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ABSTRACT

The Berk-Breizman augmentation of the Vlasov-Maxwell system is widely used to model self-consistent resonant excitation and damping of wave fields by evolving energetic particle populations in magnetic fusion plasmas. It is shown here that frequency splitting phenomena displayed by MagnetoHydroDynamic (MHD) oscillations in the Joint European Torus (JET) can be explained by a slow change in time of the Berk-Breizman model parameters. Fully nonlinear self-consistent numerical implementation of this model enables us to construct a rigorous link between the tokamak observations and the classical nonlinear phenomenology of bifurcations, period doubling and the transition to chaos.

1. INTRODUCTION

Energetic Particles (EPs) play an important role in the plasmas of magnetically-confined nuclear fusion experiments (1). Sources of EPs include Neutral Beam Injection (NBI) heating, Ion Cyclotron Resonance Heating (ICRH) and fusion-produced alpha-particles. It is critical to the success of any future fusion reactor both that the kinetic energy of these EPs is contained within the plasma sufficiently long to sustain the fusion process and also that the presence of these particles does not destabilize the plasma (2; 3). Alfvén eigenmodes are of particular interest as an example of collective wave modes that may be resonantly excited by EPs (4). In this paper we concentrate on an observation shown in Figure 1 of a toroidal Alfvén eigenmode (TAE) undergoing frequency splitting observed in Pulse No: 40332 at the Joint European Torus (JET).

This observation has been interpreted in terms of period doubling bifurcations (5). It is widely accepted that the Berk-Breizman augmentation of the Vlasov-Maxwell system (6) (henceforth "the VM(BB) model”) provides a physically motivated paradigm for the interactions between energetic particle populations and high frequency MagnetoHydroDynamic (MHD) modes in tokamaks. Recently, for example, a fully nonlinear numerical implementation (7-10) of the VM(BB) model has demonstrated that the frequency sweeping phenomenon known as chirping emerges naturally (11; 12) from the VM(BB) model with fixed parameters in a specific range. Thus the intrinsic time evolution exhibited in chirping does not require or reflect time evolution in the system parameters. Instead it reflects the intrinsic time evolution of a nonlinear system that has fixed parameters.

The following questions naturally arise. First, can the fully nonlinear VM(BB) model display frequency splitting that resembles, to some degree, the behavior seen in tokamaks? Does frequency splitting in the model occur for fixed or time-varying parameters? What are the links between observed frequency splitting and the classic nonlinear phenomenology of bifurcations, period doubling and the transition to chaos?

In the present paper we demonstrate for the first time that frequency splitting, arising from period doubling bifurcations, sometimes separated by regions of chaos, can occur naturally as the parameters of the VM(BB) model are slowly varied. This is in contrast to the frequency sweeping result (11; 12) which occurs for fixed parameters. Within the VM(BB) model, bifurcation phenomenology can be seen most clearly when field amplitudes are considered. In addition to time traces and frequency spectra, therefore, we also construct figures that display the evolution of peak field amplitude with model parameters. We quantify the region in VM(BB) parameter space for which this behavior arises, and in particular relate it to previous results (7) that categorized the periodic, multiply periodic, and chaotic regimes of the VM(BB) system. The structure of the VM(BB) model is such that, additionally, we can relate frequency splitting phenomenology to the self-consistent evolution of the energetic particle distribution function in velocity space. We shall show that the key physics is determined by the extent to which particles are captured by, trapped in, and released from the potential well of the excited mode.

