EFDA–JET–CP(13)03/44

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** See annex of F. Romanelli et al, "Overview of JET Results", (24th IAEA Fusion Energy Conference, San Diego, USA (2012)).*

Preprint of Paper to be submitted for publication in Proceedings of the
40th EPS Conference on Plasma Physics, Espoo, Finland.
1st July 2013 – 5th July 2013

1. INTRODUCTION

A new Bayesian tomographic method for soft-X and bolometer diagnostic systems has been developed. The method is non-parametric in the sense of using Gaussian processes to model ,the underlying emissivity distribution, and the regularization of such a model becomes defined by a multivariate normal distribution at the points where the emissivity distributions should be evaluated [1,2]. As opposed to currently used methods, e.g. Maximum entropy [3] (MaxEnt) and Equilibrium-Based Iterative Tomography Algorithm [4] (EBITA), to which this method is compared, this method is fully analytical, involving no nonlinear iterations, and so can be feasible for real-time applications. Additionally, uncertainties of the solution, accounting for both measurement uncertainties and ambiguities due to insufficient coverage of sight lines, can be provided by direct sampling from the posterior probability distribution. Describing the emissivity distribution by Gaussian processes [5,6] has the further advantage that regularization can be expressed in a natural way as correlation length scales of a diffusion process. In particular, the method can locally adapt the length scales to the varying smoothness of the emissivity distribution. This method has been applied to three different experiments: soft-X reconstructions for W7-AS stellarator, and bolometer reconstructions for the WEGA stellarator and the JET tokamak, comparing favourably to currently used methods.

2. METHOD

In nuclear function, a conventional way to observe the plasma is the line integral measurement across a target region of the plasma by a detector located outside the plasma region. By using multiple detector arrays, a soft X-ray diagnostic system can spatially resolve emissivity distributions with a very high time resolution. In a similar way, a bolometer system can infer the distribution of total radiated power. The emissivity/radiation distributions over a 2D poloidal cross section can be expressed as $f(\bar{r})$ and a line integral signal obtained from a

$$d_l = c_l \cdot \int_{S_l} ds \cdot f(r) + \varepsilon_l, \quad l = 1, 2, ..., M \tag{1}$$

where, the line integral along path S_l is carried out within the solid angle subtended by one of M available detectors. The calibration factors c_l relates to the slight differences in spectral efficiencies and solid angles between the detectors. ε_l denotes an error term including both random and systematic uncertainties suffered by the diagnostic system.

In this Bayesian method, the posterior probability $p(\bar{f}_N | \bar{d}_m, \bar{\theta})$ over all possible solutions is deduced from the combination of a prior $p(\bar{f}_N | \bar{\theta})$ and a likelihood $p(\bar{d}_m | \bar{f}_N, \bar{\theta})$, divided by an evidence term $p(\bar{d}_m | \bar{\theta})$ with regard to model assumption, which can be written:

$$p(\bar{f}_N | \bar{d}_M, \bar{\theta}) = \frac{p(\bar{d}_M | \bar{f}_N, \bar{\theta}) \; p(\bar{f}_N | \bar{\theta})}{p(\bar{d}_M | \bar{\theta})} \tag{2}$$

where, the prior acts as a regularization on f_N to express our knowledge of it before any measurement; the likelihood introduces the constraints from the measured data within required data fits, finally the evidence term relates directly to the prior model assumptions, and can be used to optimize the model hyper-parameters $\bar{\theta}$. In this work a Gaussian Process [5,6], is applied to construct the prior, giving for the covariance between two points $\bar{r}_i$, $\bar{r}_j$:

$$k_{SE}\left(\bar{r}_i,\bar{r}_j\right)=\sigma_f^2\exp\left(-\frac{d_{ij}^2}{2l^2}\right),\quad d_{ij}=\left\|\bar{r}_i-\bar{r}_j\right\| \tag{3}$$

where (σ, l) are the hyper-parameters $\bar{\theta}$, determining the properties of the random process, in this case the rough magnitude of the emission, and a length scale l. These hyper-parameters can be determined by maximizing the evidence term in Eq.(2) in the space of $\bar{\theta}$. For further details of this method see [1] or [2]. The posterior distribution evaluated at a number of discrete points in the emissivity region, will be a multivariate normal distribution with mean and covariance given by

