, \mu, P_\phi)/\partial E|_{\mu, P_\phi} > 0$. To ascertain whether such a drive is formed in this experiment the Monte-Carlo SELFO code was used to determine the distribution function for HFS minority proton ICRH. It was found that mirror trapped energetic protons whose turning point magnetic fields are less than the ICRH resonance magnetic field, do indeed form such a distribution.

We attribute the pronounced frequency sweeping being observed to the formation of phase space structures, whose general nonlinear theory has been discussed in references [13, 14]. It is required that the instantaneous experimental frequency be equal to an orbit resonance frequency, which in our case is the bounce orbit frequency of a magnetically trapped particle. The HAGIS code [15] was used to determine the bounce frequencies of particles with a fixed μ and P_ϕ and it was found that the experimental frequency range of the chirp (32-42 kHz) of the observed $n = 0$ mode corresponded to the bounce frequency range of particles whose energy varied from 220 to 260 keV in a region where the SELFO code indicated that the distribution was energy inverted. The highest energy corresponded to the bounce frequency of a deeply trapped particle and the lowest frequency to a particle with a turning point at the ICRH resonance layer. It should be noted that it is the nature of mirror confinement that higher (lower) energy particles correspond to lower (higher) bounce frequencies.

Non-linear Evolution. The changing mode frequency in the experiment is interpreted as a synchronism between the mode frequency and the resonance frequency as explained in reference [14]. At the phase space region associated with a given mode frequency there are particles trapped in the fields of the wave. The distribution function of these trapped particles are in the form of a clump (hole) where the wave trapped particle distribution has a larger (smaller) value than the ambient distribution. The clump (hole) cannot be stationary in time as the structure needs to be compensated for the power being absorbed by dissipation present in the background plasma which forces the structure to move in phase space in a manner that extracts energy from the distribution function. The mode frequency synchronized with this structure likewise shifts. Hence a clump (hole) must head to lower (higher) energy, and thus because of the bounce frequency energy dependence

of mirror trapped particles the frequency must increase (decrease). This description is compatible with the signals in Fig. 1 where we see the highest frequency signals stagnating in time. They can be interpreted as clumps that have been convected in phase space to the deeply trapped mirror orbits of the tokamak. Having reached the maximum bounce frequency of the particles, there is no higher frequency to shift to. Note that in this experiment the frequency increase (decrease), associated with the motion of clumps (holes), is opposite from that arising in most other chirping events reported [14, 16].

There is an aspect of the frequency chirping being observed that is in need of deeper understanding. Previously we described the energetics of frequency chirping in accord with the theory developed in reference [14]. In these references a quantitative chirping theory is developed under the assumption that the basic oscillation is primarily due to the excited linear wave and the phase space structures that are formed only perturb the basic wave dynamics. In such a description, the chirping is being described on the basis of a weak perturbation theory that appears to be incompatible with the data, where large frequency shifts are found (however previously used arguments to determine the directions of the frequency chirping are still applicable as they are based on more general energy concepts). Thus there is a need to develop a non-perturbative theory that simultaneously accounts for the kinetic phase space structures and for the fluid dynamics which is taking the form of a zonal flow that arises during the excitation of an $n = 0$ mode [4, 5, 6]. This zonal flow initially gives rise to the GGAM as described by linear theory, but as the frequency shifts the dynamics of the interaction of zonal flow with phase space structures, takes on a considerably different, and as yet undeveloped, nonlinear description.

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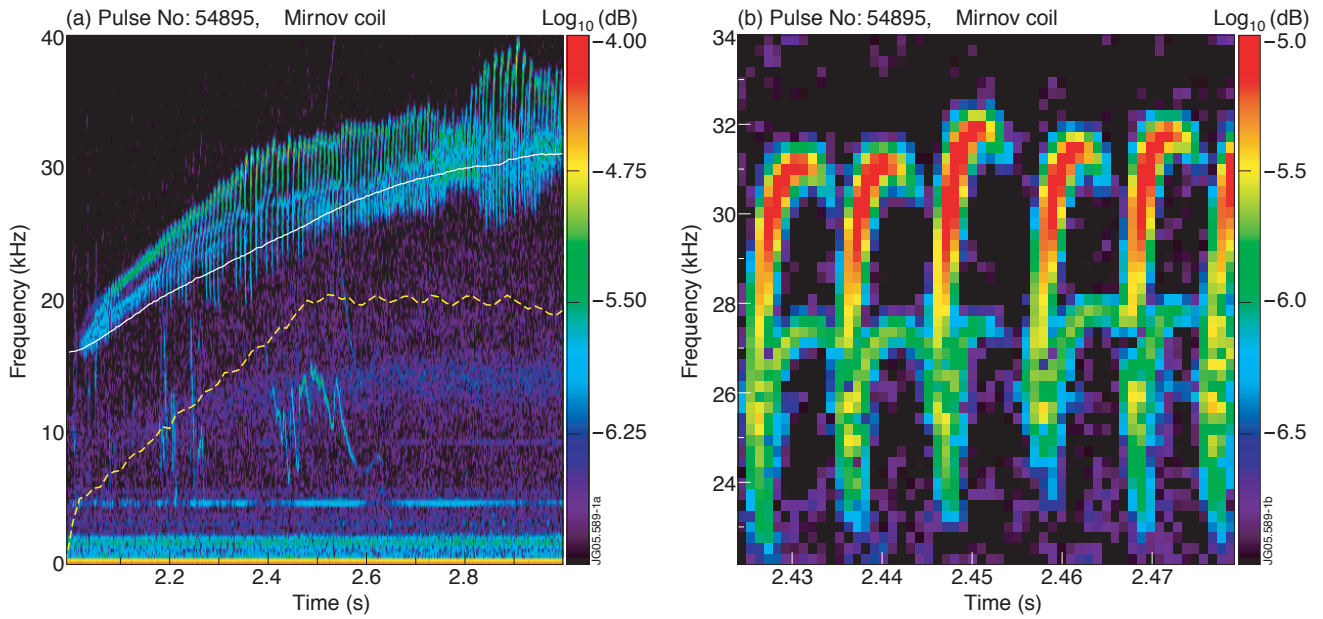


