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ABSTRACT.
A multi-channel reflectometer diagnostic probing the mid-plane plasma in the O-mode polarisation is routinely used for detection of Alfvén Eigenmodes (AEs) on the JET tokamak. The AEs are observed on the spectrogram of the homodyne signal reflected by the inner wall when the probing frequency is higher than the maximum of the plasma frequency (interferometry regime). In particular the time and frequency resolution of the Alfvén Cascades (ACs) is far better from this technique than from any other diagnostic. We show here that knowing the shape of the density fluctuations induced by an Alfvén Cascade, the level of these density fluctuations can also be inferred from the phase fluctuations of the reflectometry signal.

1. INTRODUCTION

A new class of Alfvén Eigenmodes, so-called Alfvén Cascades (ACs), was recently observed in the JT-60U [1], JET [2], TFTR [3] and Alcator C-Mod [4] tokamaks. The excitation and observation of these Alfvén Cascades are now used to diagnose advanced plasma scenarios. In addition to the determination of the reversed magnetic shear [2,5-7], the time evolution of the minimum of the safety factor $q_{min}(t)$ can be inferred from the time evolution of the AC frequency. Internal Transport Barriers (ITBs) are widely used in advanced scenarios in order to improve the plasma confinement. A good determination of $q_{min}(t)$ is crucial since the ITBs are usually triggered at rational magnetic surface corresponding to $q_{min}(t)$ [7]. Therefore the use of reliable diagnostics for the measurement of ACs is paramount for the development of advanced plasma scenarios with ITBs. Various diagnostics, such as external magnetic pick-up coils and ECE radiometry, proved to be more or less successful for the observation of Alfvén Cascades. It was shown on TFTR that X-mode reflectometry could provide more information on the Alfvén Cascades than majority of the other diagnostics generally used for their detection [3]. On JET a new approach based on O-mode measurement in the interferometry regime gives an unprecedented clear picture of the Alfvén Cascades [8]. It is shown in this paper that, in addition to a good determination of the time-frequency evolution of the ACs, this technique also allows the estimation of the level of density fluctuations associated with the ACs. The paper is organised as follows. In section 2 some theoretical considerations are developed in order to interpret O-mode reflectometry measurement of MHD modes such as the ACs. The main characteristics of the O-mode reflectometry diagnostic at JET are presented in Section 3. In Section 4 are discussed experimental results of reflectometry measurements of the ACs on JET. Finally some conclusions are drawn in Section 5.

2. BASICS ON REFLECTOMETRY DEDICATED TO FLUCTUATION MEASUREMENT

2.1. USE OF O-MODE REFLECTOMETRY

Now widely applied as diagnostic of fusion plasmas, microwave reflectometry is a radar technique, which consists of launching an electromagnetic wave in the suitable frequency range so that the plasma reflects it. The sweeping of the probing wave frequency allows measurement of the density profile whereas probing waves at fixed frequency are used for diagnosing the turbulence [9]. Unlike X-mode reflectometry whose interpretation requires a good knowledge of the magnetic field, O-mode reflectometry (obtained when the electric field of the probing wave is parallel to the applied magnetic field) depends on the electron density only. For the O-mode the refractive index N0 is then function of the frequency f of the probing wave and of the electron density profile ne(R) along the line of sight [10]:

$$N_o(f,R) = \sqrt{1 - \frac{n_e(R)}{n_c(f)}} \qquad [1]$$

The so-called critical density n_c in [1] is directly proportional to the frequency f of the probing wave:

$$n_c(f) = 4\pi^2 \varepsilon_0 \frac{m_e}{e^2} f^2 \qquad [2]$$

where ε_0, m_e and ε are the vacuum permittivity, the electron mass and the electron charge respectively. In the conventional use of reflectometry, a probing frequency with a critical density lower than the maximum of the plasma density is chosen. In this case, the refractive index equals zero when the plasma density equals to the critical density, which corresponds to a well-defined position, so called cut-off layer, where the probing wave is reflected from the plasma.