2. ANALYSIS OF FREQUENCY SPLITTING DATA IN JET PULSE No:40332

Figure 1 shows the power spectrum of a family of TAEs in JET Pulse No:40332, previously considered in Ref.5. The raw signal is from an external magnetic pick-up coil sampled at 1 MHz. The power spectrum is constructed from Fast Fourier Transforms (FFTs) of Hanning data windows that are each 4096 time steps in length. The greyscale is effective across two orders of magnitude. Each TAE begins life as a single mode which later develops sidebands. The TAEs are excited via a combination of NBI and ICRH heating. The different TAEs have toroidal mode numbers in the range 5-12 and frequencies that are Doppler-shifted due to plasma rotation. In the model analysis below, we concentrate on a single excited mode. To analyse the experimentally observed structures in more detail, the slow frequency drift of the modes (due to a slow change in the properties of the background plasma) is compensated for as follows: for each of the data windows (i.e. at each discrete time step) we pick out the power maxima as a function of frequency; we then test to see if, as time varies, there exist pixels corresponding to maxima which are connected, either side-to-side or diagonally. All connected structures of this type that have at least 100 elements are shown in Fig.2. The best resolved structure has been highlighted in bold; its frequency defines a function of time which we shall refer to as $\omega_{ref}(t_n)$, where t_n is the time coordinate of the n-th pixel. The frequency coordinate of the original power spectrum Fig.1 at each time step is then mapped according to $\omega(t_n) \mapsto \omega(t_n)\,(\omega_{ref}(t_0) = \omega_{ref}(t_n))$. The result of this procedure is shown in Fig.3. This enables the power spectrum to be compared at di®erent times on the same frequency scale. Figure 4 shows two cuts through the transformed power spectrum Fig.3 at t = 12:40s and t = 12:45s. We note that each of the dominant peaks in panel (a) has developed sidebands in panel (b).

3. FREQUENCY SPLITTING IN THE VM(BB) MODEL

The VM(BB) system self-consistently models the resonant nonlinear coupling between energetic particles and the wave modes they excite. It is based on the one-dimensional electrostatic bump-on -tail model, with particle distribution relaxation and background electric field damping. We cast the model as follows (7), in terms of the particle distribution f(x, v, t) and the electric field E(x, t):

$$\frac{\delta f}{\delta t} + v\frac{\delta f}{\delta x} + E\frac{\delta f}{\delta v} = -\nu_a (f - F_0) \tag{1}$$

$$\frac{\delta E}{\delta t} + \int \nu\,(f - f_0)\,dv = \gamma_d E \tag{2}$$

Here F_0 denotes the combined particle source and loss function, ν_a the particle relaxation rate, γ_d the combined effect of all background damping mechanisms that act on the electric field, and f_0 the spatial mean of f. Spatial lengths are normalised to the Debye length λ_D; velocities to the thermal speed ν_{th}; time to the inverse plasma frequency $\omega_p^{-1} \equiv \lambda_D/\nu_{th}$; and E to $m_e v_{th}^2/e\lambda_D$. Central to the present paper is an interpretive code (8) which uniquely solves the fully nonlinear VM(BB) system without employing analytical approximations. It allows direct numerical solutions of the fully nonlinear self-consistent VM(BB) system across the entirety of (γ_d; ν_a) parameter space for any $F_0(v)$. Application of this code (7) for a particular $F_0(v)$ shows how the behavior of the VM(BB) system depends on its parameters. Our first objective here is to investigate how far the conceptually simple physics basis of the VM(BB) model captures the observed phenomenology of frequency splitting in JET.

For both the simulations described in this paper, we initiate using a bump-on-tail distribution

$$F_0\,(v) = F_{bulk} + F_{beam} \tag{3}$$

where

$$(2\pi)^{1/2}\, F_{bulk} = (\eta/\nu_c)\, \exp\,(-\nu^2/2\nu^2_c) \tag{4}$$

$$(2\pi)^{1/2}\, F_{beam} = [(1 - \eta)/\nu_t]\, \exp\,[-(\nu - \nu_b)^2/2\nu^2_t \tag{5}$$

and we choose $\eta = 0.9$, $\nu_c = 1.0$, $\nu_b = 4.5$, and $\nu_t = 0.5$. The results presented below are not dependent on this specific choice of parameters; they are quoted for the sake of reproducibility. We note, however, that this choice of distribution has a small proportion of particles in the energetic tail and that the beam is well separated from the bulk. Space is taken to be periodic with period $L = 2\pi/4/k_0$ where $k_0 = 0.3$. The simulations are initialised with $f(x, \nu, t = 0) = F_0(\nu)(1 + \alpha \cos(k_0 x))$ where $\alpha = 0.001$; the initial electric field is provided by Poisson's equation $E(x, t = 0) = (\alpha = k_0) \sin(k_0 x)$.

