EUROPEAN FUSION DEVELOPMENT AGREEMENT
JET

EFDA–JET–PR(03)36

V.Yavorskij, V.Goloborodko, K.Schoepf, S.E.Sharapov, D. Stork
and JET EFDA contributors

Confinement Criteria for Fusion Alpha Particles in Current Hole Tokamaks

Confinement Criteria for Fusion Alpha Particles in Current Hole Tokamaks

V.Yavorskij[1,2], V.Goloborodko[1,2], K.Schoepf[1], S.E.Sharapov[3], D. Stork[3]
and JET EFDA contributors*

[1]*Institute for Theoretical Physics, University of Innsbruck, Austria, Association EURATOM-OEAW*
[2]*Institute for Nuclear Research, Kiev, Ukraine*
[3]*Euratom/UKAEA Fusion Association, Culham Science Centre, Abingdon, Oxfordshire OX14 3DB, UK*
** See annex of J. Pamela et al, "Overview of Recent JET Results and Future Perspectives", Fusion Energy 2000 (Proc. 18th Int. Conf. Sorrento, 2000), IAEA, Vienna (2001).*

Preprint of Paper to be submitted for publication in
Physical Review Letters

ABSTRACT.

An analytical approach is developed for assessing the confinement of fusion alphas born in the plasma core of a tokamak-reactor with a toroidal current hole. Confinement criteria determining the minimum plasma current requirements with respect to the alpha particle energy are derived. It is shown that the enhancement of first orbit alpha losses induced by a current hole of a radius r_* can be recompensed by an increase of the total plasma current by the factor $1/(1-\sqrt{r_*/a})$, where a is the minor plasma radius. Numerical modelling of alpha losses in an equilibrium magnetic configuration reconstructed for a high-performance JET current hole plasma validates the analytical results.

INTRODUCTION

Recent experiments on improving the plasma confinement by optimising the magnetic field topology in JET [1,2] and in JT-60U [3] have shown that regimes with a toroidal current hole can be seriously regarded as candidates for a stationary tokamak operation. The attractiveness of current-hole configurations for a stationary tokamak may be, however, deteriorated by possible negative effects of the current hole (CH) on the confinement of fusion alphas. A detailed analysis of the CH effect on alpha particle confinement in JET has been performed in [4]. In the present Letter we generalise the results of [4] to an arbitrary CH tokamak plasma in order to extrapolate these results towards reactor-type tokamaks such as ITER. Focussing on the experimentally well-tested first orbit (FO) loss mechanism [5] and on the critical plasma current required for alpha confinement, we present an analytical assessment of the quality of *a*-containment in CH tokamaks.

Considering a diminutive but finite density of the toroidal current in the near-axis region of radius r_*, we start from inspecting the drift orbits of alphas in an axisymmetric tokamak with magnetic field $\mathbf{B}(r, \chi)$, where r and χ are the radial and poloidal angle coordinates. The alpha drift orbits are determined by conservation laws [6] for the particle energy $E = m\mathbf{V}^2/2$, the magnetic moment $m = \mu[\mathbf{V}\times\mathbf{B}]^2/(2B^3)$ and the toroidal canonical momentum which, in the case of a weak poloidal magnetic field of a tokamak, can be represented as $P_\phi = e\Psi/c - mRV\xi$. Herein $\Psi = \Psi(r)$ is the poloidal magnetic flux inside a given magnetic flux surface (FS) with an equatorial distance r from the magnetic axis, further $R(r, \chi)$ denotes the distance from the tokamak axis of symmetry, and $\xi = \mathbf{V}\bullet\mathbf{B}/(VB)$ is the pitch-angle cosine. The effect of the toroidal current profile on the particle orbit is entirely described by the dependence $\Psi(r)$. For CH equilibria, this function takes on a simple form

$$\Psi(r) \cong \bar{\Psi}(x, x_*) \equiv a\,\Psi'_a\psi(x - x_*),\ \psi(y) = y\Theta(y) \qquad (1)$$

where $x = r/a$ is the normalised radius of the magnetic flux surface, Ψ_a' is a parameter determined mainly by the total plasma current and the plasma shape, Θ represents the Heaviside step function and $x_* = r_*/a$ measures the effective radial size of the current hole. Figure 1 demonstrates the agreement between the linear shape of $\Psi(r)$ given by Eq. (1) and the profiles $\Psi(r) = \Psi(r/a,\ x_m)$