However the O-mode reflectometry systems cannot probe the core region (where the density gradient is usually low) by plasma reflection, even by probing the plasma simultaneously from the Low Magnetic Field Side (LFS) and from the Figh magnetic Field Side (HFS), as done on ASDEX Upgrade for instance [11]. This is a strong limitation, in particular for the observation of Alfvén Eigenmodes that are localised in the core region. It was found on JET that the best way of detecting density fluctuations in the core region from O-mode reflectometry is to use a probing frequency corresponding to a critical density higher than the maximum of the plasma density along the line of sight. In this case the refractive index [1] remains positive all along the probing line of sight so that the probing wave propagates without being reflected by the plasma. Reflected by the inner-wall, the probing wave then comes back to the detector after a round trip along the probing line of sight in the whole plasma re‰ion. In this situation, the reflectometer acts as an interferometer (we will refer to “interferometry regime” in the following) and all the plasma (including the core and high field side regions) can be probed. As shown in a previous paper [8], the interferometry regime proves to be quite efficient to observe MHD modes, such as the Alfvén cascades. The large wavelength density fluctuations induced by the Alfvén cascades are responsible for strong fluctuations of the reflectometer signal, clearly observable in both its amplitude and its phase. While no quantitative information can be easily extracted from the amplitude fluctuations, we will see that the level of the density fluctuations can be inferred from the phase fluctuations. We show in the next sub-section that in the case of large wavelength density fluctuations the WKB approximation is fully valid to evaluate the phase fluctuations of a reflectometer signal.

2.2. WKB COMPUTATIONS OF THE PHASE OF A REFLECTOMETRY SIGNAL

The phase shift of a reflectometry signal after its round trip in the plasma can be computed under the WKB approximation [10]. Assuming that the plasma parameters vary slowly compared to the probing wavelength scale (that is typically about 1cm for reflectometry), the WKB approximation remains valid in the presence of MHD modes inducing short wave-number density fluctuations (for which no Bragg back-scattering effect occurs). The phase shift φ(f) of the probing wave after its round trip in the plasma can then be written as [12]:

$$\varphi(f) = 2\mathrm{x}\int_{R_e}^{a} k(R)\, dR - \varphi_0 = \frac{4\pi f}{c}\int_{R_e}^{a} N_0(f,R)\, dR - \varphi_0 \qquad [3]$$

provided that the emitter/detector are located at $R= R_e$. In the reflectometry regime the upper limit R=a and the constant phase φ_0 correspond to the cut-off layer position and to π/2 respectively whereas in the interferometry regime, R=a is the inner wall position and φ_0 equals to π. Assuming the presence of time-varying density fluctuations δn_e (R, t), the phase shift [3] can be written using expression [1] as:

$$\varphi(f,t) = \frac{4\pi f}{c}\int_{R_e}^{a}\sqrt{1-\frac{n_e(R,t)}{n_c}}\, dR - \varphi_0 = \frac{4\pi f}{c}\int_{R_e}^{a}\sqrt{1-\frac{< n_0(R) > +\delta n_e(R,t)}{n_c}}\, dR - \varphi_0 \quad [4]$$

where $<n_0(R)>$ represents the equilibrium density profile.

Expression [4] is valid as long as the plasma density can be considered frozen during the round trip of the probing wave in the plasma. In other words, the characteristic time T_{fluc} of the density fluctuations should be much larger than the time of flight τ of the probing wave in the plasma. To assess the validity of expression [4], we performed some full-wave computations using a 1D finite-difference code, as described in reference [13]. Unidirectional transparent signal injection [14] is used for discrimination of the emitted and the reflected waves. Thus the phase shift between the emitted wave and the reflected can be easily obtained (it is tricky to obtain the constant term contributing to the phase shift but the fluctuations of the phase shift can be evaluated). These simulations were carried out for a probing frequency of 50GHz propagating in a 30cm wide homogeneous plasma with average density $n_0 = 2.5\times10^{19}\ \mathrm{m}^{-3}$ (interferometry regime). The time of flight **τ** of the probing wave in the plasma, which can be deduced from the group velocity V_g [10], gives in this case:

$$\tau = 2\int_{w_{plasma}} \frac{dR}{V_g} = \frac{2}{c}\,\frac{W_{plasma}}{\sqrt{1-n_0/n_c}} \cong 4.65ns \qquad [5]$$

taking into account the plasma width W_{plasma}= 30cm, the velocity of the light $c = 3\times10^{8}$ m/s and the critical density $n_c = 3.1\times10^{19}\ \mathrm{m}^{-3}$, given by expression [2] with f= 50GHz.

