EFDA–JET–CP(10)03/14

F.P. Orsitto, A. Boboc, P. Gaudio, M. Gelfusa, E. Giovannozzi, C. Mazzotta,
A Murari and JET EFDA contributors

Mutual Interaction of Faraday Rotation and Cotton Mouton Phase Shift in JET Polarimetric Measurements

Mutual Interaction of Faraday Rotation and Cotton Mouton Phase Shift in JET Polarimetric Measurements

F.P. Orsitto[1], A. Boboc[2], P. Gaudio[3], M. Gelfusa[3], E. Giovannozzi[1], C. Mazzotta[1],
A Murari[4] and JET EFDA contributors*

JET-EFDA, Culham Science Centre, OX14 3DB, Abingdon, UK

[1]Associazione EURATOM-ENEA C R Frascati 00044 Frascati (Italy)
[2]EURATOM-CCFE Fusion Association, Culham Science Centre, OX14 3DB, Abingdon, OXON, UK
[3]Associazione EURATOM-ENEA Universita' Roma II Tor Vergata, Italy
[4]Associazione EURATOM-ENEA sulla Fusione, Consorzio RFX Padova, Italy
** See annex of F. Romanelli et al, "Overview of JET Results",*
(Proc. 22 nd IAEA Fusion Energy Conference, Geneva, Switzerland (2008)).

Preprint of Paper to be submitted for publication in Proceedings of the
18th High Temperature Plasma Diagnostics, Wildwood, New Jersey, USA.
(16th May 2010 - 20th May 2010)

ABSTRACT.

The paper presents a study of Faraday Rotation angle (FR), and Cotton-Mouton phase shift (CM) measurements to determine their mutual interaction and the validity of the linear models presently used in equilibrium codes. Comparison between time traces of measurements and model calculations leads to the result that only exact numerical solution of Stokes equations reproduces all the experimental data. As consequence, approximated linear models can be applied only in a limited range of plasma parameters. In general the non-linear coupling between FR and CM is important for the evaluation of polarimetry parameters.

1. INTRODUCTION

The mutual interaction of the Faraday rotation and Cotton-Mouton were documented and preliminary analyzed in the papers (1,2). As demonstrated in (1), the structure of the Stokes equations gives a reasonable framework to describe the coupling to the interaction between FR and CM. The present paper deals with a more detailed analysis of the interaction of the FR and CM in the following respects: i) determining the precise range of plasma parameters where the interaction is important; ii) documenting the presence of the interaction in the JET database; iii) discussing briefly the problem of models of FR and CM more suitable to be used as constraints in equilibrium codes in particular for plasma parameters where the interaction of FR and CM is important. The present paper is organized as follows: in sec.2 a brief summary of the concepts basis for the analysis of polarimetry is given and a short theoretical discussion is presented about the range of plasma parameters where the interaction of FR and CM is important;. in sec.3 we present the evidence of the mutual interaction of FR and CM in a limited dataset representative of JET plasmas and the implications of the present analysis on the mathematical formulation of the polarimetric constraints into equilibrium codes are outlined; in sec.4 the conclusions and lines for the future work are given.

2. BASIC CONCEPTS OF POLARIMETRY: STOKES EQUATIONS AND THEIR APPROXIMATE SOLUTIONS.

The considered geometry includes the propagation of a laser beam along a vertical chord (taken as z-axis) in a poloidal plane of a Tokamak. The toroidal magnetic field ($\vec{B}_t$) is perpendicular to this plane and the angle of the electric field vector of the input wave with $\vec{B}_t$ is 45^o. The polarisation of a beam can be described using the Stokes vector $\vec{s}$, whose components areexpressed in terms of the ellipticity angle (χ) and Faraday angle (ψ), or in terms of the ratio of the components of the laser beam electric field (α) and their phase shift angle (φ).