reconstructed for a typical CH JET equilibrium [4], where $x_m = (\Phi_m/\Phi_a)^{1/2} = 0.45$ and 0.60 with Φ_m and Φ_a representing the toroidal fluxes at the radial position of maximum toroidal current and, respectively, at the plasma edge. For the examples illustrated in Fig.1 the values $x_* =$ 0.41 and 0.57 provide the best fit $\overline{\Psi}(x, x_*) \approx \Psi(x, x_m)$. For the general case of an arbitrary x_m we assume the best approximation of the shape $\Psi(x, x_m)$ be given by the model $\overline{\Psi}(x, x_*)$ with $x_* = x_*(x_m)$ minimising the functional

$$W(x_*, x_m) = \int_0^1 dx\, [\overline{\Psi}(x, x_*) - \Psi(x, x_m)]^2. \tag{2}$$

Within an accuracy of a few percents for $x_*(x_m)$, we obtain from Eq.(2)

$$x_* \cong 1 - \sqrt{6\int_0^1 dx\,(1-x)\,\Psi(x, x_m)\,/\,\Psi(1, x_m)}. \tag{3}$$

Using results of Ref. [4] we approximate the CH size by $x_* \cong 0.2+1.1x_m^2-0.3x_m^4$. Strictly, x_* should be considered as the effective hole size of the poloidal flux $\Psi(r)$ rather than the hole size of the toroidal current $j(r)$. Hence note that, even for a monotonic $j(r)$ corresponding to x_m=0, the above approximation yields $x_*>0$. Therefore the model shape $\overline{\Psi}(x, x_*)$ may be used as well for the description of fast ion orbits in tokamaks with a monotonic current (MC). This is confirmed also by Fig.1(a) that indicates $\Psi(x)/\Psi(1)\sim x^2\sim 10^{-2}$ in the plasma core $x<0.1$-0.2 for a typical monotonic current associated with the safety factor profile

$$q_a\,/\,q = 1 + 0.5x\,(1-x^2), \tag{4}$$

where $s=2(q_a/q_0-1)$ is the shear ($s=d\ln q/d\ln x$) at the plasma edge. Note that satisfactory agreement is observed also between the shape of safety factor q corresponding to a linear $\Psi(r)$ and the CH q-profile experimentally measured on JET, as seen from Fig.1(b). Following [6] we suppress the weak contributions of the poloidal magnetic field and of poloidal plasma currents to $B(r, \chi)$, and also neglect the small contribution of the Shafranov shift, $\Delta(r)$, to $R(r, \chi)$. Taking $\overline{\Psi}(x, x_*)$ from Eq. (1) and introducing ξ as well as the normalised poloidal gyro radius $d = mcVa/(e\Psi_a')$ as the new variables for the particle velocity, we derive the orbit equation

$$(\psi - \psi_0 + dh_0\xi_0)^2 - d^2h\,[h-(1-\xi_0^2)h_0] = 0 \tag{5}$$

where $h = A + x\cos\chi$ with the plasma aspect ratio $A = R_a/a$ and the subscript "0" denotes the value that corresponds to the initial point ($x=x_0$, $\chi = \chi_0$) in the toroidal tokamak cross section, which for all orbits can be chosen as the crossing point with the equatorial plane ($\cos\chi_0 = \pm 1$) at their respective minimum FS radius x_0. Introducing $\psi(x)$ from Eq. (1) into Eq. (5) makes the latter quadratic both in x as well as in $\cos\chi$, and thus yields the simple single-valued explicit analytical expressions

$$x = X(\chi, x_0, \chi_0, \xi_0; d, x_*) \tag{6}$$

used here for investigating the orbit topology. Following the conventional orbit analysis [6-9] one readily obtains from Eq. (6) that, for any given d and x_*, the maximum FS radii $X_m(x_0, \xi_0; d, x_*) = max\{X(\chi, x_0, \xi_0; d, x_*)\}$ are reached by the particles trapped in the region of a weaker toroidal field (in the outer midplane of the tokamak) within the pitch-angle cosine band $-1 < \xi_s(x_0, d, x_*) \le \xi_0 \le 0$. Here x_s separates the counter-circulating particles, $-1 \le \xi_0 < \xi_s(x_0, d, x_*)$, from the trapped ones and is obtained from Eq. (6) as

