EUROPEAN FUSION DEVELOPMENT AGREEMENT

EFDA–JET–PR(03)37

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** See annex of J. Pamela et al, "Overview of Recent JET Results and Future Perspectives", Fusion Energy 2000 (Proc. 18th Int. Conf. Sorrento, 2000), IAEA, Vienna (2001).*

Preprint of Paper to be submitted for publication in
Nuclear Fusion

ABSTRACT.
Plasma parameters, like the internal inductance l_i and the diamagnetic poloidal beta β_{DIA}, are of particular relevance for a reliable real-time control system of next step Tokamaks. These and other quantities have been obtained at JET (Joint European Torus) with a method that uses the Shafranov integrals S_1, S_2 and S_3. Indeed they allow the direct calculation of the Shafranov parameter $\Lambda = \beta_{MHD} + l_i/2$ and of β_{DIA}. Moreover, in discharges with a sufficiently high elongation (k > 1.3 typically), the internal inductance can be separated from the MHD poloidal beta β_{MHD} and calculated independently, through the Shafranov integrals, with a precision which is more than satisfactory for real-time applications. It is worth mentioning that, since S_1, S_2 and S_3 are integrals defined on the plasma boundary, a specific algorithm, depending on the fast code XLOC, has been expressively developed to determine this quantity.
The method to determine the plasma parameters has been verified off-line, benchmarking its outputs with the estimates of the most sophisticated equilibrium codes available. The results of this systematic comparison have been very encouraging both in the limiter and x-point phases of the discharges and on all the investigated plasma configurations. The computational time, necessary to determine the plasma boundary and about 50 signals, is only about 1.5 ms on a PC equipped with a 400 MHz Pentium II, well below the 10 ms constraint of JET real-time applications. The code has therefore been implemented on-line and its outputs have already been exploited to achieve the feedback control of some plasma parameters.

1. INTRODUCTION

In the last years the need of a reliable real-time control system of Tokamak plasmas has become increasingly evident. The new perspectives opened by the advanced scenarios have emphasised the requests on the feedback control of several plasma parameters. This necessity is of particular relevance in the case of plasma scenarios aimed at the study of Internal Transport Barriers (ITBs), which are very promising in terms of performances and could benefit from the use of feedback control tools [1]. Both the achievement and the sustainment of these ITBs can be very demanding and a reliable feedback control of these plasma scenarios would be a major breakthrough in this research programme. In addition, there is also the need to develop new techniques for the avoidance of MHD instabilities, the control of the edge localised modes (ELMs) and of the plasma density. With all these long term programmes in view, not only local values, like the current density and the q profiles [2], but also integral quantities like the internal inductance and the confinement parameters are of great interest. In JET it has therefore been decided to develop a fast algorithm, BetaLi, to determine these parameters in real-time and make them available to the applications that need them for the feedback control.

Since the plasma boundary, or last closed flux surface (LCFS), is a prerequisite to the determination of the integral parameters, a new algorithm has been developed in order to compute it. This algorithm is based on the poloidal magnetic flux Ψ reconstruction obtained by the external magnetic measurements (section 2). Once the boundary has been identified, crucial surface-dependent quantities can be calculated in addition to obvious geometric parameters like the elongation k or the poloidal cross-section area A. In particular, the Shafranov integrals (S_1, S_2, S_3) and moments (Y_i, i = 1..n) can be evaluated (section 2). The formers are integrals of the poloidal magnetic field on the LCFS and constitute a prerequisite to the determination of the diamagnetic poloidal beta β_{DIA} and of the plasma internal inductance l_i (section 3), quantity needed also to calculate other main global parameters, like the stored MHD energy W_{MHD} and the confinement time τ_E (section 3).

The results of the described application have been extensively compared with the estimates of more complex off-line programs, like the equilibrium code EFIT [3], which solves the complete Grad-Shafranov equation, and FAST [4], and the agreement has been more than satisfactory, as described in section 4. In this section the computational performance of BetaLi is also reported, showing the compatibility of this code with JET real-time requirements.

An example of BetaLi real-time application is reported in section 5. The BetaLi diamagnetic normalised beta has been given as input to a model based feedback code. Using the additional heatings as actuators, it has been possible to obtain the real-time control of this quantity for all the discharge.

After a brief summary, the future prospects and applications of the described algorithms are discussed in section 6.