The phase fluctuations of the reflected wave were induced by oscillations of the plasma density:

$$n_e(R,t) = n_0 \left[1 + a_f \sin(2\pi \frac{t}{T_{fluc}}) \right] \quad [6]$$

with fluctuation amplitude $a_f = 2.5$ % and for different values of fluctuation period $T_{fluc} = 30, 60, 90, 120, 150$ns. In Figure 1 expression [4] is compared with the phase shift evaluated from full-wave computations for $T_{fluc} / \tau = 30/4.65 \approx 6$ and $T_{fluc} / \tau = 150/4.65 \approx 32$. We can notice some discrepancies between the WKB expression [4] and the phase fluctuations deduced from the full-wave computations for $T_{fluc} / \tau \approx 6$. On the other hand, the discrepancies are almost insignificant for $T_{fluc} / \tau \approx 32$. The time shift between the full-wave computation and the WKB expression is due to the fact that expression [4] should consider the density fluctuations at the half time of the round trip of the probing wave δn_e (R,t-τ/2). This effect becomes completely insignificant for $T_{fluc} >> \tau$ and the time shift is already almost unnoticeable for Tfluc = 150 ns. From the relative error between the WKB and the full-wave phases that is depicted on Figure 2 we can see that the amplitude of the phase fluctuations is accurately obtained from the WKB expression for $T_{fluc} > 20\ \tau$ (less than 1% discrepancies with full-wave computations). This is fully realistic in most reflectometry experiments where the time of flight of the probing wave is usually in order of magnitude of 10-100 ns and the characteristic time of density fluctuations associated with Alfvén Eigenmodes is around 10 microseconds (i.e. frequency of 100kHz).

2.3. EFFECT OF DENSITY FLUCTUATIONS ASSOCIATED WITH ALFVÉN CASCADES

The use of a reflectometer in the interferometry regime requires that the plasma does not refract the probing beam, which can then be detected after reflection by the inner wall. In other words it means that this kind of measurement can be interpreted with a one-dimensional approach, thus avoiding the need for heavy and time-consuming 2D or 3D full-wave computations. We have shown that WKB computations are good enough to evaluate the phase shift of the reflected signal in the presence of large wavelength density fluctuations such as those due to Alfvén Cascades. Since it is more delicate to extract quantitative information from the amplitude of the reflected signal, in the following we will restrict ourselves to assessing the effect of Alfvén Cascades on the phase of the reflected signal.

The shape of the density fluctuations induced by the Alfvén Cascades can be obtained in the case of weakly reversed shear by using the JET equilibrium reconstructed from the MSE data and the MISHKA-H [15] and NOVA-K [16] codes. An example of density fluctuations induced by an Alfvén Cascade on JET is depicted on Figure 3, which shows that the level of density fluctuations is larger in the HFS side region than in the LFS side region. These AC fluctuations can be accurately fitted by the following analytical expression:

$$\frac{\delta n_e(R)}{n_0} = a_1 \exp\left(-\left[\frac{R-R_1}{w_1} \right]^2 \right) + a_2 \exp\left(-\left[\frac{R-R_2}{w_2} \right]^2 \right) \quad [7]$$

with a_1 to be determined, $R_1 = 2.85$m and $w_1 = 6$cm for the density fluctuations localised in the HFS region and $a_2 = a_1/4$, $R_2 = 3.65$m and $w_2 = 5$cm for the density fluctuations localised in the LFS region.

To illustrate the sensitivity of O-mode reflectometry used in the interferometry regime, some WKB computations were carried out. In these computations, AC density fluctuations given by [7] were considered and the equilibrium plasma was assumed homogeneous in the radial region 1.9m < R < 3.9m (which is not so different from flat density profiles observed in some JET advanced scenarios displaying ACs). Figure 4 represents the amplitude of the phase fluctuations of the reflected signal as a function of the amplitude a_1 of the AC density fluctuations for different ratios n_c/n_0 between the critical probing frequency and the plasma density (let us note that $n_c/n_0 > 1$ in the interferometry regime). As expected, the higher the amplitude of the density fluctuations, the higher the amplitude of the phase fluctuations. Moreover, it is noticeable that the amplitude of the signal phase fluctuations increases when the probing critical density approaches the plasma density. This means that the probing critical density should be only slightly higher than the plasma density maximum for better measurement of Alfvén Eigenmodes in the interferometry regime. Finally, we can also conclude that assuming a given shape of density fluctuations (as for instance expression [7]) the level of these can be inferred from the level of phase fluctuations of the reflectometry signal.