$$\xi_s = -dH - 2\sqrt{Hx_0 / (A + x_0)},\ 1/H = 1 + \sqrt{1 - d^2} \tag{7}$$

for orbits outside the current hole, $x_0 > x_*$, and as

$$\xi_s = -dH - \sqrt{2H(x_* \pm x_0) / (A \pm x_0)} \tag{8}$$

for particles crossing the current hole area, $x_0 < x_*$. Hence, for particles crossing the plasma centre, $x_0 = 0$, the pitch-angle cosine corresponding to the boundary between trapped and circulating particles, $\xi_{s0} = \xi_s(x_0 = 0)$, takes on the value $-(2x_*/A)^{1/2} \neq 0$ if the particles energy tends to zero. This is contrary to the case of a monotonic q-profile, where $\xi_{s0} \to 0$ if $E \to 0$. Consequently, the current hole results in an enlargement of the population of toroidally trapped particles in the plasma core region. This constitutes the most essential effect for particles with low and moderate energies, for which $d \ll 1$. Correspondingly, the maximum radial coordinates along the orbits, x_{max}, achieved by barely trapped particles are given by

$$x_m = X_m(x_0, \xi_s; d, x_*), \tag{9}$$

where $\xi_s = \xi_s(x_0)$ is determined by the expressions of Eqs. (7,8). Using Eqs. (7,9) we find the maximum values of poloidal gyro radii, $d = d_{cr1}$, below which all particles that cross the current hole area are confined in the plasma. The corresponding confinement condition is $X_m(x_0 = x_*, \xi_s; d_{cr1}, x_*)$ that can be rewritten within an accuracy of a few percents as

$$d < d_{cr1} \cong \hat{d}(A)(1 - \sqrt{x_*}),\ \hat{d} \cong \frac{1}{\sqrt{2A}}\left(1 - \frac{1}{16A}\right). \tag{10}$$

Analogously, the condition $X_m(0, \xi_{s0}; d_{cr2}, x_*) < 1$ gives the criterion for confinement of particles crossing the plasma centre, which, with the same accuracy, is written as

$$d < d_{cr2} \cong d_{cr1} f(x_*),\ 1 \le f(x_*) \le 1.2 \tag{11}$$

where $f(x_*) \approx (1 + 0.285\sqrt{x_*} - 0.107x_*)$. Orbits of marginally confined fattest bananas corresponding to $d = d_{cr1}$ and to $d = d_{cr2}$ for JET CH equilibria with $r_* = 0.407a$, $x_m = 0.45$, are displayed in Fig.2. Considering the explicit dependence of the normalised poloidal gyroradius d on the plasma current and on the particle energy

$$d = \frac{mcVa}{e\Psi'_a} \equiv \frac{mcVq_a g}{ed\Phi\,(a)/da} = \frac{mc^2 Vq_a g}{2eIA}\,Fg\,, \tag{12}$$

where I denotes the total plasma current, $g = d\ln\Psi(a) / d\ln\bar{\Psi}(a) \approx 1$ is a factor accounting for the deviation of $\Psi(r)$ from $\bar{\Psi}(r)$ of Eq. (1), and $1.5>F>1$ is a geometrical factor determined by plasma non-circularity [10], the criteria of Eqs. (10) and (11) can be represented in the two alternative forms

$$I > \{I_{cr1}, I_{cr2}\},\ E < \{E_{cr1}, E_{cr2}\} \tag{13}$$

with the critical values

$$I_{cr1} \cong \frac{Fg}{Z_i}\sqrt{\frac{\mu_i E}{A}}\,\frac{1}{1-\sqrt{x_*}} = I_{cr2}f,\ E_{cr1} \cong \frac{A}{\mu_i}\left[\frac{Z_i I}{Fg}(1-\sqrt{x_*})\right]^2 = \frac{E_{cr2}}{Z_i} \tag{14}$$

Here the particle energy E is in MeV, the plasma current I is in MA, Z_i and m_i represent the fast ion charge and mass numbers. For 3.5MeV alphas, Fig 3a displays the critical currents I_{cr1} and I_{cr2} corresponding to the criteria determined by Eqs. (10,11) as functions of the effective current hole size for a JET-like flux surface geometry with F=1.4 and g=1 [4]. Also shown are the critical currents $I_{cr1}(x_*)$ and $I_{cr2}(x_*)$ both calculated using the numerical JET-like CH equilibrium [4]. Satisfactory agreement is seen between the confinement criteria obtained for the qualitatively modelled and the numerically reconstructed CH configuration. Further, it follows from Fig.3(a) that in an ITER-like tokamak, a total plasma current $I \geq 10MA$ will provide good alpha particle confinement for CH scenarios with an effective current hole radius up to $x_* \sim 0.7$. Figure 3(a) also indicates that in present-day JET experiments with $I \leq 3MA$, even medium size current holes $x_* \sim 0.3$ would result in a significant enhancement of FO alpha losses.