2. IDENTIFICATION OF THE PLASMA BOUNDARY AND OF THE RELATED INTEGRAL QUANTITIES

The boundary algorithm developed to determine the LCFS is based on the knowledge of the Ψ function on a cross-section of the machine. The spatial distribution of this quantity is calculated in real-time in JET by the code XLOC, using the least square method to fit the available magnetic measurements with the model [5,6]. Since this application is aimed only at the identification of the plasma boundary and at measuring its distance from the first wall, the flux equation can be solved in the vacuum region outside the plasma. The real-time requirements are matched by approximating Ψ with a sixth order Taylor expansion [5,6]. Once the spatial function $\Psi(R,Z)$ has been determined, it is quite straight to find out the value of Ψ_{LCFS}, the poloidal flux at the boundary. Indeed in the limiter configurations Ψ_{LCFS} is given by the maximum value of Ψ at the first wall, while in the case of x-point plasmas the LCFS is defined as the flux surface of the x-point and so Ψ_{LCFS} is the x-point flux [6].

Using Ψ given by XLOC and solving the equation

$$\Psi(R, Z) = \Psi_{LCFS} \tag{1}$$

on the vessel cross section allows then to find the points $P(R, Z)$ belonging to the LCFS.

It has been decided to discretise the plasma boundary, dividing it in 100 points, in order to simplify its calculation. This has been achieved considering 100 radial segments, called Gaps, whose layout is shown in fig.1. The chosen number of segments is due to a compromise between accuracy and computational speed. Equation 1 is then solved along each of the Gaps. The univocity of the solution of equation 1 on each Gap is guaranteed by the reason that *Ψ(R, Z)* is monotonic from the centre of the plasma to the outside (although the values of *Ψ(R, Z)* inside the plasma are not correct due to the vacuum region approximation).

There is however an exception, which is due to the non-monotonicity of the flux in the region near the x-point. To solve this problem, the solution adopted consists in using the information about the position of the x-point (also supplied by XLOC). The nearby segments are clipped by taking the orthogonal projection of the x-point on them [7]. From the projection point on, the flux is monotonic and, due to the Gap layout, no point of the LCFS can be below the projection points.

The bisection method has been exploited to find the solution of equation 1 in a fast way suitable to the real-time constraints. It foresees to compare, on each cycle, the flux on the current segment centre with Ψ_{LCFS}, and to discard the part of the Gap where Ψ_{LCFS} is not contained. The length of the investigated segment halves as the number of cycles increases, which, as far as it is concerned, depends on the discretisation degree of the Gap (in this case the Gaps have been subdivided in N = 150 equispaced points each; the reason is always that of a compromise between accuracy and short computational time). The search terminates when the length of the segment to check is equal to one point: this means the flux of the remaining point is equal to Ψ_{LCFS}, and so its coordinates are the coordinates of the LCFS along that segment. This search method is fast because on average its efficiency (i.e. the order of magnitude of the number of searches for each Gap) is equal to O(logN), whereas the efficiency of a linear search would be equal to O(N). Two results of this procedure can be examined in fig.2, where a surface for the limiter configuration (dotted line) and another for the divertor configuration (solid line) are shown.

Once the points of the LCFS have been determined, it is immediate to provide the required geometrical quantities like the plasma volume *V*, the minor and major radii *a* and R_0, the elongation *k* and the triangularity.

The next fundamental step in BetaLi is the calculation on the LCFS of the integral quantities used in the determination of the plasma confinement properties. In particular, the so-called Shafranov integrals S_1, S_2 and S_3 [8,9], are necessary to determine l_i and some of the main plasma confinement quantities (see next section). S_1, S_2 and S_3 are integrals of the various components of the poloidal field on the LCFS and are defined as [9]:

$$S_1 = \frac{1}{B_{pa}^2 V} \int_S B_\theta^2 \left(R\overline{e_R} + Z\overline{e_Z} - R_c\,\overline{e_R}\right) \cdot \bar{n}\, dS \tag{2}$$

$$S_2 = \frac{1}{B_{pa}^2 V} \int_S B_\theta^2\, R_c\, \overline{e_R} \cdot \bar{n}\, dS \tag{3}$$

$$S_3 = \frac{1}{B_{pa}^2 V} \int_S B_\theta^2\, Z\overline{e_Z} \cdot \bar{n}\, dS \tag{4}$$

where B_θ is the poloidal magnetic field, *S* is the plasma surface, *R* and *Z* the radial and vertical coordinates ($\boldsymbol{e_R}$ and $\boldsymbol{e_Z}$ the respective vectors), R_c a constant major radius (taken equal to 2.96 m, the radial coordinate of the vessel geometrical centre), B_{pa}[1] the poloidal field for normalisation and $\boldsymbol{n}$ the vector normal to the plasma surface. To assess the quality of the results, our evaluation of the Shafranov integrals has been compared with the same quantities provided by EFIT, used generally in JET to obtain information on the magnetic configuration. In fig.3 a typical example of S_1, S_2 and S_3 evolution during a discharge is reported, showing the good agreement between our calculation and EFIT outputs. Other essential quantities, which can be determined from the knowledge of the magnetic field at the LCFS, are the Shafranov moments [10,11,12], whose definition is:

$$Y_m = \frac{1}{\mu_0 I_P} \oint_S F_m B_p \, dl \tag{5}$$

where Γ is the contour of the plasma cross-section, and F_m is a weighting function satisfying $\nabla \times \nabla \times F_m \boldsymbol{e_\varphi} = 0$, where $\boldsymbol{e_\varphi}$ is the toroidal coordinate vector.