Figure 1: Spectrograms of JET Mirnov coil data. (a) Rapidly chirping $n = 0$ mode. (b) Zoom showing frequency chirping in detail. In both plots the white line is proportional to $\sqrt{T_e}(r/a = 0.15)$. At 2.0 s the electron temperature is 1.0keV and at 3.0s it is 5.1keV. Yellow curve in (a) indicates the applied ICRH power where the power is off at 2.0s and is 5MW at 3.0s

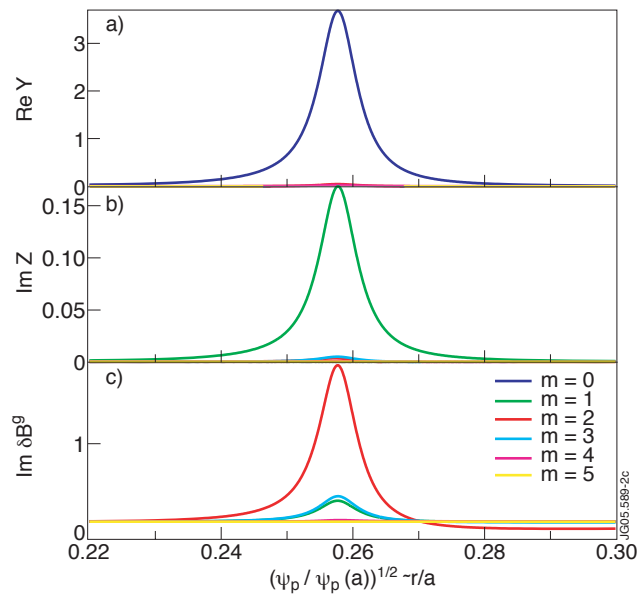


Figure 2: Eigenmode structure for the global geodesic acoustic mode for a reversed shear, circular JET-like equilibrium. (a) $\text{Re } Y_m(r)$, (b) $\text{Im } Z_m(r)$, (c) $\text{Im } \delta B_m^g(r)$. Note that $\text{Re } Y_m(r) = \text{Re } Y_{-m}(r)$, $\text{Im } Z_m(r) = -\text{Im } Z_{-m}(r)$, $\text{Im } \delta B_m^g = -\text{Im } \delta B_{-m}^g$ and that $\text{Im } Y_m(r) = \text{Re } Z_m(r) = \text{Re } \delta B_m^g = 0$.

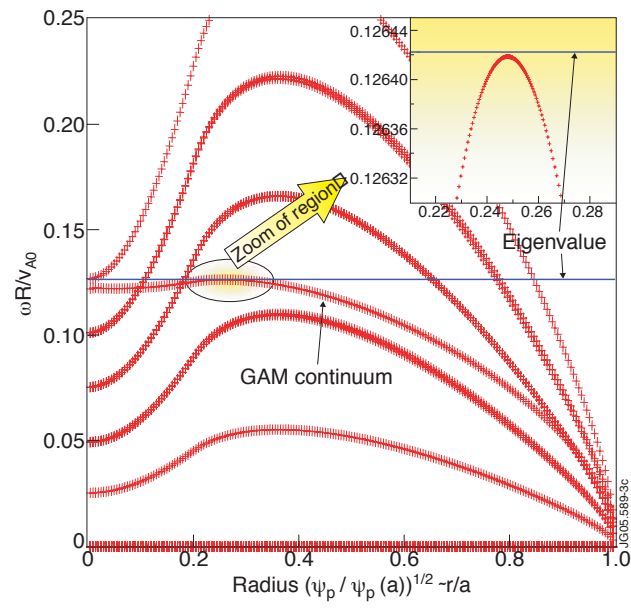


Figure 3: Plot of the $n = 0$ GAM and acoustic continuum showing that the global mode (GGAM) is situated just above the GAM continuum. Note, the GGAM experiences continuum damping well away from its region of localization