3. DESCRIPTION OF THE FIXED FREQUENCY MULTI-CHANNEL REFLECTOMETER DIAGNOSTIC ON JET

A twelve-channel narrow-band reflectometer system probing the mid-plane plasma with the O-mode polarisation was used for density profile measurement on the JET tokamak [17]. This system now works with ten channels at fixed frequencies (from 18.6 up to 69.6GHz corresponding to the critical density range $0.43 - 6\times10^{19}\,m^{-3}$) and is purely dedicated to study of the density fluctuations. All of these ten channels work in the same way as depicted on Figure 5. Each channel uses two Gunn oscillators, whose frequency difference is maintained equal to 10.7MHz by a phase-locked loop, for heterodyne detection. Each channel is also equipped with I/Q detection, thus allowing the determination of the amplitude and phase signals. I/Q data from up to seven channels can be acquired with a digital converter at a frequency rate of 1MHz and for a maximum of 3s. In addition another fast digitisation system offering the ability to record up to sixty-four channels for up to 32 s at a frequency rate of 2MHz was recently implemented at JET. The O-mode reflectometer data can be then recorded on the same time base as other diagnostics such as the magnetic coils and the ECE radiometer, thus allowing cross-correlation analysis with these diagnostics to be performed.

The advanced scenarios developed at JET can lead to different values of plasma density depending on the level of heating power. The use of a ten-channel reflectometry diagnostic ensures that at least a channel works in the interferometry regime for majority of the scenarios (as long as the maximum of plasma density does not exceed $6\times10^{19}\,m^{-3}$), thus allowing routine measurement of Alfvén Eigenmodes.

4. CHARACTERISATION OF ALFVÉN CASCADES FROM REFLECTOMETRY MEASUREMENT IN THE INTERFEROMETRY REGIME

As reported in [18] different classes of Alfvén Eigenmodes – such as the Alfvén Cascades (ACs) and the Toroidal Alfvén Eigenmodes (TAEs) in the 40-150kHz range and the Elliptical Alfvén Eigenmodes (EAEs) in the 300-400 kHz range - can be observed with very high time and frequency resolution from reflectometry data. It is particularly important to get a good detection of the Alfvén Cascades for diagnosing the time evolution of safety factor minimum qmin(t), which in turn is used for the triggering of Internal Transport Barriers in advanced scenarios. An example of characterisation of the Alfvén Cascades is discussed in the following. In this example the JET reflectometer channel working at 50.5GHz was used to probe a Helium plasma heated with 5MW of ICRF heating and 1.7MW of LHCD and presenting strong Alfvén Cascades (Pulse No: 63100). The electron density profile measured by the JET Thomson scattering diagnostic [19] is represented on Figure 6, then showing that the critical density at 50.5GHz is higher than the maximum of the plasma density (i.e. interferometry regime). The spectrogram (sliding FFT) of the reflectometer homodyne signal A(t).cos(ϕ(t)) shown on Figure 7 indicates the presence of ACs in this shot. On Figure 8 is represented a zoom of Figure 7, showing the presence of an AC with a frequency of 55kHz around t= 5.585s. On the bottom of Figure 8 are represented respectively the amplitude and the phase of the homodyne reflectometry signal. After calibration of the I(t) = A(t).cos(ϕ(t)) and Q(t) = A(t).sin(ϕ(t)) homodyne signals to remove the offsets and the amplitude and phase unbalance, these amplitude A(t) and phase ϕ(t) signals were computed as:

$$A(t) = \sqrt{I(t)^2 + Q(t)^2} = \sqrt{[A(t)\cos(\phi(t))]^2 + [A(t)\sin(\phi(t))]^2} \qquad [8a]$$

$$\phi(t) = Arc\tan\left[\frac{Q(t)}{I(t)}\right] = Arc\tan\left[\frac{A(t)\sin(\phi(t))}{A(t)\cos(\phi(t))}\right] \qquad [8b]$$

We can notice that the AC induces significant fluctuations of both the amplitude and the phase of the reflected signal. On Figure 9 a zoom over a narrow time window highlights clear periodical fluctuations on the homodyne signal and on its amplitude and its phase. The fact that the frequency of these fluctuations is equal to the AC frequency, i.e. 55 kHz, clearly indicates that they are induced by this AC. The fluctuations of the phase are particularly clear so that their amplitude can be estimated with a good precision. In this case, we can see that amplitude of the phase fluctuations is about 1 radian.