In BetaLi a particular role is assumed by the first two moments Y_1 and Y_2. The expression for Y_1 is the following:

$$Y_1 = \frac{\oint_\Gamma \left\{ \left[R - R_c + \frac{1}{2R_c}(R-R_c)^2 \right] B_\tau + \frac{RZ}{R_c} B_n \right\} dl}{\mu_0 I_P} \tag{6}$$

where B_τ and B_n are respectively the tangential and normal coordinates of the poloidal magnetic field. This quantity allows the determination of the radial (R_t) and vertical (Z_t) coordinates of the current centroid [10]. The expression for Y_2 is given by:

$$Y_2 = \frac{\oint_\Gamma [f_2 B_\tau + R g_2 B_n] dl}{\mu_0 I_P} \tag{7}$$

where f_2 is:

$$f_2 = \left[(R - R_t)\left(1 + \frac{R-R_t}{2R_t}\right) \right]^2 - \left[(Z - Z_t)\left(1 + \frac{R-R_t}{R_t}\right) \right]^2 \tag{8}$$

and g_2 is:

$$g_2 = \frac{2(R-R_t)(Z-Z_t)}{R_t} + \frac{(R-R_t)^2(Z-Z_t) - \frac{2(Z-Z_t)^3}{3}}{R_t^2}. \tag{9}$$

Y_2 is important because can be directly correlated to l_i (see below).

3. INTERNAL INDUCTANCE AND CONFINEMENT PARAMETERS

If the plasma is sufficiently elongated ($k > 1.3$ in JET — see section 4), the Shafranov integrals lead to the calculation of the plasma internal inductance l_i [9] according to the relation:

[1] $B_{pa} = \sqrt{\frac{\mu_0^2 R_C I_P^2}{2V}}$, where I_p is the current and μ_0 is the diamagnetic parameter in the vacuum

$$l_i = \frac{1}{\alpha - 1}\left[S_1 + S_2\delta - 2S_3\right] \quad (10)$$

where

$$\delta = 1 - \frac{R_t}{R_c} \quad (11)$$

R_t is the radial current centroid and the parameter α is defined as

(12)

In the determination of the internal inductance the most delicate point is the evaluation of the α volume integral: since XLOC, and so BetaLi, cannot get the values of the magnetic field inside the plasma, (12) is unusable. So another expression of α becomes indispensable. Even if this quantity can be approximated by [9]

$$\alpha \cong \frac{2k^2}{k^2 + 1} \quad (13)$$

expressing the volume integral in terms of the elongation, unfortunately this expression turned out to be not completely satisfactory for JET plasmas. In order to reduce the errors on the internal inductance below 10%, a better estimate of this quantity proved to be necessary. To this end, a systematic analysis of the parameters influencing α has been performed. In this perspective, a so-called ideal parameter α_{ID} has been defined and it has been calculated from (10) using all BetaLi quantities except l_i, that has been taken from EFIT, since this is the value that has to be reproduced. From a careful correlation analysis, it has been recognised that α_{ID} shows a strong and clear dependence on k, Y_2, α_S and on their mixed products. α_S has been defined as the analogous of (12) but computed on S instead than in V:

$$\alpha_S = 2\int_S \left(\overline{B}_\theta \cdot \overline{e}_Z\right)^2 dV \,/\, B_\theta^2 dV \quad (14)$$

The α_{ID} parameter has been approximated as a linear combination of the three main quantities k, Y_2, α_S and of their mixed products. Its dependence on the six aforementioned quantities is testified in table 1, where the correlation coefficients are shown: the closer to one the absolute value of a coefficient, the more the related quantities are correlated. The expression of α_{ID} becomes the following:

$$\alpha_{ID} = c_1 k + c_2 Y_2 + c_3\alpha_S + c_4 kY_2 + c_5 k\alpha_S + c_6 Y_2\alpha_S \quad (15)$$

A database of 21 pulses, representative of the most typical plasma configurations, has been used in order to derive the linear combination coefficients c_i by inversion of the following over-determined matrix equation:

$$. \quad [\alpha_{ID}] = [X] \cdot [C]. \tag{16}$$

In (16), [X] is the 6-column matrix containing the parameters on which α_{ID} depends and [C] is the vector containing the 6 unknown coefficients. The number of rows, i.e. the number of patterns considered, is about 3000. The inversion of the equation and the determination of the coefficients have been performed with the Singular Value Decomposition method. The coefficients determined in this way have then been used to perform the calculation of α in real-time. In particular, it is worth mentioning that the adopted method allows an optimisation of the coefficients valid both for the limiter and the x-point phase of the discharge.