Figure 3(b) displays the critical currents I_{cr1} and I_{cr2} in comparison with the critical current I_{cr0} required in a tokamak reactor with a MC profile to confine fusion alphas produced in the paraxial area. The critical value of the poloidal gyro-radius, d_0, not to be exceeded in order to confine paraxial orbits can be obtained from the orbit analysis using Eq.(5) with the normalised flux $\psi_{MC}(x,s)=x^2(1+s/2-sx^2/4)/2$, corresponding to the q-shape of Eq.(4). Neglecting the high order toroidal correction, this value is given by $d_0 = 0.25\delta_{cr}A^{-1/2}$ with δ_{cr}=1+0.28s and results in the following critical current and critical particle energy, respectively:

$$I_{cr0} = \frac{2.9F}{\delta_{cr} Z_i}\sqrt{\frac{\mu_i E}{A}}\,,\quad E_{cr0} = 0.12\,\frac{A}{\mu_i}\left(\frac{Z_i\delta_{cr} I}{F}\right)^2. \tag{15}$$

Note that similar expressions for I_{cr0} and E_{cr0} follow from Eq. (14) for g = d$ln\psi_{\rm MC}(1,s)$/d$ln\psi(1, x_*)$ = 2(1− x_*) / (1+s/4) and x_* = 0.2. From Fig.3(b) it is apparent that a CH tokamak exhibiting a moderately sized current hole (x_* ~0.3-0.4) is equivalent – from the point of view of fast particle confinement – to a MC tokamak with a moderate shear $s \leq 2$. The scenario with a larger current hole, x_* ~0.5-0.6, is comparable with a shear-less (s=0) or a reversed shear (s<−1) field qualitatively corresponding to the magnetic configuration of a stellarator of multipolarity l = 2 and l = 3. For 3.5MeV alphas confined in the shear-less magnetic field with circular and concentric flux surfaces (F = 1), a well-known criterion follows from the first formula of Eq. (15), namely $I_{cr0} = 5.4A^{-1/2}MA$ (e.g. Ref. [11]). Further shown in Fig.3(b) is the critical current obtained in [12] for good confinement of 3.5MeV alphas in CH configurations. This latter criterion was based on the small banana-width approximation and works well only at relatively large x_* > 0.8-0.9 and/or relatively high currents (I >10 MA).

We note that the fulfilment of our new confinement criteria, Eqs. (13,14), guarantees rather good confinement of fast ions at arbitrary tokamak operational conditions. Fig.4 displays the first orbit loss fraction, L_{FO}, of fusion alphas in JET current hole equilibria with x_* = 0.41 as it varies with I and the fusion source shape. For $I > I_{cr1}$ the FO loss level is L_{FO} <5% even in the case of the flat fusion source term $S_{\rm THERMAL}(x < x_*) = S_{\rm THERMAL}(0)$ expected in future steady-state tokamaks [4]; for $I > I_{cr2}$ similar small loss levels occur only in the case of peaked source terms, e.g. $S_{\rm JET}$ [4] and $S_{\rm TFTR}$ [5] sketched in Fig.4 . However, note that L_{FO} <10%, even for the flat source term $S_{\rm THERMAL}(x)$, if $I_{cr1} > I > I_{cr2}$.

Inspecting the confinement criteria of Eqs. (11,12), we see that an enlargement of the current hole, Δx_*, demands for an increase of the plasma current, DI, according to

$$\Delta\,[I\,(1-\sqrt{x_*})] = 0 \qquad (16)$$

in order to sustain the same alpha confinement performance. Indeed this was observed in our numerical modelling of the FO alpha loss in JET CH equilibria [4] and is confirmed by Fig.5 displaying the FO loss distribution of 3.5MeV alphas over poloidal and pitch angles at the first wall of JET. The loss distribution was obtained using the orbit following loss simulation for the CH JET equlibrium used in [4]. It is seen that both loss level and loss distribution for the case I/x_m = 2MA/ 0.45 [$I_* \equiv I\,(1- x_*^{1/2})$ = 0.72] are remarkably similar to those in the case I/x_m=3MA/0.6 (I_* = 0.74). However, the confinement appears essentially improved for a regime with I/x_m=3MA/ 0.45 (I_* = 1.08), whereas it is drastically worse for I/x_m=2MA/0.6 (I_* = 0.49) when compared to the cases I_* = 0.72, 0.74 shown in Figs.5(a), 5(d). Moreover, also the poloidal and pitch-angle distributions of the FO loss vary significantly as _ is changed.

