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ABSTRACT

Fourier Transformation is one of the workhorses in time series analysis. If nonlinear effects deform the wave profiles of measured data, FFT needs additional harmonic components to simulate these profiles. Non-stationarity and nonlinearity therefore induce spurious harmonic components. These short-comings can also be found in wavelet analysis if the popular Morlet wavelet is used since it is Fourier based. In order to overcome these problems, the adaptive timefrequency data analysis method "Empirical Mode Decomposition" (EMD) is suitable. EMD acts as a dyadic filter bank to decompose the series into a finite set of so-called Intrinsic Mode Functions (IMF). Each IMF reflect the data on a different time-scale and admit well-behaved Hilbert transforms.With the Hilbert transform, the IMF yield instantaneous frequencies as functions of time that give sharp identifications of imbedded structures.

We applied EMD on MHD and ECE "Palm Tree Mode" (PTM) data. Hilbert amplitude spectra of data from the fast poloidal coil arrays show that the PTM is a coherent structure. The results prove in an independent way that the higher harmonics as seen in the FFT are a consequence of the high localization of the PTM structure. The effect of the mode on the temperature measured by ECE is twofold. One is the increase in temperature measured as temperature inside the filament. The other is the change in temperature perceived through the change in geometry by the magnetic perturbation of the PTM. These two contributions have been separated for the first time. EMD from a single signal thereby consistently supports the assumption that the PTM is indeed a current filament. Comparisons between FFT, wavelet analysis and EMD will be presented to highlight strengths and weaknesses of the individual methods.

INTRODUCTION

Recent works report the existence of coherent long-lived structures or current filaments in the edge of tokamak plasmas. These are found as confined current ribbons (outer modes) [1] or Palm Tree Modes (PTM) [2] in the edge of JET for example. An intuitive model was proposed in [3] which describes the genesis of these structures by a blob/hole creation mechanism during the ELM. ELM filaments with excess current [4], temperature and density are treated as blobs whereas it is believed that they leave corresponding holes behind due to current conservation considerations on short time scales [5]. It therefore complements the MHD description by a blob/hole or "quasi-particle" picture.

From pellet injection experiments in DIII-D and Tore Supra it is known that maxima of the mass deposition profiles are close to the magnetic $q = 2$ and $q = 3$ surfaces. Low order rational surfaces appear to slow down or even stop the polarization drift towards the low field side of the plasma [6]. Similarly it is to be expected that the lifetime of blobs and holes is increased in the vicinity of rational surfaces. The ∇B polarisation of holes is opposite to blobs. Under the assumption that the interchange drive of a polarised ELM-induced hole moves it into the proximity of a rational surface with low q, it should be slowed down or even stopped. Furthermore, large holes and blobs could close on themselves in the vicinity of a rational magnetic field line. For a closed filament, charge interchange imbalance would be short-circuited by parallel currents which would stop the filament completely. The result would be a closed, well localized filament with a signicantly increased lifetime which can only be filled by slow perpendicular transport. Figure 1 illustrates the proposed events before the holes and blobs reach the resonant surfaces. In this way of thinking outer modes are ELM filaments which were not able to propagate to the scrape-off layer whereas palm tree modes are signatures of ELM-induced holes. Nevertheless, the probability to create PTMs is smaller because holes travel up the temperature gradient into regions with increasing parallel transport. If this is true one can ask why the spectra of outer modes and PTMs show rich harmonics in the Fourier spectra? Fourier spectral analysis uses linear superpositions of unlocalised trigonometric functions. Therefore FFT can only be applied to linear and stationary time series. If nonlinear effects deform the wave profiles towards localisation, FFT needs additional harmonic components to describe these profiles. Non-stationarity and nonlinearity can induce spurious harmonic components that cause energy spreading. An example for such a behaviour would be a Fourier decomposition of a flash light, represented by a delta function in the time domain. Localised in time functions produce less localised FFTs. Spatial localised structures are therefore not independently traveling waves, but highly coherent wave packages. The non-stationary time series may be represented as follows [7].

$$X(t) = \int_{j=1}^{n} a_j(t)\ exp\ (i, \int w_j(t)dt) \tag{1}$$

In order to extract signals from nonlinear or non-stationary time series X(t), the adaptive timefrequency data analysis method “Empirical Mode Decomposition” (EMD) [7] is suitable. EMD acts as a dyadic filter bank to decompose the series into a finite set of so-called Intrinsic Mode Functions (IMF). Each IMF reflects the data on a different time-scale and admits well-behaved Hilbert transforms. EMD ables examinations of noisy linear and nonlinear processes. In contrast to Fourier spectral analysis the amplitudes a_j (t) and frequencies ω_j (t) obtained by EMD are functions of time (Eq.1). Amplitude and frequency modulations described by the IMFs are now clearly separated.