Previous work (7) has classified how the system's behavior depends on the parameters (γ_d, ν_a). Specifically it has been shown that, for different values of (γ_d, ν_a), the system may be damped, steady state, periodic or chaotic. Let us now study how the system proceeds from steady state (i.e. the field is a single mode with constant amplitude) to chaos. This is achieved by choosing a cut in (γ_d, ν_a) parameter space, where one end corresponds to steady state and one to chaotic behavior. Simulations are then performed where we travel slowly along the cut, at a rate sufficiently slow that the results are independent of the speed of travel. The cut chosen is the straight line segment $\gamma_d = 1.0$,

$0.022 \le v_a \le 0.05$: with γ_d unchanged as we move along the cut, $v_a = 0.05$ and $v_a = 0.022$ correspond to steady state and chaotic system behavior, respectively. It is found that it is possible to move through some sections of parameter space more quickly than through others; intervals that are rich in bifurcations must be traversed slowly. We have therefore performed two numerical simulations, travelling along sections of the same cut at two different speeds: (a) $v_a = 0.06 - 10^{-7}t$ and (b) $v_a = 0.0265 - 10^{-8}t$. The second simulation therefore provides a more detailed picture of the end of the cut corresponding to smaller v_a, in the chaotic regime. Each simulation ran until $t = 5\times10^5$ and took approximately 120 hours running in parallel on four processors each capable of 5.2 GFLOPS.

During each simulation we record the (complex) component of the electric field in the first spatial mode $E_1(t) = \int E(x, t)\, exp(-ik_0x)\mathrm{d}x$.

Spectra $|\tilde{E}_1(\omega)|$ of the electric field component $E_1(t)$ are shown in Fig.5. These spectra are generated by linearly interpolating the simulation time series onto a regular temporal grid with spacing $\Delta t = 0.1$, taking windows of 4096 points, applying a Hanning windowing function, and then performing a Fast Fourier Transform (FFT) on each of these windows. The results shown in Fig.5 use a greyscale scheme across five orders of magnitude.

To examine the spectra in detail we take cross-sections of Fig.5 at different fixed values of v_a; a selection is shown in Fig.6. The first plot (a) ($v_a = 0.05$) confirms that the system is dominated by a single mode; the Fourier spectrum is of the shape one would expect for a single mode of frequency ω_0 viz. $\tilde{E}_1(\omega) \sim \omega/(\omega^2_0 - \omega^2)$. The second plot (b) ($v_a = 0.045$) shows the appearance of distinct sidebands; the envelope of the spectrum is still dominated by the central mode. The third plot (c) ($v_a = 0.04$) corresponds to a regime of complex behavior; there is no clear structure to the spectrum. Structure returns in plot (d) ($v_a = 0.03$) although here there are a much larger number of significant side-bands. Moreover, the envelope of the maxima in ω behaves as exp $(-|\omega - \omega_0|)$. Between plot (d) and plot (e) ($v_a = 0{:}0245$) there is a period-doubling bifurcation, which corresponds to a halving of the distance between sidebands. The consistency in sideband separation between plot (e) and plot (f) ($v_a = 0.02375$) implies that there is no period-doubling bifurcation between these values of v_a. There is however a period doubling going from plot (f) to plot (g) ($v_a = 0.023$). Plot (h) ($v_a = 0.022$) corresponds to the chaotic regime.

4. THE BIFURCATORY ORIGIN OF FREQUENCY SPLITTING

The VM(BB) model defined by Eqs.1 and 2 can be said to capture frequency splitting in JET Pulse No: 40332 insofar as, for example, Fig.5 resembles Fig.3; see also Ref.5 which uses an analytic approximation to the fully nonlinear VM(BB) model deployed here. The results in the preceding section suggest that the classic nonlinear phenomenon of period doubling bifurcation lies at the heart of the observed plasma behavior. This conjecture is strengthened by the following analysis, which provides a rigorous account of the role of period doubling bifurcations in the fully nonlinear model.