The equations defining the Stokes vector $\vec{s}$ are:

$$\begin{cases} s_1 = \cos(2\chi)\cos(2\psi) = \cos(2\alpha) \\ s_2 = \cos(2\chi)\sin(2\psi) = \sin(2\alpha)\cos(\phi) \\ s_3 = \sin(2\chi) = \sin(2\alpha)\sin(\phi) \end{cases} \tag{1}$$

$$s_1^2 + s_2^2 + s_3^2 = 1$$

The JET polarimeter system measures primarily the two components of the electric field (Ex and Ey, in a plane orthogonal to the propagation direction) of the laser beam emerging from the plasma and the phase shift (φ) between these components. This feature gives the possibility of determining the values of the components of the Stokes vector, using the measurements and the definitions [2.1].

The spatial evolution along the z-axis of the polarization of a beam is given by the Stokes equation:

$$\frac{d\vec{s}}{dZ} = \vec{\Omega} \times \vec{s} \tag{2}$$

where

$$\begin{aligned} &\vec{\Omega} = ka(\Omega_1,\Omega_2,\Omega_3)\,,\Omega_2 = 2C_1\; nB_x B_t \text{ and} \\ &\Omega_1 = C_1 n(B_t^2 - B_x^2);\; \Omega_3 = C_3 nB_z \end{aligned} \tag{3}$$

Here B_t (T) is the toroidal magnetic field, B_Z the component of the poloidal magnetic field along the propagation axis, B_X the component of poloidal magnetic field orthogonal to the propagation axis $n(m^{-3})$ is the electron density, $C_1 = 1.8\times10^{-22}$ and $C_3 = 2\times10^{-20}$ constants calculated for the laser wavelength of 195μm, and Z = *z*/*ka* is the normalized coordinate along a vertical chord, where k is the elongation and a the minor radius. The relations between the Faraday rotation y and the Cotton-Mouton phase shift   angles and the corresponding components of Stokes vector follow from [2.1]:

$$\frac{s_2}{s_1} = \tan 2\psi \quad ; \qquad \frac{s_3}{s_2} = \tan\varphi \tag{4}$$

Equation [2] is solved with the initial condition $s_0 = (0,1,0)$ corresponding to 45^o angle between the electric field vector of the input wave and $\vec{B}_t$. In the present work data related to the channel #3 (corresponding to the vertical line with coordinate R = 3.04m, r/a~0.04) are presented. The values of the vector $\vec{\Omega}$ are obtained1 using the values of $\vec{B}$ as calculated by the EFIT equilibrium reconstruction and the LIDAR Thomson Scattering measurements of plasma density n projected along the line of sight of the vertical channels on the basis of the reconstructed equilibrium.

The type I solution1, to the Stokes equations [2.2], is found if the quantities $W_i = \int\Omega_i\, dz$ satisfy to the conditions $W_i^2 \ll 1$. In this approximation :

$$s_1 = -W_3 = C_3\int n_e B_z dz = 1/\tan 2\psi \tag{5}$$

$$s_2 = 1 - (W_1^2 + W_3^2)/2 \approx 1 \tag{6}$$

$$s_3 = W_1 = C_1\int B_t^2 n_e dz = \tan\varphi \tag{7}$$

Relations [2.5-7] are the key equations used for evaluating the polarimetric measurements linking the Faraday rotation to the component of the poloidal magnetic field along the direction of propagation of the laser beam (and then to the plasma current profile), and the Cotton-Mouton phase shift angle to the line–integral of the electron density. **The expressions in [2.5-7] are valid only for W_i^2<<1**. For large Faraday and Cotton-Mouton angles other methods must be used to find solutions to the Stokes equations. The term of 'linear approximation' will be used in this paper with reference to formulas [2.5-7]. The term 'numerical solution' is referred to the numerical solution of Stokes eq.[2.2].