$$\bar{m}_f^{post}=\bar{m}_f+\left(\bar{\bar{R}}^T\bar{\bar{\Sigma}}_d^{-1}\bar{\bar{R}}+\bar{\bar{\Sigma}}_f^{-1}\right)^{-1}\bar{\bar{R}}^T\bar{\bar{\Sigma}}_d^{-1}\left(\bar{d}_M-\bar{\bar{R}}\bar{m}_f\right) \tag{4}$$

$$\bar{\bar{\Sigma}}_f^{post}=\left(\bar{\bar{R}}^T\bar{\bar{\Sigma}}_d^{-1}\bar{\bar{R}}+\bar{\bar{\Sigma}}_f^{-1}\right) \tag{5}$$

Equation (4) will give a single most likely reconstruction and Eq.(5) will give the uncertainty of the reconstruction. Uncertainties of functions of the distribution (such as total power) can be calculated by repeated sampling from the multivariate normal distribution with (4) as mean and (5) as covariance matrix, and forming a histogram of the function values calculated from each sample.

3. PERFORMANCES AND RESULTS

3.1 APPLICATION TO SOFT X-RAY SYSTEMS AT W7-AS STELLARATOR.

In W7-AS, the dependence of the maximum achievable thermal/magnetic pressure ratio β on the equilibrium magnetic flux surface has been extensively investigated [7]. Since the emission relevant parameters e.g. plasma density, temperature are expected to be approximately constant within the equilibrium flux surfaces calculated by the Variational Moments Equilibrium Code [8] (VMEC), the basic features of the reconstructed emissivity distribution should approximately agree with the equilibrium flux, hence the β induced effects on equilibrium flux surfaces can be investigated by tomographic analysis. The reconstructions by this method in Fig.1(b) shows a strong outward shift due to high β , and structures consistent with VEMC equilibrium analysis, with the exception of a large indentation in the inboard side, which may arise from an increased peaking of the pressure profile in the plasma center. Figure 1(c) shows reconstructions of an m = 3 mode structure which distributes symmetrically around the axis of the flux surface.

3.2 APPLICATION TO BOLOMETER SYSTEMS AT WEGA STELLARATOR AND JET TOKAMAK.

WEGA is a five period stellarator with a major radius of 0.72m and the coverage of lines of sight from the bolometer system is shown in Fig.2(a). When plasma axis is at R = 702mm, the center of reconstruction (Fig.2(b)) has a consistent location. In a different pulse when the plasma axis is shifted to R = 720mm, the center of reconstruction (Fig. 2(c)) moves to the same position. For both pulses an OXB heating (with a deposition region r ≈ 10mm) is applied, the size of reconstructions also coincides with the localized feature of such a heating approach.

Another implementation was carried out in JET when the massive gas injection was tested as a protection system by increasing the radiative power. Reconstructed radiation in Fig.3 verifies the concentration of the radiation in main plasma.

CONCLUSION

The work aims to develop a new method for the reconstructions with unsymmetric and localized features. Through implements on different systems and comparisons with different inversion methods using both simulated and experimental data, this method has proved to give convincing results, which is further confirmed by a good agreement between reconstructions from other methods, and also good correspondence with equilibrium flux surfaces. Additionally, this method provides uncertainties on the reconstructions, taking into account both measurement uncertainty and line of sight coverage. Without any nonlinearity and numerical iteration, this calculation is also fast enough for real time applications.