The time evolution of a BetaLi estimate of l_i and the corresponding signal from EFIT is reported in fig.4, showing the good agreement between the two values during the entire shot. It can be seen that the time variations ofl_i during the discharge are well reproduced and this is a fundamental point for the real-time control purposes. Once the plasma internal inductance has been properly estimated, it can be used to determine the MHD beta, β_{MHD}, from the following relation [8]:

$$\beta_{MHD} = \frac{S_1}{2} + S_2\left(1 - \frac{\delta}{2}\right) - \frac{l_i}{2} \tag{17}$$

and to provide the MHD energy, W_{MHD}, calculated from [13]:

$$W_{MHD} = 4.71e^{-7} R_0 I_p^2 \beta_{MHD} \tag{18}$$

In its turn β_{DIA} can be expressed just in terms of S_1, S_2 and of the plasma diamagnetic parameter μ, which is supplied directly by XLOC, as follows [8]:

$$\beta_{DIA} = S_1 + S_2\delta + \mu \tag{19}$$

whose related diamagnetic energy, W_{DIA}, is given by [13]:

$$. \quad W_{DIA} = 4.71e^{-7} R_0 I_p^2 \beta_{DIA} \tag{20}$$

From W_{DIA} several important confinement parameters can be derived, like the diamagnetic toroidal beta β_{TOR} and the diamagnetic normalised beta β_{NORM} [14]:

$$\beta_{TOR} = \frac{19.123e^{-6} \cdot W_{DIA} R_0^2}{B_{VAC} V} \tag{21}$$

$$\beta_{NORM} = \frac{56.605 \cdot W_{DIA}\, a R_0}{I_p B_{VAC} V} \tag{22}$$

where B_{VAC} is the toroidal field in the vacuum computed at R_c and supplied by XLOC. Combining at last the results from l_i and β_{DIA}, also τ_E can be evaluated [4]:

$$\tau_E = \frac{W_{DIA}}{P_T - \frac{dW_{DIA}}{dt}} \tag{23}$$

where P_T is the total input power, given by the sum of the ohmic input power and the powers coming from the additional heatings, namely the neutral beam injection (NBI), the radio-frequency heating (ICRH) and the lower hybrid (LH). The ohmic input power itself can be evaluated by means of l_i [4]. As an example, in fig. 5 the time evolution of β_{NORM} during a shot is compared with the same quantity given by EFIT. Even in this case the agreement is more than satisfactory for real time applications.

4. OVERVIEW OF THE RESULTS AND PERFORMANCES OF THE CODE

4.1. OVERVIEW OF THE RESULTS

A summary of the quantities calculated with the described application is given in table 2. To give some indications about the accuracy of our estimates, in fig. 6 and 7 a statistical comparison between the results of the BetaLi and EFIT codes are shown for l_i and β_{NORM}. It must be pointed out that in fig.6 the points most far from the bisectrix belong to the final part of the pulse, when both the current and the elongation are low. The database comprises 54 discharges, mainly of the kind used in optimised shear, ITB, high triangularity, and next-step Tokamak configurations, and that cover a wide spectra of the main plasma parameters, such as the plasma current and the toroidal magnetic field.

The application has been benchmarked successfully on a large variety of discharges and can be considered of general validity. The important condition for the applicability of these routines is on the elongation: it must be sufficiently high (a typical value for JET is $k > 1.3$) in order to have a reliable value of S_3, an α greater than 1 (for circular plasmas $k = 1$ and α equals 1) and therefore a consistent value of l_i. The mean relative error (MRE[2]) and the mean squared error (MSE[3]), with respect to EFIT, of some of the evaluated plasma parameters, are summarised in table 3. Error bands of a few percent, with respect to the off-line analysis code, must be considered more than satisfactory for real-time control applications, keeping in mind that the reference EFIT values have by themselves an error band of a few percent. Similarly table 4 shows the errors with respect to FAST for two others confinement parameters, namely the total input power and the confinement time. In this case the values are worse, but two issues must be pointed out: a) the determination of these parameters is more difficult since it involves the real-time calculation of derivatives; b) FAST itself gives, for these signals, a level of accuracy of 20-25%.

