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ABSTRACT

The Alfvén wave Cascade (AC) is known to exist in plasma configurations with shear reversal and to be localized at the shear reversal point. However, when the core shear is $\gtrsim 0$ a core-localized Alfvén wave cascade can exist at $r/R_0 < 0.2$. This core-localized AC was found to exist for a narrow range of toriodal (n = 1, 2) and poloidal (m = 3, 4) numbers and even then only for specific values of q on axis. Due to the sensitivity of the mode existence on the q profile and the fact that only the low n and m modes were found to exist, it is believed that most observed ACs to date have not been the core-localized AC but instead the reversed shear AC.

I. INTRODUCTION

In a recent paper by Breizman et al.[1] it was noted that it is not always necessary to have magnetic shear reversal to establish an Alfvén wave cascade. This statement arose from the inspection of equations (22) and (29) in the absence of hot ions. Rewriting equation (22) from reference [1], applying WKB analysis, and neglecting the hot ion term yields

$$k_r^2 = -\frac{m^2}{r^2}\left\{1 + \frac{\omega^2}{\omega^2 - \omega_A^2}\left[\frac{(2\epsilon^2 + 4\Delta')}{4\,(nq - m)^2 - 1}\right]\right\}, \tag{1}$$

where k_r is the radial wave number, $\epsilon \equiv r/R_0$, R_0 is the major radius of the plasma, Δ' is the Shafranov shift factor, m and n are the poloidal and toroidal mode numbers, q is the safety factor, $\omega_A^2 = \overline{V}_A^2\,(n - m/q)^2/R_0^2$, and $\overline{V}_A$ is the flux surface averaged Alfvén velocity. It can be seen that in equation (1) an interplay between the radial dependence of the terms and the radial dependence of the q(r) terms can create a "potential well" to support a localized eigenmode. This is made clearer by expanding equation (1) around $q = q_0$,

$$k_r^2 = -\frac{m^2}{r^2}\left\{1 + \frac{1}{1 - \frac{\omega_0^2}{\omega^2} - \frac{2m}{\omega^2 q_0^2}\,(n - \frac{m}{q_0})\,(n - q_0)}\;\frac{2\epsilon^2 + 4\Delta'}{1 - 4\,(nq_0 - m)^2 - 8n\,(nq_0 - m)\,(n - q_0)}\right\}, \tag{2}$$

where q_0 is q on axis and ω_0 is the Alfvén continuum frequency on axis given by $\omega_0 = V_A(n - m/q)/R_0$, where V_A is the Alfvén velocity and R_0 is the major radius of the plasma. In equation (2) when $q = q_0$, $m/n \geq q_0 \geq m/n + 1/2n$, and $\omega \gtrsim \omega_0$ then k_r^2 is negative and farther away from the axis when $(q - q_0)$ is large k_r^2 is also negative allowing for a region in between where k_r^2 can be positive.

Equations (1) and (2) are valid for both monotonic q profiles and reversed shear profiles, where in the reversed shear case q_0 is replaced by the the minimum q. In the reversed shear case there are two re ection points on either side of the minimum q surface where $k_r^2 = 0$ due to the increasing q away from the minimum q surface, which is needed for a standing wave to exist. Previous analyses

[1–5] of the Alfvén wave cascades have all focussed on the reversed shear case, and experimentally the identification of Alfvén wave casades has been used to determine when the plasma is in reversed shear and the value of the minimum q [4]. In this paper it will be shown that it is not necessary for the plasma to have a reversed shear profile for the Alfvén wave cascades to exist, but that the operational space of these monotonic Alfvén wave cascades is so small that it is unlikely that such a mode has been observed.

2. EQUILIBRIUM SETUP AND PARAMETER SCAN

Since the existence of Alfvén wave cascades is expected to arise from considering $O(\in^2)$ ideal MHD effects in a toroidal geometry, MISHKA-1 [6], a full geometry, ideal MHD stability code, was used to investigate the existence of this mode.