In order to estimate the amplitude of the density fluctuations associated with the AC observed in this example, WKB computations were performed using the density profile depicted on Figure 6 (inferred at the time when the AC is observed) and the shape of density fluctuations represented on Figure 3. A level of density fluctuations of 0.8% in the HFS region and of 0.2% in the LFS region was found to get phase fluctuations of 1 radian. This is in agreement with the previous experimental results and theoretical expectation discussed in [8].

CONCLUSIONS

It was already shown in a previous work [8,18] that an O-mode multi-channel reflectometer used as an interferometer is a powerful diagnostic to detect the Alfvén Cascades with high frequency and time resolution. This can be achieved by computing the spectrogram of the reflectometer homodyne signal. We show in this paper that in addition, the level of density fluctuations induced by an Alfvén Cascade can be inferred from the phase fluctuations of the reflectometry signal. This only requires an assumption on the shape of the density fluctuations, which can be obtained from equilibrium reconstruction and MISHKA-H and NOVA-K computations. It was found that about ten samples per period of the density fluctuations ensure a good measurement of the phase of the reflected signal. The results presented here were obtained with data sampled at 1MHz, which proved to be enough for analysis of AC with frequency in the order of magnitude of 50-100kHz. However, a higher acquisition frequency may be required in the presence of NBI heating, since the frequency of the ACs can be Doppler up-shifted by a factor of 2 due to strong plasma toroidal rotation [8].

The main limitation of using O-mode reflectometry in the interferometry regime is that the ACs cannot be localised directly from the measurements. The use of X-mode reflectometry or cross-correlation between reflectometry data and other diagnostic data, such as electron temperature from ECE radiometry will be explored in the near future in order to address this limitation.

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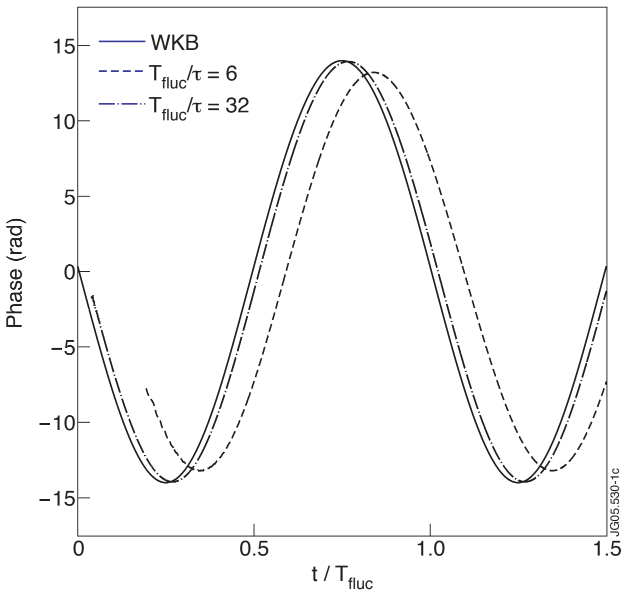


Figure 1: Comparison between WKB and full-wave computations of the phase fluctuations of a reflectometer signal induced by plasma oscillations