RESULTS AND DISCUSSION

Phenomena like PTMs exhibit a current, density and a temperature perturbation. Therefore EMD was applied to MHD and ECE data of JET. Electron temperature fluctuations of the PTM measured by ECE show different behaviour in different channels of the diagnostic (Fig.2a). The investigated channels B2:001, B1:012 and B1:011 cover parts of the pedestal region of JET for Pulse No: 52011, t = 19:164–19:166 s. Channel B1:011 (green) shows large positive temperature spikes as the PTM filament enters the detection volume. In contrast, channel B2:001 (orange) shows negative spikes compared to the background temperature. Channel B1:012 (blue) suggests a symmetric modulation of the electron temperature without temperature spikes as in channel B1:011 or B2:001. This observation was quantified in PDFs. Figure 2 shows the result of EMD. The red signal is the superposition of the first five IMFs, where the black line shows the detrended raw data (green, Fig.2) minus the red signals. Although ECE data sets are pretty noisy, signs of temperatures spikes can be observed in the red signal. It is assumed that these spikes represent the temperature inside the filament. The black line is interpreted as distortion of the local equilibrium due to the current perturbation of the PTM and is in phase with the positive temperature spikes. With the Hilbert transform it is possible to compute Hilbert Amplitude Spectra (HAS) from the IMFs [7]. This is demonstrated in Fig.3. EMD was applied on magnetics data when a PTM was present (coil T002, JET Pulse No: 52011). The obtained spectrum shows only one dominant mode and some signs of intra-wave modulations. A similar behaviour is observed in HAS of ECE data although the noisy data sets aggravate the interpretation. Usually, in Fourier spectra of PTMs up to seven harmonics are observed. These are also visible in wavelet analysis if the popular Morlet wavelet is used since it is Fourier based. A drawback of HAS is that the frequency of the PTM cannot be as easily determined as in FFTs or wavelet scalograms. It is therefore useful to combine several methods to make use of all advantages.

CONCLUSIONS

EMD and HAS from single signals thereby consistently supports the assumption that the PTM is indeed a current filament. The magnitude of the current-induced temperature fluctuations can also be deduced and allows an estimate of the influence on the equilibrium.

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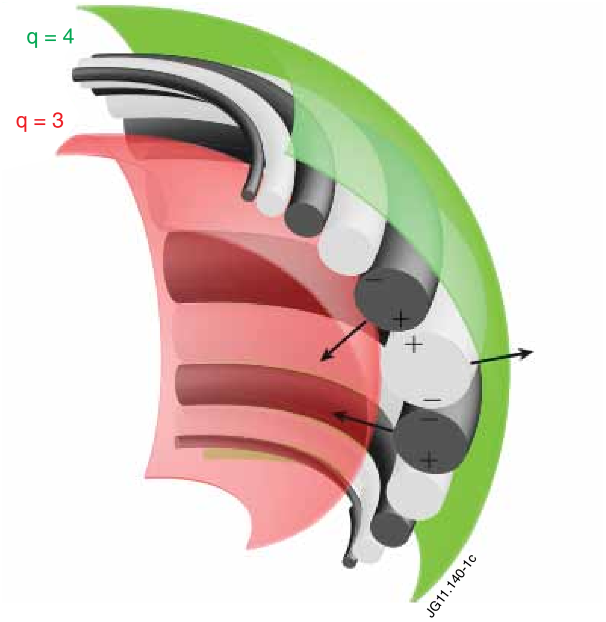

Figure 1: Illustration of an ELM in the exhaust phase at the outboard side of a tokamak. If an ELM filament (white) is stopped and captured at e.g. the q=4 surface (green) an outer mode or current ribbon could be created [1]. Similarly a hole (black) captured on a q = 3 surface could create a palm tree mode (PTM) [2]. PTMs and outer modes are possibly "twins".