In conclusion, an analytical model based on physically reasonable approximations was used to derive compact efficacious confinement criteria determining the minimum plasma current required for confining fusion alphas in the paraxial area of a tokamak with hollow current profiles. From the point of view of fast ion loss, the presence of a current-less area in the plasma core, $x < x_*$, is shown to be equivalent to the reduction of the total plasma current in accordance with the rule $I \rightarrow I(1- x_*^{1/2})$. These results are a good starting point for investigating the effect of confined alphas on the CH operation regime both due to alpha heating of the bulk plasma as well as due to the alpha bootstrap contribution to the plasma current.

ACKNOWLEDGEMENT

The authors are grateful to C.D. Challis for instructive discussions on shear-reversed scenarios and thank N.C. Hawkes for providing MSE data for the relevant *q*-profile.

This work has been partially carried out within the Association EURATOM-OEAW and under the European Fusion Development Agreement. It was funded jointly by the U.K. Eng. and Phys. Sciences Research Council and by EURATOM.

REFERENCES

[1]. C.D.Challis et al., Plasma Physics Controlled Fusion**44**, 1031 (2002).

[2]. N.C. Hawkes *et al.*, Phys. Rev. Lett., **87**, 115001 (2001)

[3]. T. Fujita *et al.*, Phys. Rev. Lett.**87**, 245001 (2001)

[4]. V.Yavorskij,*et al.*, Preprint EFDA-JET-PR(03)01, (2003); Nucl. Fusion, **43**, 1077 (2003)

[5]. S. Zweben, Nucl. Fusion **40,** 91 (2000)

[6]. T.E. Stringer, Plasma Phys. **16**, 651(1974)

[7]. J.A.Rome, Y-K.M.Peng, Nucl. Fusion, **19**, 1193 (1979)

[8]. L.M.Hively *et al.*, Nucl. Fusion, **21**, 1431 (1981)

[9]. V.Ya.Goloborod'ko *et al.*, Nucl. Fusion, **23**, 399 (1983)

[10]. V.A. Yavorskij, *et al.*, Plasma Phys. Contr. Fusion, **43,** 249 (2001)

[11]. S.V. Putvinskij, Rev. Plasma Phys., **18,** 239 (1992), Ed. by B.B.Kadomtsev, Consultants Bureau, New York-London

[12]. Ya.I. Kolesnichenko *et al.*, Proc. 9th IAEA Conf. Pl. Phys. & Contr. Nucl. Fus. Res., Baltimore, Sept. 1982, Vol III, p 349