A period doubling bifurcation is a global bifurcation that occurs when a periodic orbit becomes unstable and the system moves to a nearby periodic orbit of double the length; this new orbit traverses twice around in a neighborhood of the original orbit before joining back on itself. Such orbit splitting can manifest itself in the splitting of extrema in the time series of macroscopic quantities, such as $|E_1(t)|$ in the VM(BB) system considered here. For example, if we write *a(t)* = $|E_1(t)|$, and consider a projection of the system onto the (a, $\dot{a}$) plane, then extrema in a correspond to the Poincaré section $\dot{a}$ = 0. This motivates the diagrams shown in Fig.7, which plot extrema in time of $|E_1(t)|$ as functions of ν_a, by inverting the mapping t $\rightarrow$ ν_a for the two simulations (a) and (b).

Since the bifurcation sequence proceeds from right to left with decreasing ν_a, let us discuss Fig. 7 in that direction. For values of $\nu_a > 0.046$, the system is dominated by a single mode. The parameter range over which a steady state is observed is extensive: $0.1 > \nu > 0.046$, although for the sake of clarity it is not shown on this diagram. Despite the mode amplitude being nearly constant in this range, small fluctuations render it visible in this plot of extrema. At $\nu_a = 0.046$ the time series develops a well separated maximum and minimum. The behavior in the interval $0.044 > \nu_a > 0.038$ is complex, and appears to display regions of chaos alternating with regions of order. In the relatively large interval $0.038 > \nu_a > 0.025$, the time series is periodic with two maxima and two minima in each cycle. Bifurcations are then observed at $\nu_a = 0.025$ and $\nu_a = 0.0236$. The onset of chaos occurs at $\nu_a = 0.0228$. It appears that a rapid quantitative change in system behavior occurs at $\nu_a = 0.0239$. However, this is only a change in the size of the orbit, and no qualitative change (such as a change in the number of extrema) occurs; this does not constitute a bifurcation. The analysis of the VM(BB) model from this perspective establishes the quantitative correspondence between the evolving Fourier spectrum (Fig.6) and the nonlinear systems plot of bifurcations (Fig.7).

CONCLUSIONS

The results presented in this paper suggest that key aspects of the frequency splitting observed in JET Pulse No:40332 are captured by the VM(BB) model in its fully nonlinear self-consistent form (Eqs.1 and 2) as implemented in the code of Refs. 7; 8. Our results also provide strong support for the conjecture that period doubling bifurcation underlies the observed plasma phenomenology. This follows both from analysis of the power spectrum of the dominant mode as it evolves in time, and from a non-linear systems approach to plots of field extrema. These results open the way to relating observations of frequency splitting to the two key model parameters (γ_d, ν_a) of the VM(BB) system, subject to signal-to-noise constraints.

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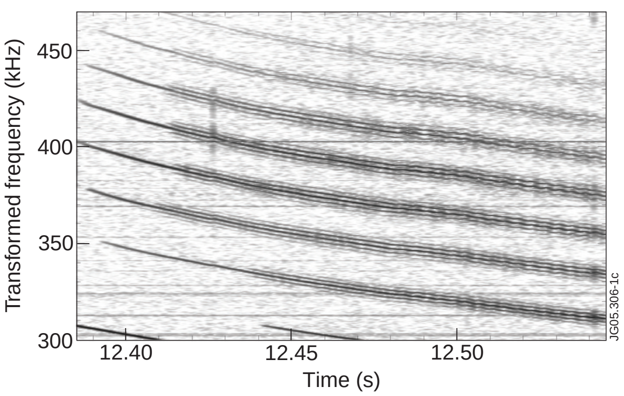

Figure 1: Experimental observation of TAEs in JET Pulse No: 40332 undergoing frequency splitting. The slow frequency drift is due to slow variation of macroscopic plasma parameters. Logarithmic greyscale plot of the fast Fourier transform (FFT) across two orders of magnitude.

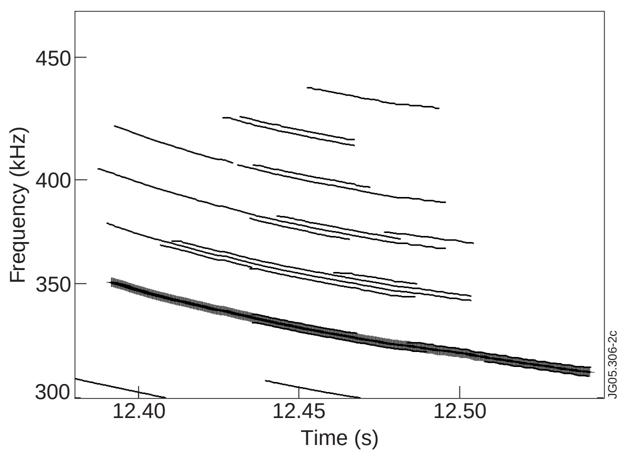

Figure 2: Connected structures defined by pixel-by-pixel analysis of the power spectrum shown in Fig.1. The structure highlighted in bold provides the basis for the frequency map by which Fig.3 is obtained.