4.2. COMPUTATIONAL PERFORMANCES

BetaLi has been written in C++ and the machine used to test it has been a PC with a 400MHz Pentium II as processor and Windows NT as operative system. The calculation of the LCFS is very fast with the described code, taking on average 600 µs per cycle. The accuracy in the determination of the

[2] $MRE = \left(\sum_{i=1}^{N_I} \left\| \frac{S_i^R - S_i^B}{S_i^R} \right\| \right) / N$ where S^R is the reference off-line signal, S^B is the analogous realtime signal from BetaLi, and N is the total number of samples

[3] $MSE = \left(\sum_{i=1}^{N_I} (S_i^R - S_i^B)^2 \right) / N.$

boundary spans from ~1 cm near the equatorial plane at Z = 0m, to ~4cm in the upper and lower regions of the plasma. This is mainly due to two reasons: the approximation of Ψ with a Taylor expansion and the uncertainty in the field measured by the magnetic probes. On the other hand, all the parameters listed in table 2, plus other temporary parameters necessary for the calculations (altogether about 50 signals), can be evaluated in about 900μs per cycle. It turns out that the overall updating time is about 1.5ms. This value is well below the sampling period of 10ms at which BetaLi is requested to supply data to the other JET control applications.

5. EXAMPLE OF REAL-TIME CONTROL OF THE DIAMAGNETIC NORMALISED BETA

Due to the positive indications obtained from a wide database of discharges, an online version of BetaLi has been implemented into the JET real-time control system and is currently running on a PC with the same characteristics as the one used in the test phase.

The outputs of the code have already been used in the last experimental campaigns. In particular, the estimate of the diamagnetic energy has been given as input to a feedback control code aimed at controlling this quantity using NBI heating systems as actuators.

5.1. RESULTS

The control model implemented to drive the actuators foresees a traditional proportional-integral gain, and can be expressed with the formula:

$$P(t) = P_0 + G_P \Delta\rho + G_I \int_{t_0}^{t} \Delta\rho \, dt, \tag{24}$$

where P is the heating power at time t, P_0 is the power at the initial control time t_0 and $\Delta\rho$ is the difference between the target value and the computed one.

The achievements have been encouraging and the results are shown in the following. In fig. 8a there are the estimates of BetaLi and EFIT W_{DIA}, while in fig. 8b the NBI power is reported; the pulse is the 58955 one.

CONCLUSIONS AND FURTHER DEVELOPMENTS

Given the accuracy in the results and the computational speed reported in section 4, the performances of BetaLi can be considered suitable to real-time control applications in big Tokamaks like JET. If the plasma elongation is above 1.3, the presented algorithm is potentially capable of providing in real-time the main topological and confinement quantities of a Tokamak plasma, also for those scenarios that can present quite involved profiles, as the reversed shear discharges. It is also worth mentioning that the code is very robust and applicable to both limiter and x-point configurations.

The real-time control of some plasma parameters has already been positively tried using feedback control algorithms and the heating systems as actuators. For example, as shown is section 5, the feedback control of the diamagnetic normalised beta has been successfully achieved and this result is very encouraging for the future, especially when having in mind the Tokamak advanced scenarios and ITER.

The BetaLi performance in the calculation of the internal inductance and of the magnetic axis radial position has been compared to that of two neural networks, expressively developed to determine in real-time the aforementioned quantities. The relevance of this alternative approach resides mainly in the generalisation capabilities of neural networks, which could be reflected in an improvement of the accuracy in the determination of l_i and R_m, especially in particular discharges with ample margins of variation of these parameters. The performance of the neural networks proved to be better than the

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TYPE OF CORRELATION COEFFICIENT	***VALUE***
Between k and α_{ID}	0.96
Between Y_2 and α_{ID}	−0.82
Between α_S and α_{ID}	0.83
Between $k*Y_2$ and α_{ID}	−0.87
Between $k*\alpha_S$ and α_{ID}	0.94
Between $Y_2*\alpha_S$ and α_{ID}	−0.85

Table 1: Correlation coefficients between the parameters k, Y_2, α_S, their mixed products and α_{ID}.

Shafranov integrals and internal inductance parameters	first Shafranov integral S_1 second Shafranov integralS_2 third Shafranov integral S_3 internal inductance l_i
Geometric parameters	minor radius a major radius R_0 elongation k triangularity δ boundary length L poloidal cross-section area S volume V radial current centroid R_t vertical current centroid Z_t
Confinement parameters	radial magnetic axis R_m diamagnetic poloidal beta β_{DIA} diamagnetic energy W_{DIA} diamagnetic toroidal beta β_{TOR} diamagnetic normalised beta β_{NORM} MHD beta β_{MHD} MHD energy W_{MHD} ohmic input power P_Ω total input power P_T confinement time τ_E
Other parameters	Greenwald density limit N_{GW} first Shafranov moment Y_1 second Shafranov moment Y_2 toroidal magnetic field in plasma B_0 loop voltage at plasma boundary V_p

Table 2: list of the main quantities calculated by BetaLi.