The fig.1 shows a comparison between measurements(blue continuous line, error bars are given for few points), the numerical solution of Stokes equations for Faraday rotation (crosses) and the linear approximation W3(circles) in the high current Pulse No: 79697. The comparison is done by a rigid shift of the magnetic surfaces by DR=0.02m (see ref. 2). While the numerical solution of Stokes equations is consistent with measurements, the linear approximation W_3 underestimates the Faraday rotation by ≈40%. More general approximate solutions [1,2] to the equations [2.2] can be found, observing that the following inequalities between the components of the vector $\vec{\Omega}$ hold for Tokamak plasmas:

$$|\Omega_3| \geq \Omega_1 > |\Omega_2| \tag{8}$$

As a consequence of the condition [2.8], the Stokes equations can be integrated analytically and the Type II solutions for the Faraday angle and Cotton-Mouton phase shift can be obtained [1]:

$$\begin{aligned} \frac{s_2}{s_1}(z) &= \tan 2\psi = -\frac{1}{\tan(W_3)} \\ \frac{s_3}{s_2}(z) &= \tan\varphi = \frac{\int_{-z}^{+z} dy\,\Omega_1(y)\cos(W_3(y))}{\cos(W_3(z))} \end{aligned} \tag{9}$$

From the [2.9] it can be easily noticed that the Cotton-Mouton phase shift increases with W_3. In practice, for Faraday rotation angles higher than 12^o (corresponding to cos(W_3) <1) the Cotton-Mouton increases due to the enhancement linked with Faraday rot (W_3>>π/15=0.2). Recalling the formulas given in ref. 3 we give an expression for Ω_3 (for a circular tokamak) :

$$\Omega_3 = PX\frac{n(r)}{n0}\frac{qa}{q(r)}; \quad where \quad P = 0.395 * ne20 * IMA$$

where *qa* is the edge safety factor, *q(r)* is the safety factor spatial profile, *n(r)* is the density profile and n_0 the centre value of density, *Rch* is the coordinate of the considered polarimetry channel along the major radius, R0 is the major radius of the torus, a is the minor radius, *X=(Rch-R0)/a, ne20* is the electron density in units of $10^{20}\,\mathrm{m}^{-3}$, IMA is the plasma current in units of MA. For the channel #3 the value of X≈0.04.

The maximum value of Ω_3 is given by :

$$\Omega_3 \max = 0.4 * 0.04 * ne20 * IMA * (qa / q0) \tag{10}$$

and an approximate value of W_{3Max} can be deducted:

$$W_{3_{Max}} = \int \Omega_3 \, dZ \approx 0.016 \; ne20 \; IMA \; (qa / q0) * 2ka \tag{11}$$

where k is the elongation .The condition W3≥0.4 (see ref 2), where the effect of interaction between Faraday and Cotton-Mouton becomes sensible, implies the following condition:

$$ne20 * IMA(qa / q0) * ka \geq 12.5 \tag{12}$$

For example corresponding to (qa/q0)=3 and an elongation k=1.75 the condition for W3 is given by the following expression:

$$ne20 * IMA \geq 2.38 \tag{13}$$

In the Table 1 two reference shots are given,as examples for the calculations of the interaction of Faraday and Cotton-Mouton .

3. THEORETICAL ANALYSIS OF THE COUPLING BETWEEN FARADAY AND COTTON-MOUTON AND EXAMPLES

In the previous section a simple rule has been derived which helps in deriving the range of plasma parameters where the interaction between Faraday and Cotton-Mouton could be relevant. Moving to a more general analysis , we start from the Stokes equations [II.2] to derive equations where the coupling terms between Faraday and Cotton-Mouton can be clearly identified. Defining:

$$F = \frac{s_1}{s_2} = \frac{1}{\tan 2\psi} \quad and \quad C = \frac{s_3}{s_2} = \tan \varphi$$

from the Stokes equations [2.2] the following system can be derived :

$$\frac{dF}{dZ} = -\Omega_3 - \Omega_3 F^2 + \Omega_1 \, F \, C + \Omega_2 \, C \qquad [3.1]$$

$$\frac{dC}{dZ} = \Omega_1 + \Omega_1 C^2 - \Omega_3 \, F \, C - \Omega_2 \; F \qquad [3.2]$$

The magnitude of the terms at the right hand side of [3.1-2] can be estimated solving directly the Stokes equations. The fig.2 show how the non-linear terms play in the determination of Cotton-Mouton effect: the values of dC/dZ and Ω_1 are plotted together versus the normalized coordinate along the beam path for the channel #3 at the time t = 13s, for the high current Pulse No:79697: it appears that the non-linear terms become important for Z>0. The fig.3 shows the calculation of dF/dZ: non-linear terms become important for Faraday rotation at high current.