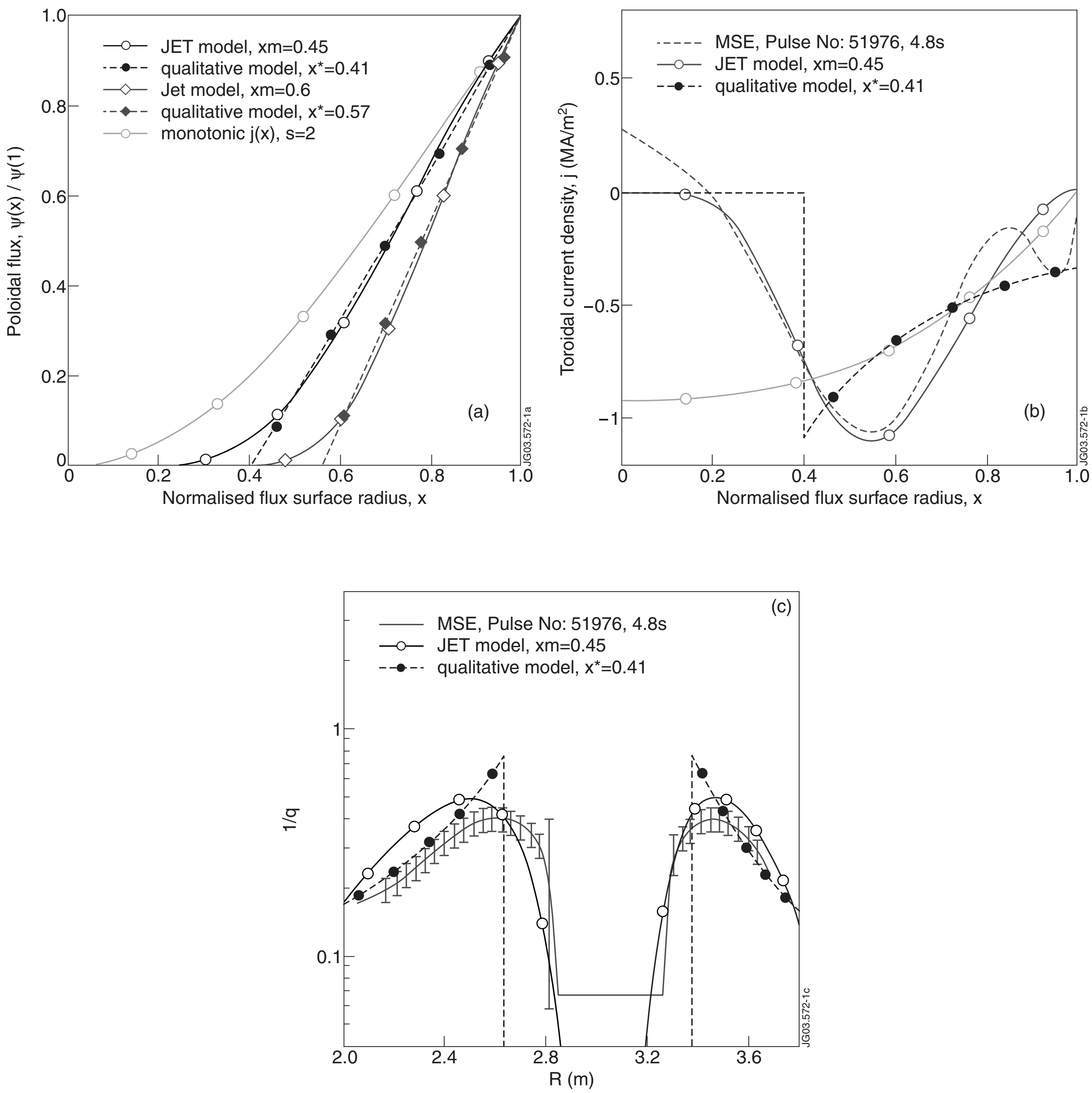

Figure 1: Profiles of poloidal flux and safety factor in a tokamak with hollow toroidal current. The symbol s denotes the shear at the plasma edge.

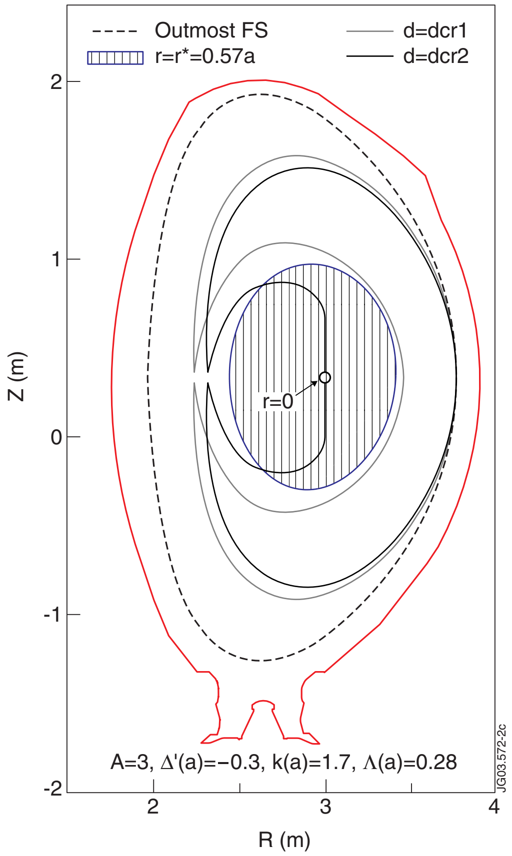

Figure 2: Orbits of marginally confined, barely trapped particles (fattest bananas) corresponding to $d = d_{cr1}$ *and to* $d = d_{cr2}$ *for a JET-like CH equilibrium with* x_m*=0.6 (*$x_* = 0.57$*), A = 3.2, elongation* $k_a = 1.7$*,*$\Delta'_a = -0.33$ *and triangularity* $\Lambda_a = 0.28$ *[4].*