Plasma parameters	MRE (%)	MSE
l_i	2.59	0.0012
S_1	3.3	0.001
S_2	4	$1.87e^{-4}$
S_3	2.82	0.0001
β_{DIA}	10.79	0.0007 $[\%]^2$
β_{TOR}	11.14	0.0023 $[\%]^2$
β_{NORM}	10.96	0.0029 $[\%]^2[m]^2[T]^2/[MA]^2$
W_{DIA}	11.43	$1.96e^{10}$ $[J]^2$
R_t	0.2	$4.2e^{-5}$ $[m]^2$
Z_t	0.86	$1.1e^{-5}$ $[m]^2$
R_m	0.33	0.0001 $[m]^2$
R_0	0.16	$2.98e^{-5}$ $[m]^2$
a	0.98	0.0002 $[m]^2$
k	1.26	0.0007

Table 3: mean relative errors (MRE) and mean squared errors (MSE), with respect to EFIT, of some of the plasma parameters evaluated by BetaLi.

Plasma parameters	MRE (%)	MSE
P_T	7.42	$2.82e^{11}$ $[W]^2$
τ_E	17.16	0.022 $[s]^2$

Table 4: mean relative errors (MRE) and mean squared errors (MSE), with respect to FAST, of some of the plasma parameters evaluated by BetaLi.

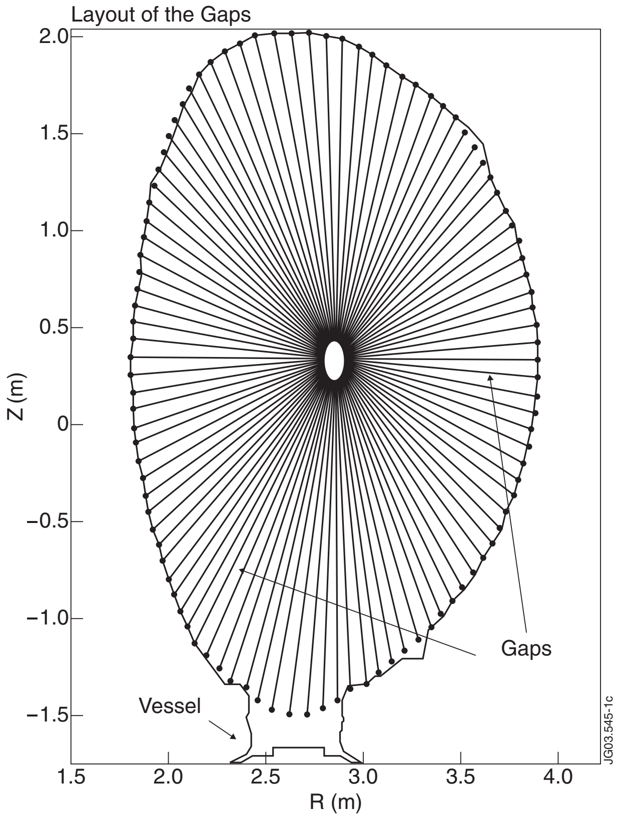

Figure 1: Layout of the 100 Gaps used to determine the last closed flux surface.

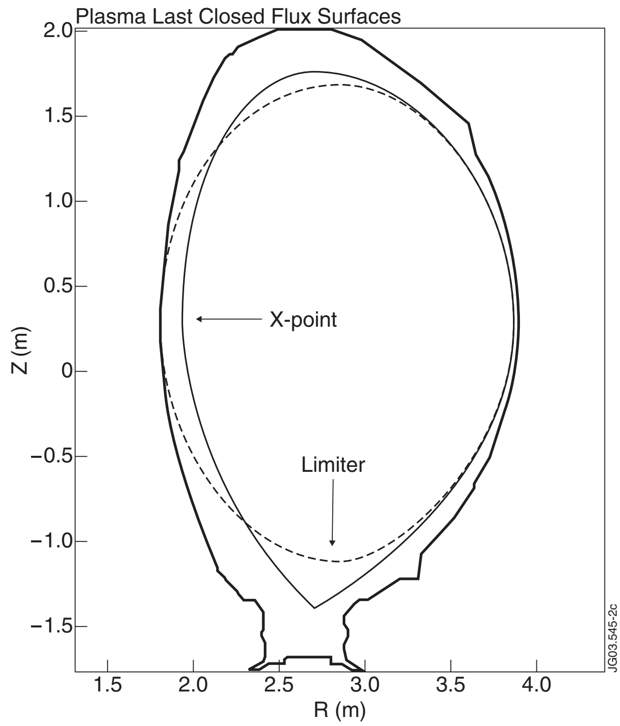

Figure 2: Example of two last closed flux surfaces determined by the boundary code: a limiter surface (dotted line) and an x-point one (solid line).