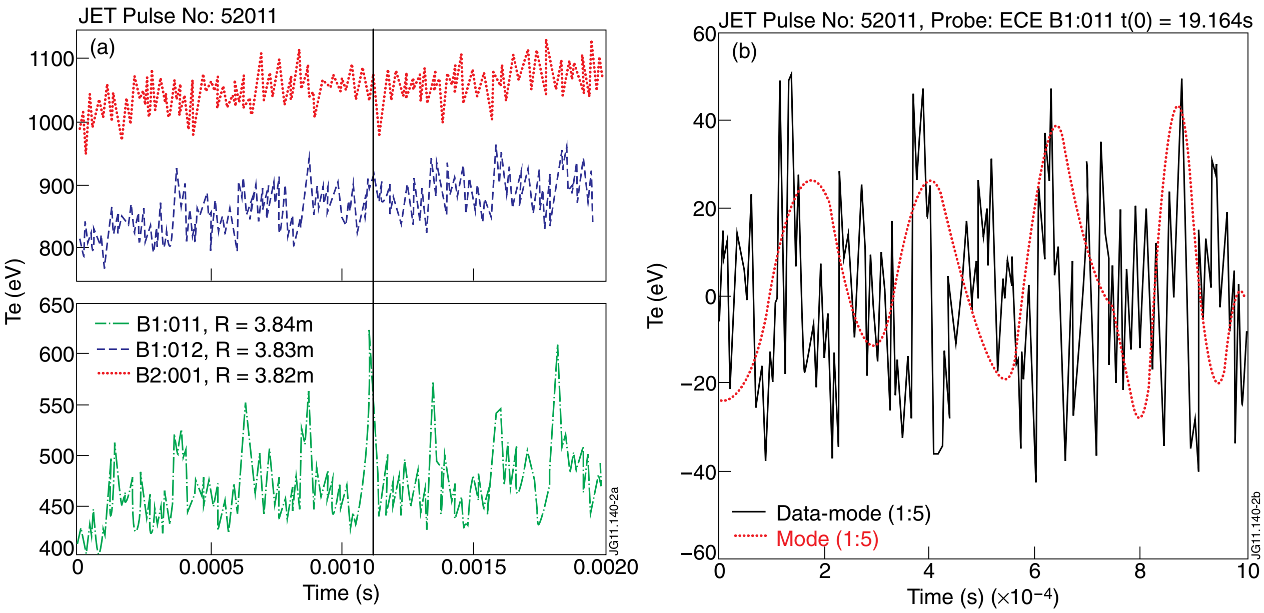

Figure 2: (a) Electron temperature fluctuations measured by three different ECE channels at different radial positions. The vertical red line gives insight into the phase relation between the three signals (JET Pulse No: 52011, t = 19:164-19:166 s). (b) EMD of channel B1:011. The red signal shows spiky temperature peaks slightly advanced with respect to the maxima of the black line. The black line gives the fraction of the current perturbation. A superposition of these two signals yields the detrended green measured signal from a).

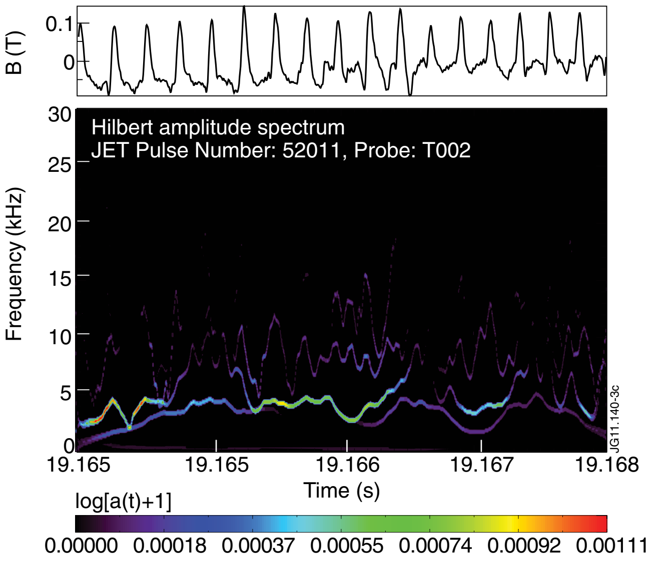

Figure 3: Hilbert amplitude spectrum of coil T002 (JET Pulse No: 52011)