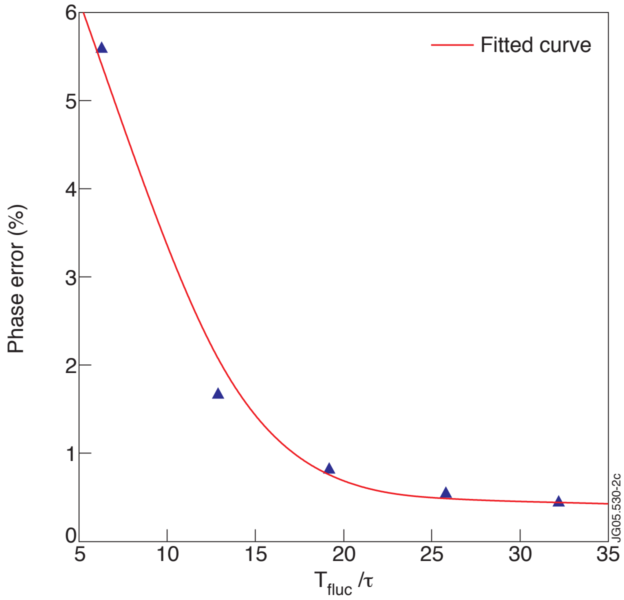


Figure 2: Relative error between WKB and full-wave computations of the level of phase fluctuations of a reflectometer signal

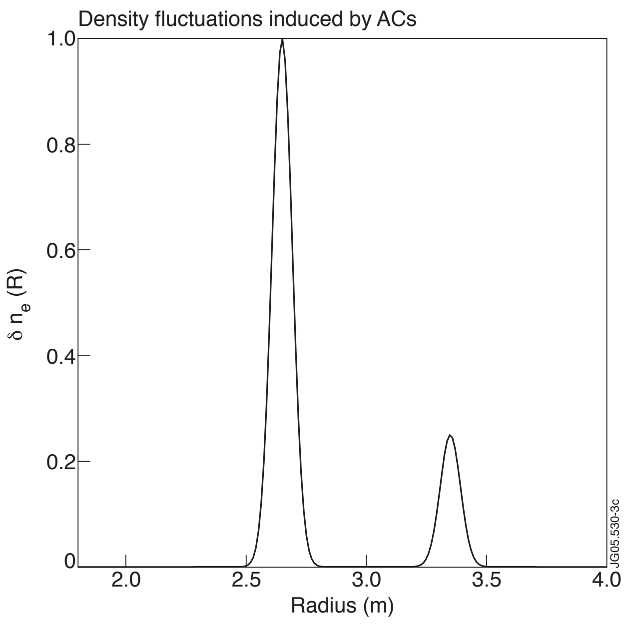


Figure 3: Typical shape of density fluctuations induced by an Alfvén Cascade (AC)

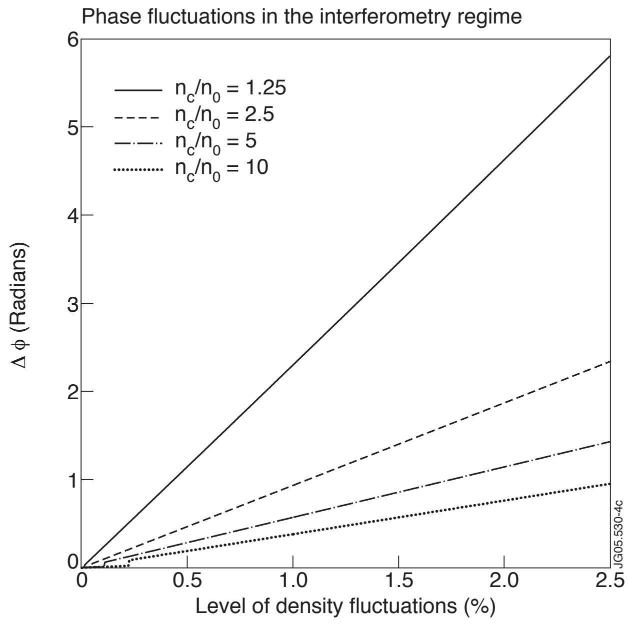


Figure 4: Amplitude of the phase fluctuations of the reflectometry signal (interferometry regime) induced by AC in the case of homogeneous plasma