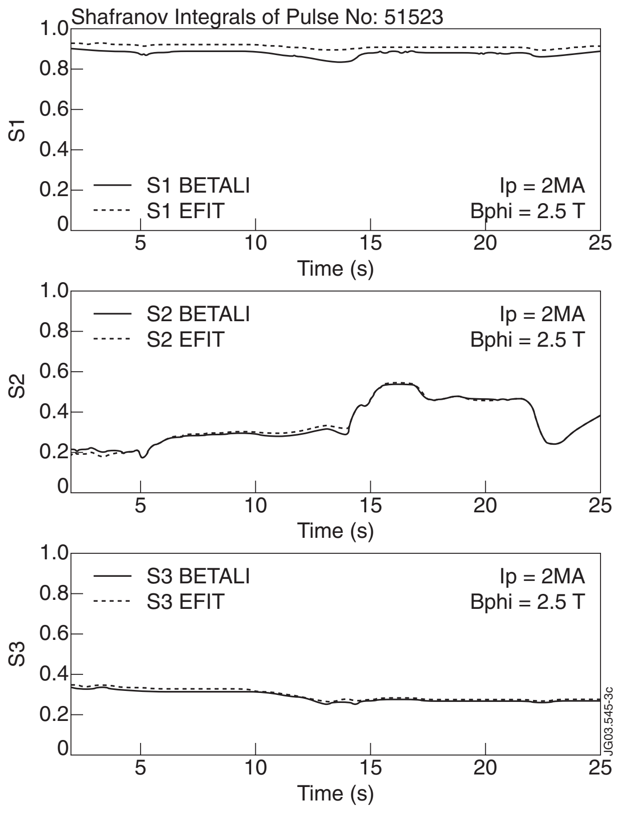

Figure 3: Estimates of the Shafranov integrals as computed by BetaLi and EFIT for Pulse No: 51523; Ip is the plasma current and Bphi is the toroidal magnetic field.

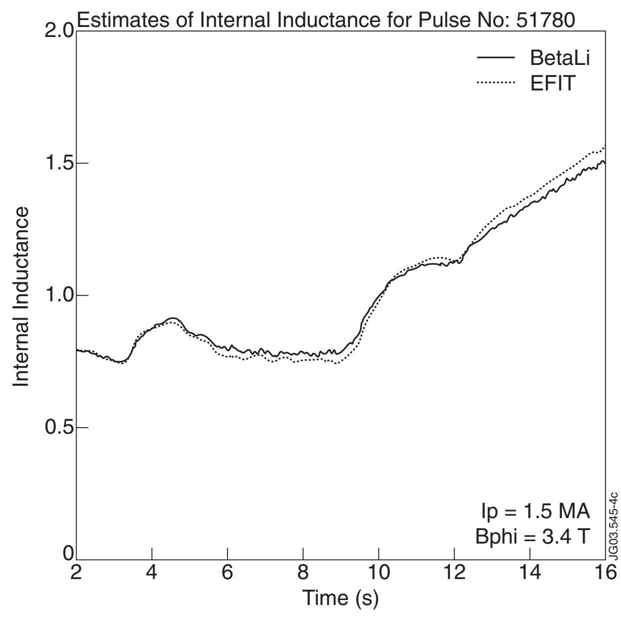

Figure 4: Estimates of the internal inductance as computed by BetaLi and EFIT for Pulse No: 51780; Ip is the plasma current and Bphi is the toroidal magnetic field.

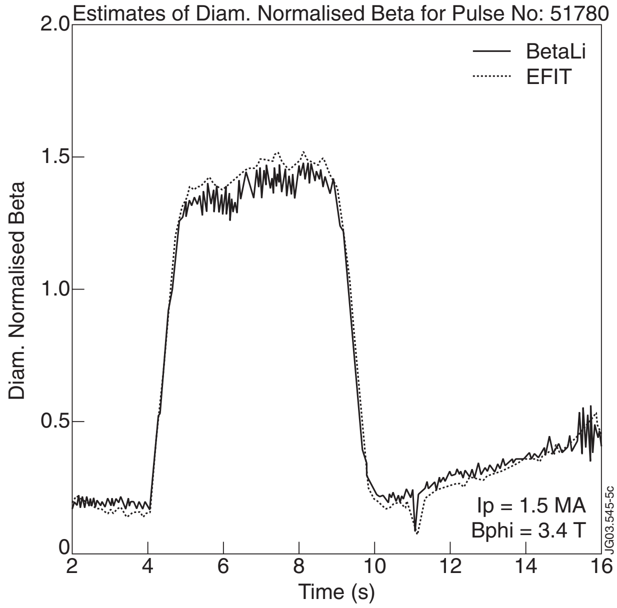

Figure 5: Estimates of the diamagnetic normalised beta as computed by BetaLi and EFIT for Pulse No: 51780; Ip is the plasma current and Bphi is the toroidal magnetic field.