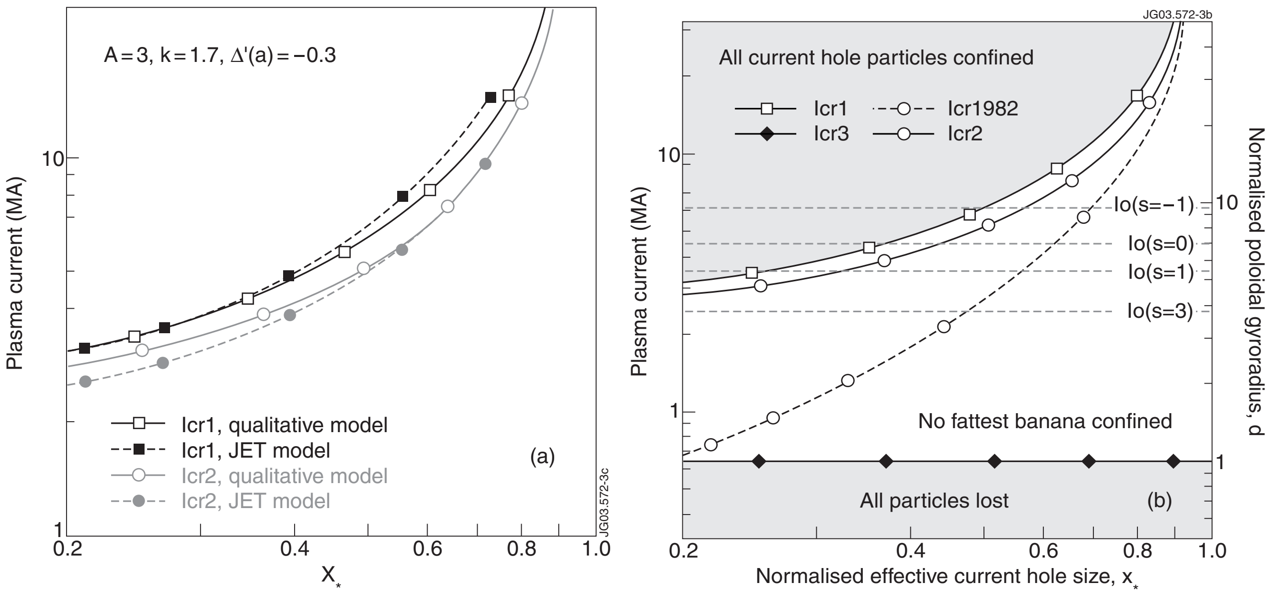

Figure 3: Critical currents required to confine all fusion alphas passing the current hole region,I_{cr1}*, and, respectively, to confine only alphas crossing the plasma centre,* I_{cr2}*, as functions of the current hole size.*