It has been verified that the most important non-linear term for Faraday is the term $\mathbf{\Omega_3 F_3}$, while for Cotton-Mouton is the term Ω_3 FC, in eq.[3.1-2]. A Similar plot is given for Pulse No:70004 (fig.4 for Cotton-Mouton), the non-linear term are less relevant as it was predicted by the condition (2.13). In fig.5 the calculated phase shift (Cotton-Mouton) is plotted for Pulse No: 79687, together linear expression W_1: the mutual interaction of Faraday and Cotton-Mouton is clearly shown.

The examples reported in this paper show that the condition [2.13] could be taken as a rough guideline for determining the plasma parameters where non-linear effects are sensible and (in particular) the linear approximation used in the equilibrium code EFIT for the determination of polarimetry constraints is not valid.

CONCLUSIONS

The paper is dedicated to the analysis of non-linear effects related with the interaction between Faraday rotation and Cotton-Mouton. The Stokes model is adequate to describe this interaction. An approximate rule can be derived analytically to determine the plasma parameters where the non-linear effects are sensible. An important point of the analysis is that the mathematical formulation of the polarimetry constraints used in the EFIT equilibrium code which uses the linear model must be reconsidered testing more adequate models.

ACKNOWLEDGEMENTS

the authors are grateful to S Segre , L Appel, M Brix, V Drozdov and E Solano for interesting observations. This work, supported by the Euratom and carried out within the framework of the European Fusion Development Agreement. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

REFERENCES

[1]. F.P. Orsitto *et al.,* Plasma Physics and Controlled Fusion **35** (2008)1261

[2]. F.P. Orsitto *et al.,* EFDA-JET-PR(09)50

[3]. S.E. Segre *et al.,* Plasma Physics and Controlled Fusion **35** (1993)1261

Pulse No:	nmax/1.10^{20} m^{-3}	IP(MA)	BT	n_e20*IMA
79697	1.1	4	3.6	4.4
70004	0.62	2.52	2.55	1.5

Table I - reference shots

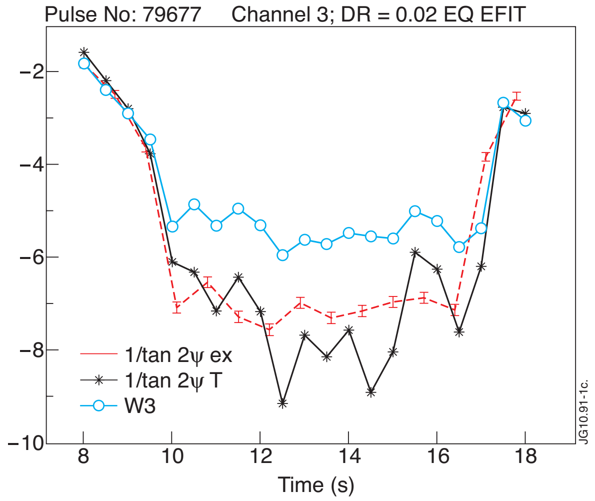

Figure 1: Faraday rotation measurement (blue line) is compared with numerical solution of Stokes equation (crosses) and linear Type I approximation W3.

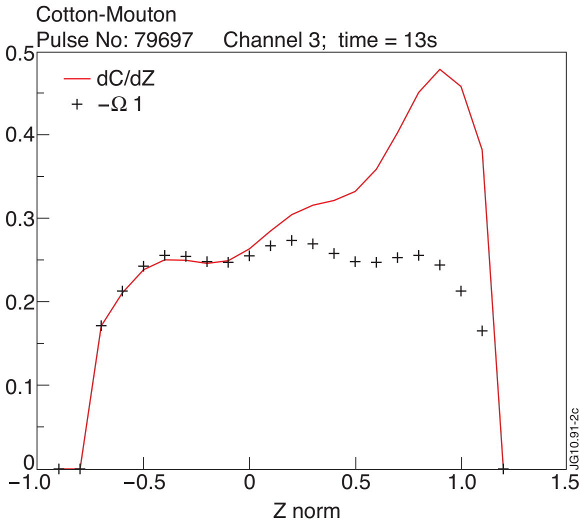

Figure 2: dC/dZ versus Z is plotted for Pulse No: 79697 together with Ω_1.