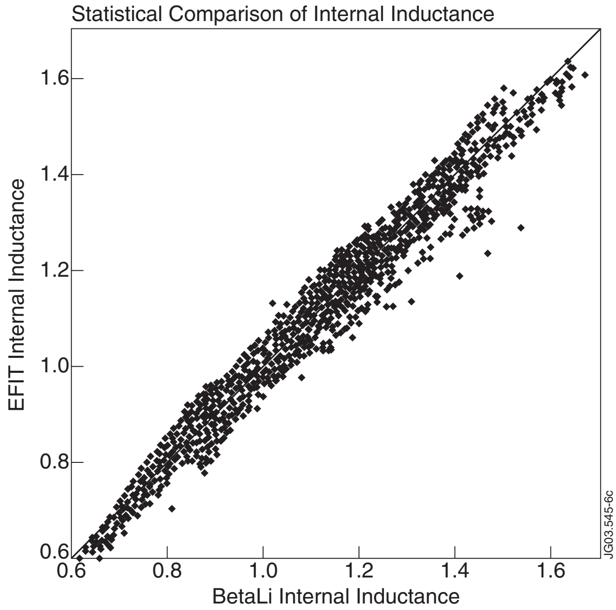

Figure 6: Statistical comparison of the estimates of the internal inductance as computed by BetaLi and EFIT.

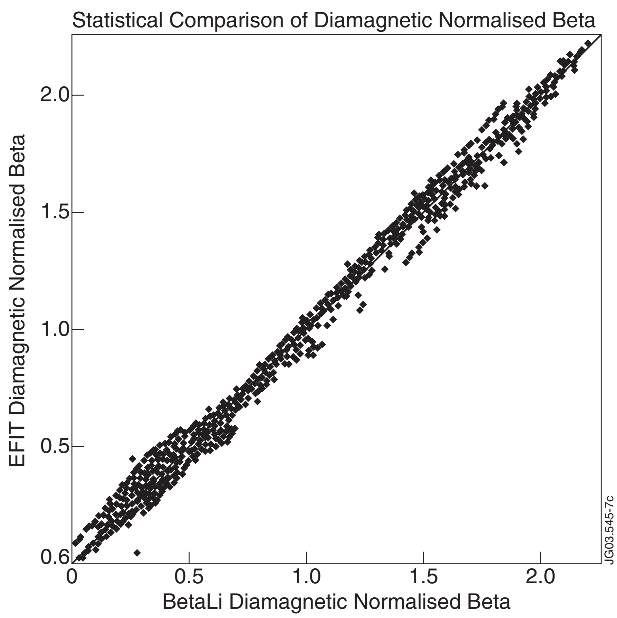

Figure 7: Statistical comparison of the estimates of the diamagnetic normalised beta as computed by BetaLi and EFIT.

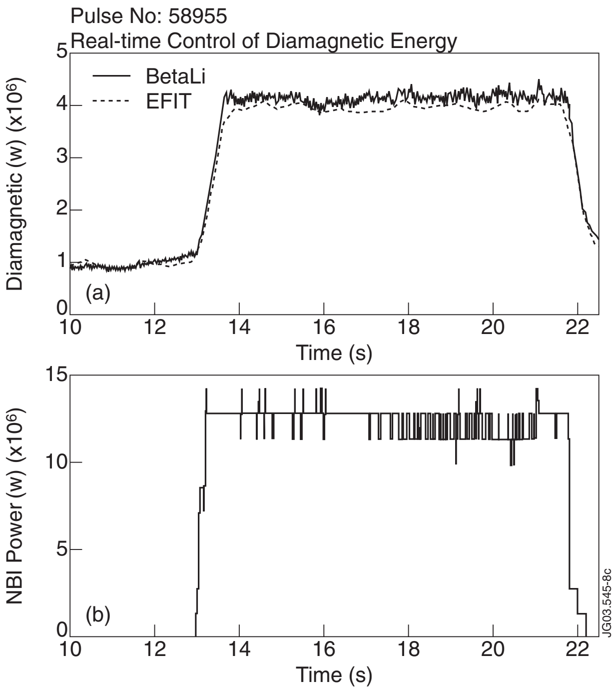

Figure8: a) Estimates of the diamagnetic energy computed by BetaLi and EFIT for Pulse No: 58955; Ip is the plasma current and Bphi is the toroidal magnetic field; b) the NBI heating power used to feedback control the energy.