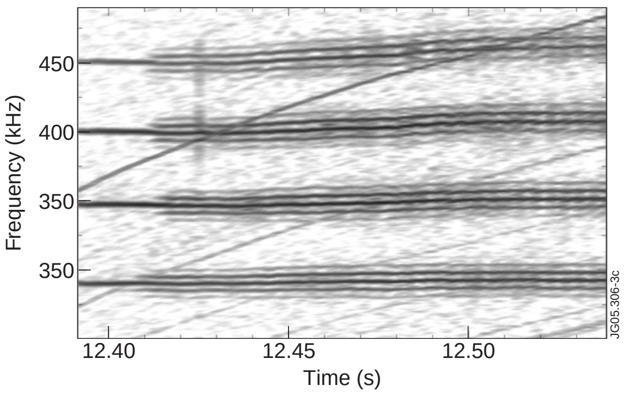

Figure 3:Transformed power spectrum of JET Pulse No: 40332. Frequency has been nonlinearly stretched to eliminate the slow frequency drift using a map based on the structure identified in Fig.2. Time is unaffected by this map. This transformation allows cross sections corresponding to different times (as in Fig.4) to be compared on the same frequency axis. Logarithmic greyscale across two orders of magnitude.

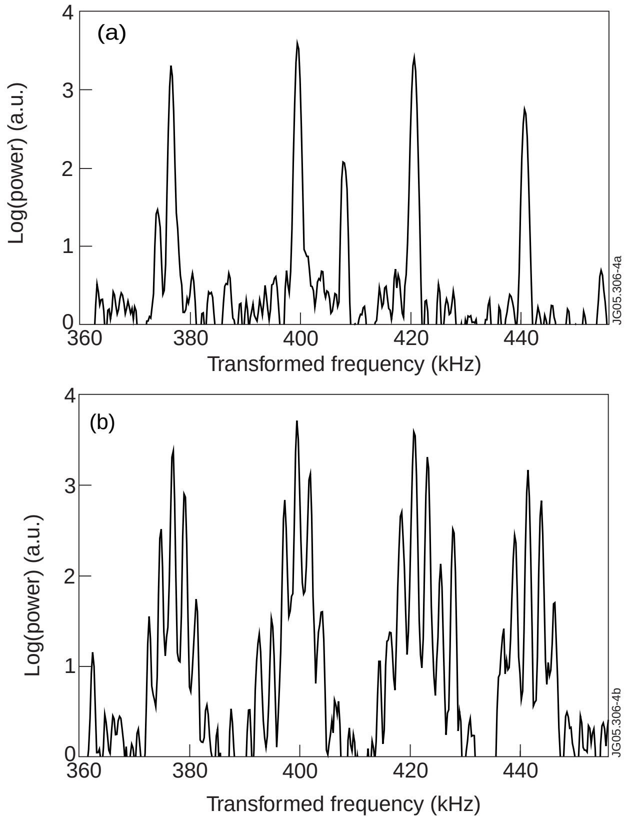

Figure 4: Cuts through the transformed power spectrum shown in Fig.3 at times (a) t = 12.40s and (b) t = 12.45s. Panel (a) displays four distinct modes. Panel (b) shows that side-bands have formed beside the dominant modes. It is not clear whether only a small number or many significant sidebands are formed. From t = 12.45s until at least the end of the observation there is no qualitative change in the power spectrum. The feature at 408kHz in panel (a) is an unrelated mode.