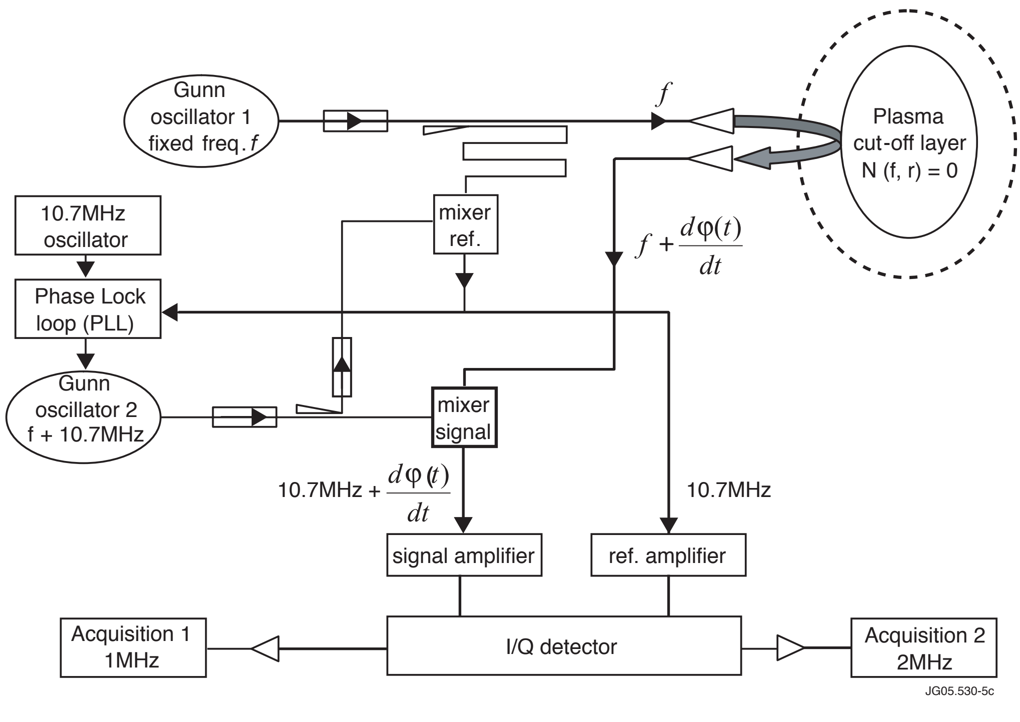
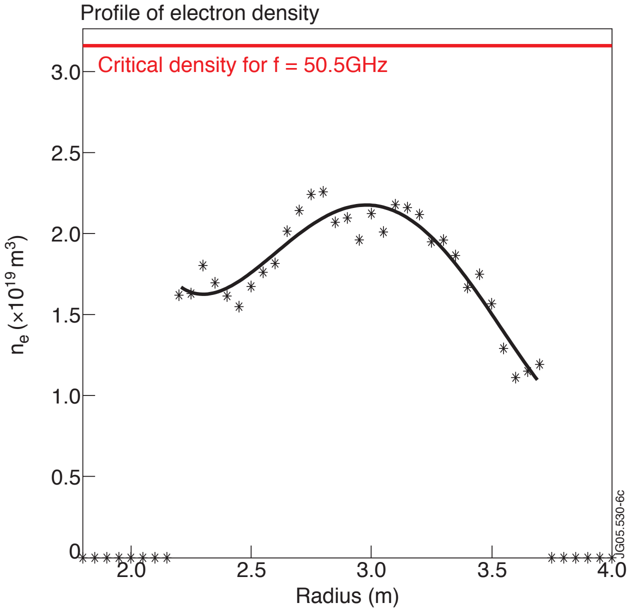


Figure 5: Diagram of one of the fixed frequency channels of the O-mode reflectometer on JET