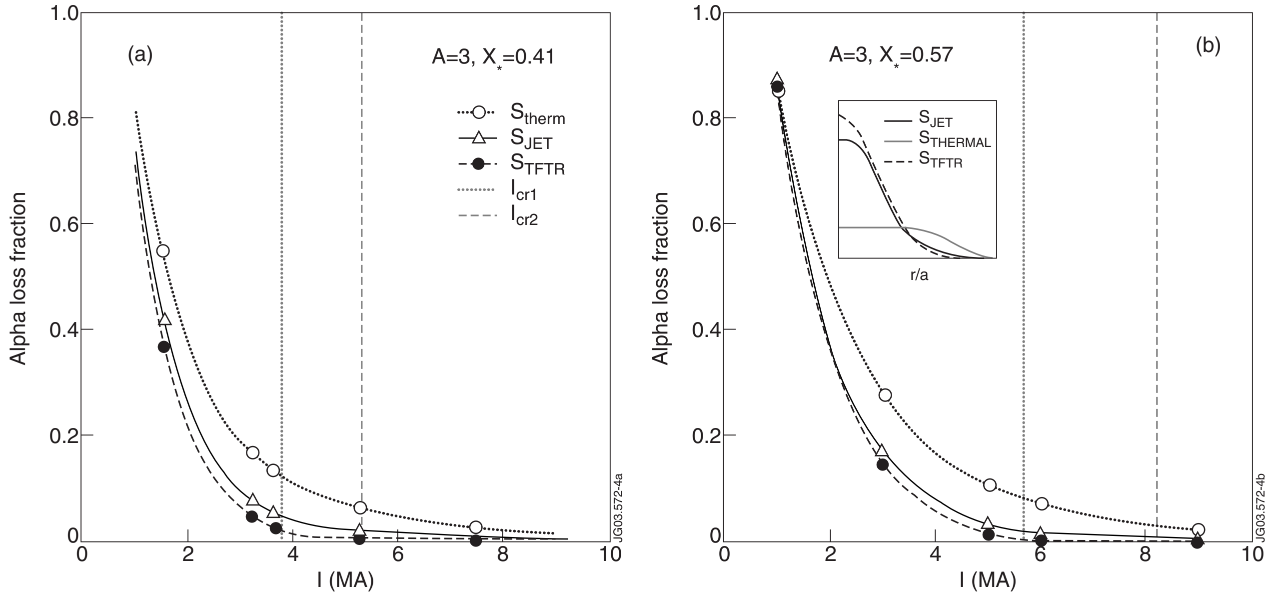

Figure 4:Alpha loss fractions versus total toroidal current in the JET CH equilibrium with $x_* = 0.41$.

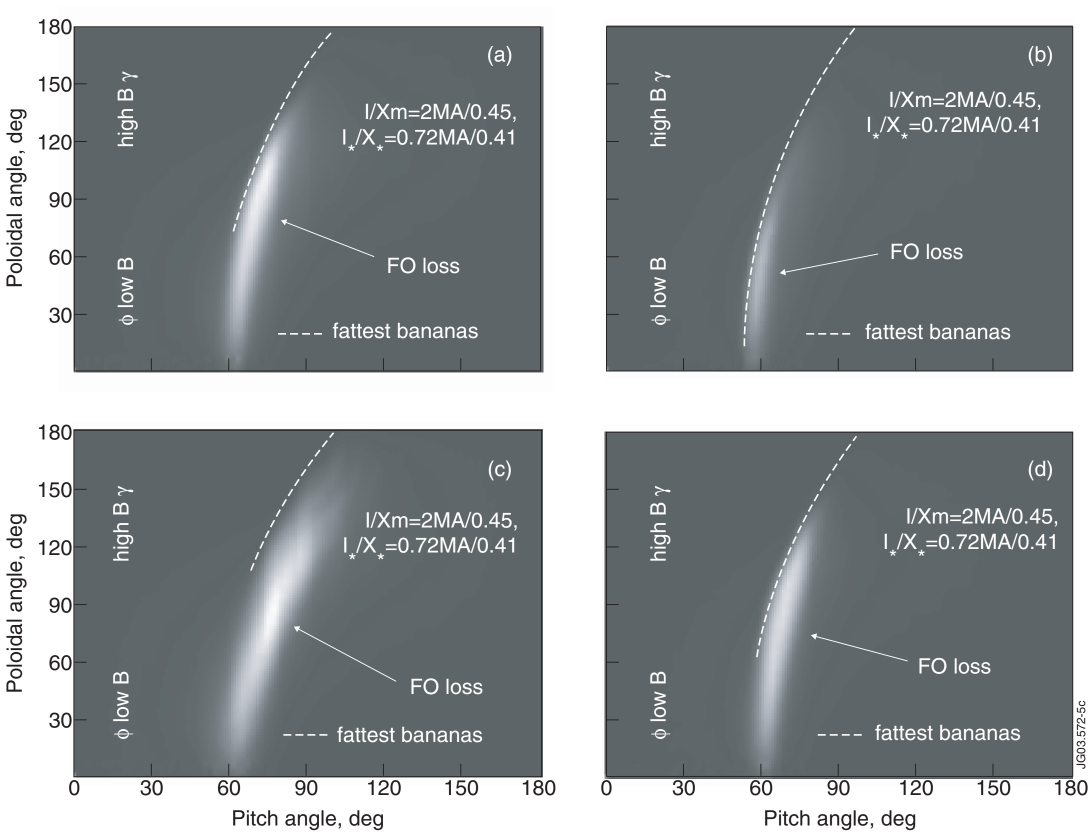

Figure 5: Distribution of FO alpha flux over pitch and poloidal angles in JET-like CH equilibria for various total toroidal currents and different current profiles.