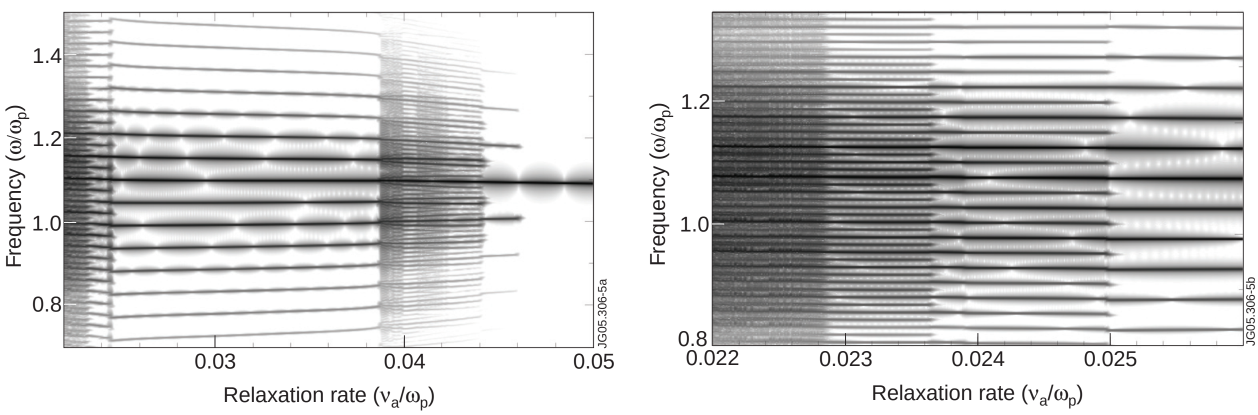

Figure 5: Field spectra $|\tilde{E}_1(\omega)|$ *as a function of time for the two simulations (a) and (b) described in the text. Plot (a) shows, as* ν_a *decreases, the transition from a single mode, through a region of complicated behavior, to a significant interval with a number of modes with well-defined spacing, to a region where separate modes are indistinguishable. Plot (b) shows details that cannot be observed in plot (a). It shows two period doublings, at* $\nu_a = 0.0250$ *and* $\nu_a = 0.0236$. *At* $\nu_a \approx 0.02284$ *either a further period doubling or transition to chaos occurs. Cross sections of these plots are shown in Fig.6. (Greyscale plots over five orders of magnitude)*