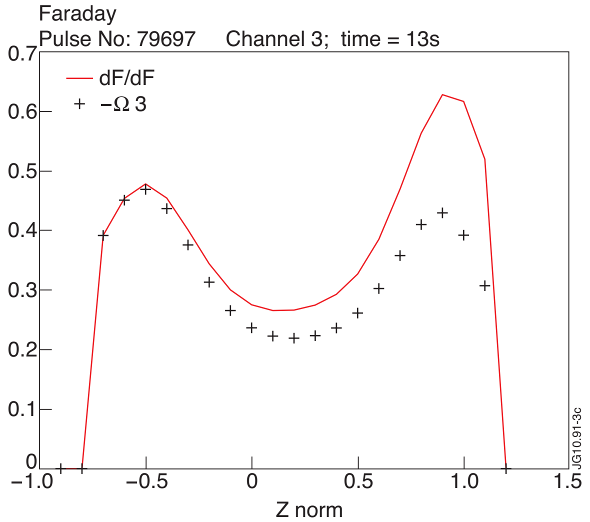

Figure 3: dF/dZ is plotted versus Z for Pulse No: 79697 together Ω_3.

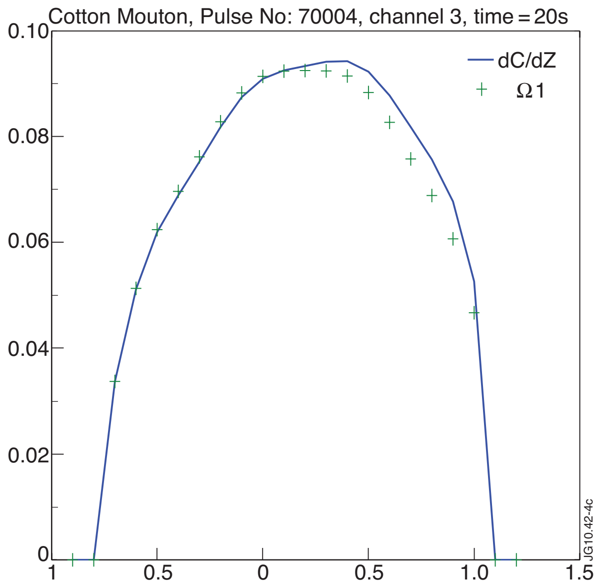

Figure 4: dC/dZ versus Z is plotted for Pulse No: 70004 together with Ω_1.

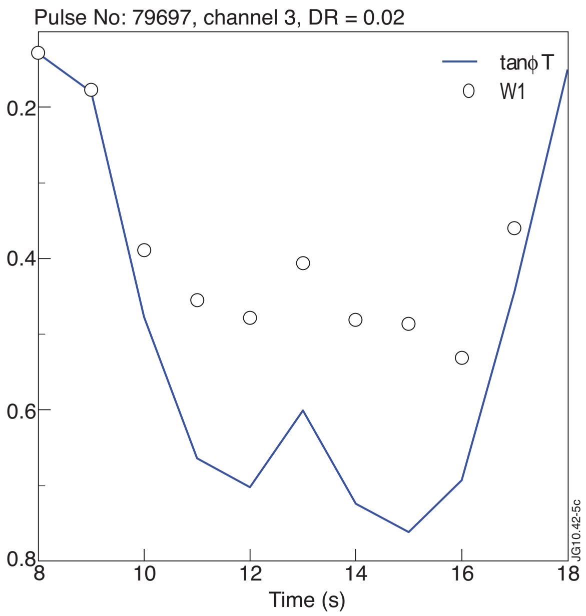

Figure 5: The Cotton-Mouton phase shift versus time(s) calculated numerically(line) is shown together the linear expression (circles) W_1 for Pulse No: 79697.