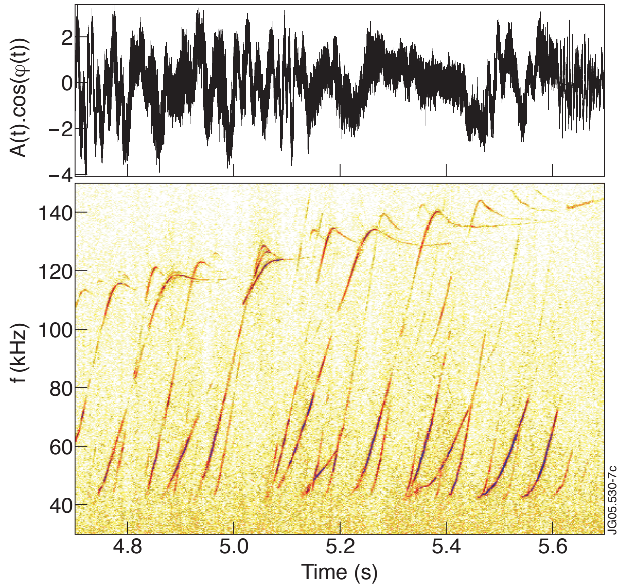
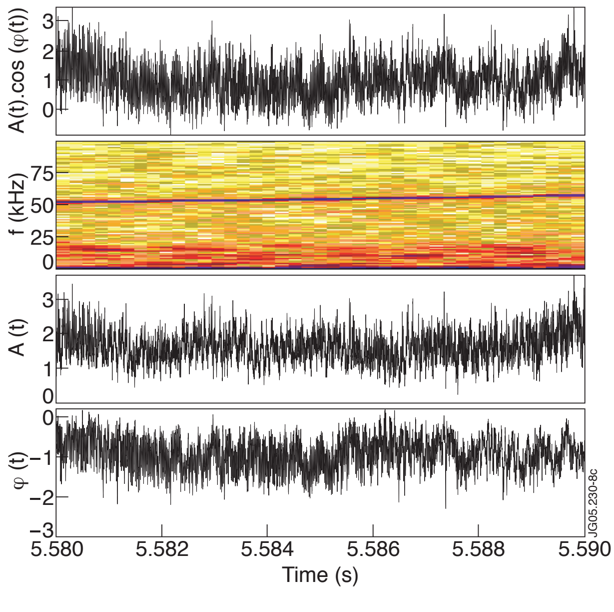


Figure 6: Radial density profile inferred from Thomson Scattering data (points) in JET plasma with Alfvén Cascades (Pulse No: 63100 at t = 5.6s). As shown by the horizontal line the channel of the O-mode reflectometry diagnostic working at 50.5GHz probes the plasma in the interferometry regime.

Figure 7: On the top is represented the O-mode reflectometer homodyne signal A(t).cos(φ(t)) obtained in the conditions depicted in Figure 6 (Pulse No: 63100 and probing frequency of 50.5GHz working in the interferometry regime).On the bottom, the time-frequency evolution of Alfvén Cascades (ACs) is clearly observed from the spectrogram (sliding FFT) of the homodyne signal.



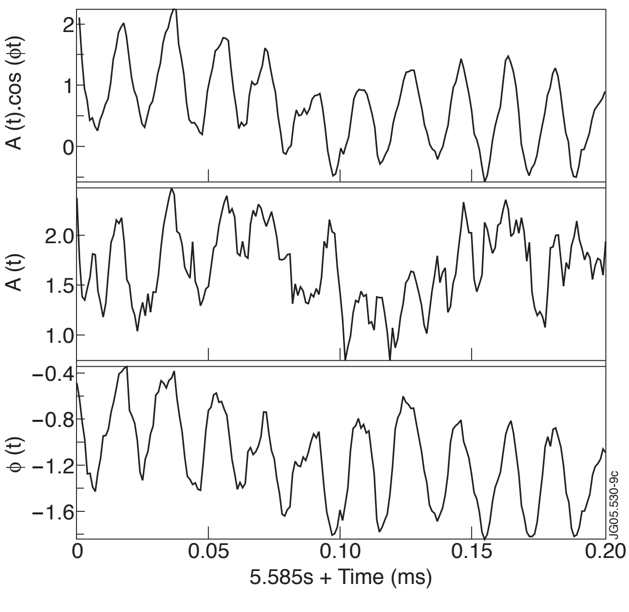


Figure 8: Zoom of Figure 7 where are represented, from the top to the bottom, the homodyne signal, its spectrogram, its amplitude and phase obtained from the I/Q detection. The clear Alfvén Cascade with a frequency of about 55kHz observed on the spectrogram induces significant fluctuations on the amplitude and the phase of the reflected signal. It can be noticed that the amplitude of the phase fluctuations is in the order of magnitude of 1 radian.

Figure 9: Zoom of Figure 8 showing the clear periodical fluctuations at the frequency of the AC (i.e. 55 kHz) of the homodyne signal, of its amplitude and of its phase (respectively from the top to the bottom). As already noticed on Figure 8, the amplitude of the phase fluctuations is about 1 radian.