The toroidal equilibrium used in this analysis has a nearly at q profile near the axis and has $dq/dr \geq 0$ everywhere. Figure 1 shows the equilibrium q profiles used in this analysis. The plasma shape and dimensions where for a typical Alcator C-Mod discharge ($\in = 0.343$, $k = 1.27$ and $\partial = 0.277$), but the analysis is valid for most tokamaks. The only parameter varied in this study was the value of q on axis, which then produced a DC shift in the q profile thus maintaining the shape of the profile.

q_0 was scanned from 4 to 1.25 and Alfvén wave cascades of low n numbers ($n \leq 3$) were solved for. During this scan only an n = 1 and an n = 2 mode was found to exist during an entire cascade cycle, from near zero to the middle of the TAE gap ($q_0 = 2 \rightarrow 1.5$ for n = 1 and $q_0 = 1.5 \rightarrow 1.25$ for n = 2). For all other n values the mode either did not exist or would disappear as the q on axis was lowered.

3. n = 1 CORE-LOCALIZED ALFV ÉN WAVE CASCADE

In this section the discussion will focus on the n = 1, m = 3 Alfvén wave cascade. Other modes behave similarly but have different windows, in q_0, of existence. For example, the n = 1, m = 4 mode is found to exist in the between $q_0 = 4$ and 3.946, the n = 2, m = 4 mode is not found to exist for any value of q_0, while the n = 1, m = 2 and the n = 2, m = 3 modes are found to exist for their entire scan of $q_0 = 2 \rightarrow 1.5$ for the n/m = 1/2 mode and $q_0 = 1.5 \rightarrow 1.25$ for the n/m = 2/3 mode.

The n = 1, m = 3 mode was found to exist, for the equilibrium discussed above, between q_0 values of 2.75 and 3, but not for $q_0 < 2.75$. While the mode exists the mode frequency has the same behavior as a normal reversed shear Alfvén cascade, in that it follows $\omega = V_A/R(m/q_{min} - n)$ (see figure 2), where in the monotonic case $q_{min} = q_0$. The radial structure of the mode does change during as the q on axis changes. Figure 3 shows this structure change as the q value decreases from 2.95 to 2.75, the lowest q_0 value where the mode was found to exist. As can be seen in figure 3 the mode both narrows and moves toward the axis and the q_0 value is decreased. This is consistent with the mode "barrier" moving towards the axis.

The Alfvén wave cascades presented here are for a specific case where the q profile on axis is at, but it should be noted that as the q profile takes on a more typical monotonic profile the Alfvén wave cascade no longer exists. In fact, the q profile was changed by less than 2% over the entire profile and this was sufficient for the cascade to no longer exist.

CONCLUSIONS

The core-localized Alfvén wave cascade can exist for a monotonic q profile, provided the profile is at in the core. It was found for the case described here that only low m (m ≤ 4) and low n (n ≤ 2) Alfvén wave cascades did exist and that only the n/m = 1/2 or the n/m = 2/3 mode existed for its entire scan from a near zero frequency to the middle of the TAE gap. The existence of a given mode was highly sensitive to the q profile, given that a less than 2% increase across the entire radius could cause the mode to no longer exist. Although it is true that it cannot be said that the existence of an Alfvén wave cascade guarantees that the q profile is reversed shear, it is unlikely that the q profile in any given discharge is in the narrow parameter region where the q profile is monotonic and the Alfvén wave cascades exist.