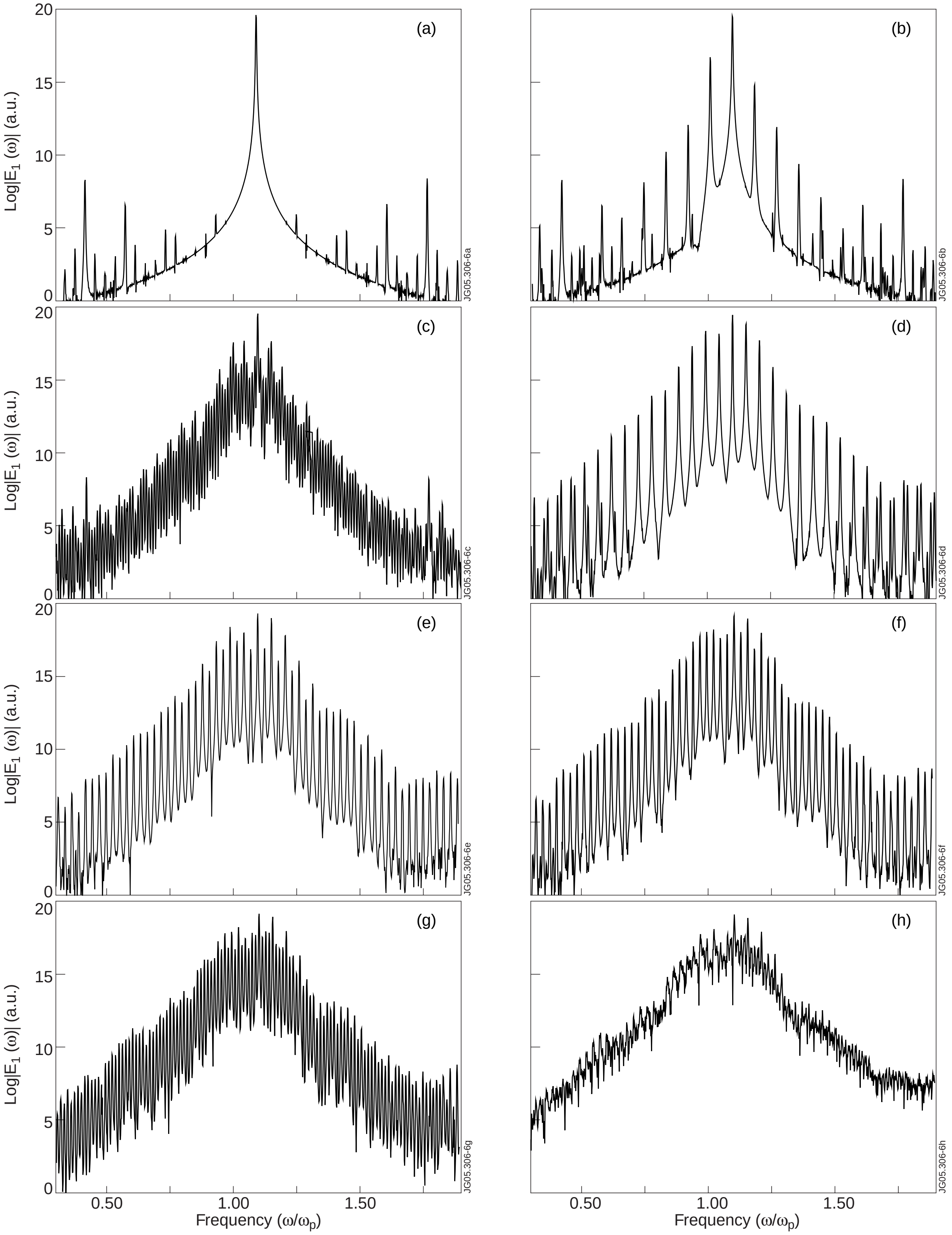

Figure 6: Cross sections through Fig.5 showing power spectra for various values of the model parameter ν_a. The bifurcation sequence is discussed in Section IV.

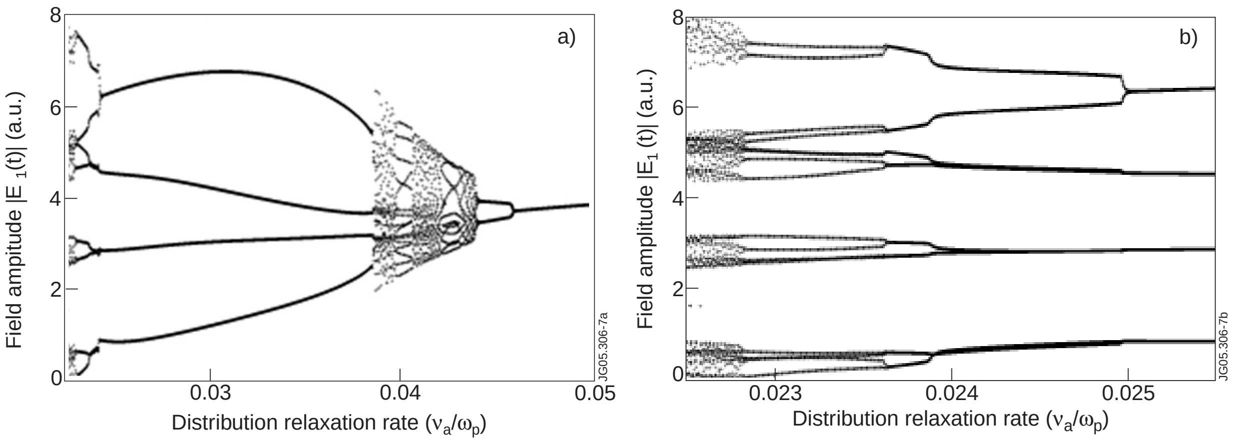

Figure 7: Plots of extrema in the electric field energy A(t) observed in the solution of the VM(BB) system as a function of particle relaxation rate ν_a (the parameter γ_d is fixed at unity) for the two simulations (a) and (b) described in the main text. The splitting of extrema corresponds to bifurcations in the time series A(t). Plot (ii), which corresponds to simulation (b) carried out at high time resolution, is a magnification of a section of plot (a); it shows the bifurcation path of the system to chaos through a series of bifurcations.