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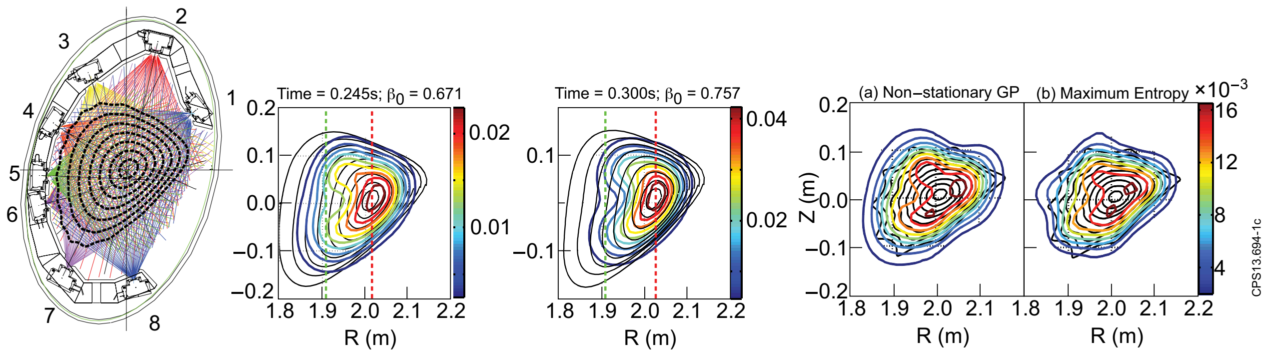

Figure 1: (a) A schematic view of the miniature soft X-ray system (MiniSoX diagnostic system) in W7-AS with eight compact detector arrays of a total 256 sight lines. (b) Reconstructions by this (left) and MaxEnt (right) methods in high β performance. (c) Reconstructions by two methods with complex mode structures.

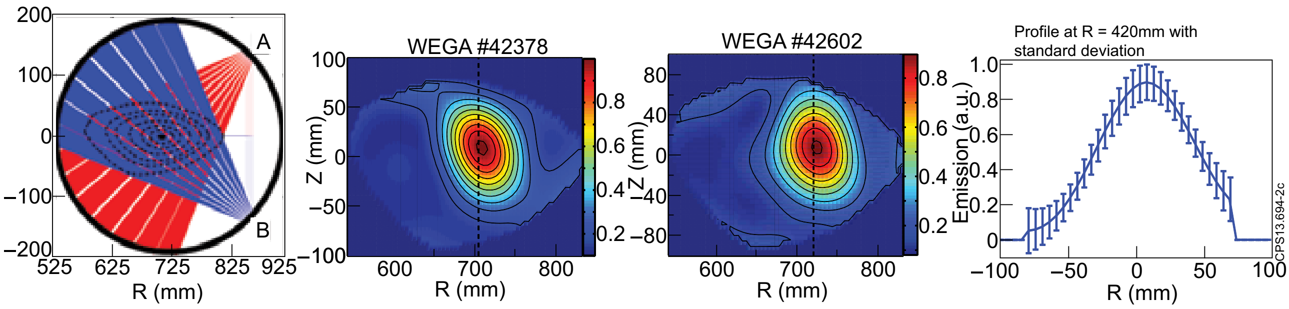

Figure 2: (a) Coverage of 16 lines of sight from two detector arrays (A and B) and typical flux surface (contours). (b) Reconstruction of radiation when plasma axis is located at R ≈ 702mm(dashed line). (c) Reconstruction of the radiation when plasma axis is shifted to R ≈ 720mm(dashed line). (d) Uncertainties of the reconstruction become larger around the edge where we have lower line sight coverage.

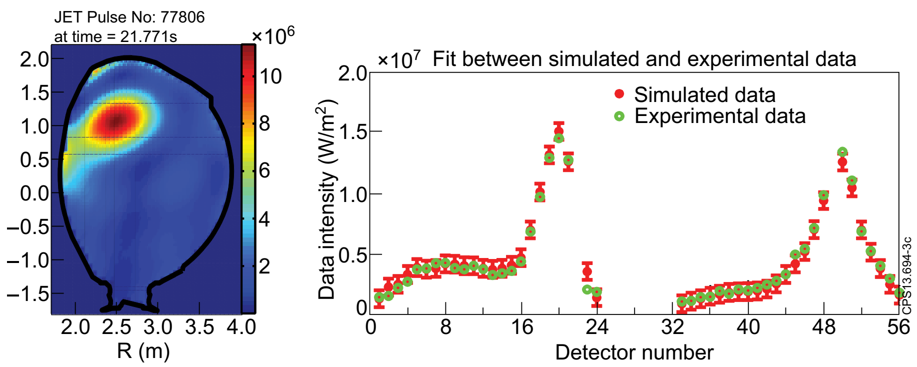

Figure 3: Reconstruction of the total radiation and the data fit with error bars.