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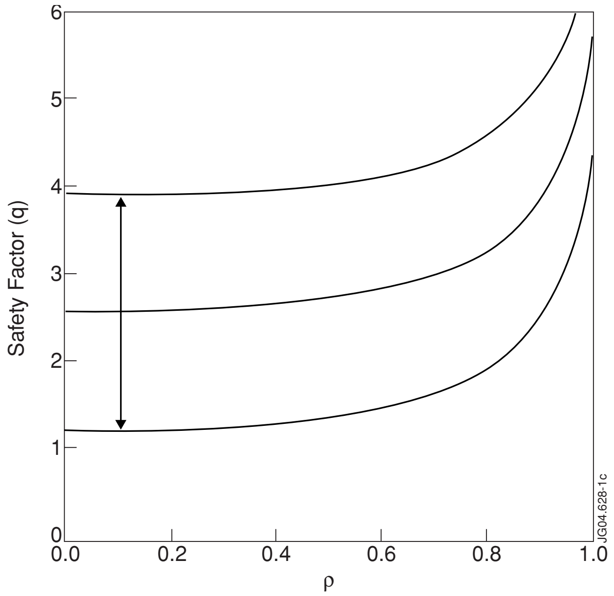
Figure 1: Equilibrium profiles of the safety factor as a function of $\rho = \psi_T^{-1/2}$, where ψ_T is the normalized toroidal flux. During the parameter scan the entire q profile was varied by an offset that changed the on axis q between 1.25 and 4.



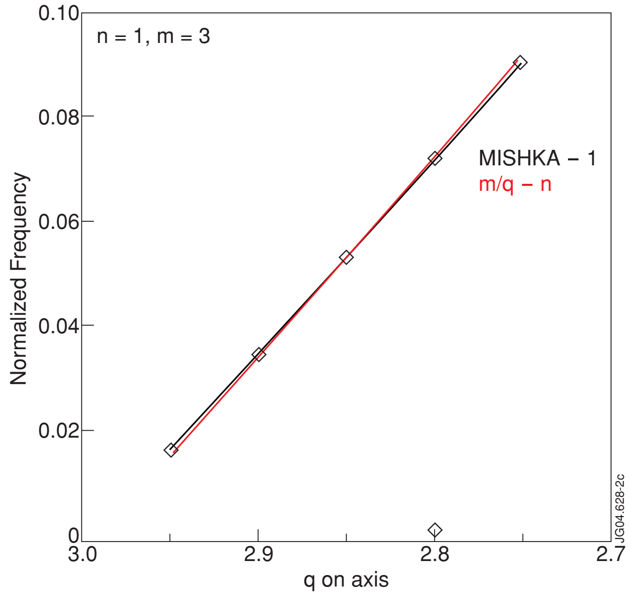


Figure 2: Plot of the frequency of the n = 1, m = 3 Alfvén wave cascade normalized to V_A/R as a function of qmin. Plotted in red is the function of $\omega_R/V_A = \omega = m/q_{min} - n$ for comparison.

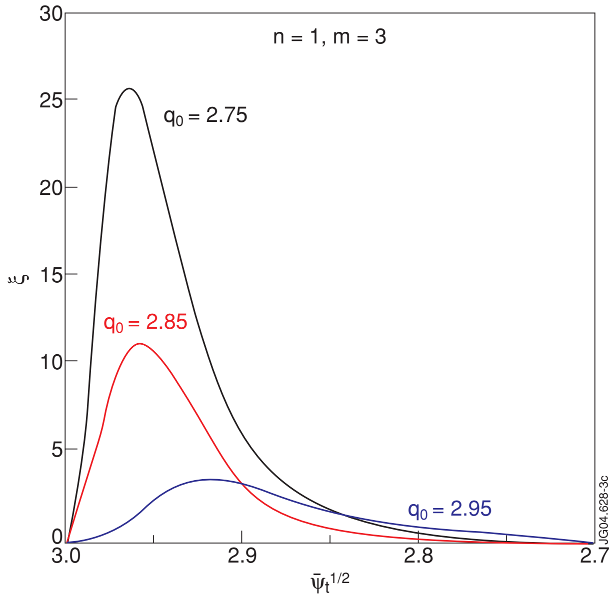


Figure 3: Plot of radial mode structure (ξ is the displacement vector) as a function of normalized toroidal magnetic ux of the n = 1, m = 3 Alfvén wave cascade for three values of q_0, 2.95, 2.85, and 2.75.