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ABSTRACT
Recently a complete model of JET polarimeter calibration has been developed. In this paper, this model has been checked using residual analysis. The differences between the estimates of the calibration code and the experimental measurements, for both manual and automatic calibrations, have been calculated and tested statistically. The correlation analysis of the residuals, performed with methods valid also for nonlinear systems such as JET polarimeter, confirms the quality of the model developed to interpret the calibration of the diagnostic.

1. INTRODUCTION
In Magnetic Confinement Nuclear Fusion (MCNF) the topology of the confining magnetic fields is essential. A careful determination of the magnetic configuration is crucial to the control of the plasma, the achievement of the required performance and the understanding of the physics. One of the only two measuring techniques which can provide information on the internal topology of the magnetic field in a high temperature plasma is polarimetry. A laser beam, linearly polarized, traversing a plasma experience a rotation of its plane of polarization according the Faraday effect, which can be written as:

$$\Psi \propto \lambda^2 \int n_e \cdot B_p \cdot dz \tag{1}$$

In (1) the various symbols indicate: λ the laser wavelength, $\boldsymbol{n_e}$ the plasma electron density and $\boldsymbol{B_p}$ the parallel component of the magnetic field.

Since the plasma, in addition to being optically active, is also birifringent, it affects the two components of the linear polarization in different ways. In particular it introduces a phase shift between the two which can be represented by the relation:

$$\Phi \propto \lambda^3 \int n_e \cdot B_t^2 \cdot dz \tag{2}$$

where λ is the laser wavelength, $\boldsymbol{n_e}$ the plasma electron density and $\boldsymbol{B_t}$ the perpendicular component of the magnetic field.

Relation (1) and (2) show clearly how the Faraday rotation and the phase shift are linked to the magnetic fields along the optical path of the laser beam inside the plasma. On the other hand, these measurements are line integrals and therefore do not provide local measurements. Their more typical use therefore consists of providing them to general inversion codes as global constraints. Once they are provided as input to the magnetic reconstruction codes, they influence their final output in a very involved way which makes it practically impossible to disentangle their effect from the one of the other measurements. Therefore it is extremely important to make sure that the measurements are correct and do not present hidden systematic errors.

In this paper a systematic process of model validation, based on correlation analysis, is applied for the first time to the calibration procedure of the diagnostic. The main idea behind the approach is the fact that, in the case of a perfect model, the residuals, i.e. the differences between the experimental measurements and the estimates of the model, should be completely random sequences. This condition is relatively easy to assess in the case of linear systems. Unfortunately, the calibration procedure is strongly nonlinear and therefore more sophisticated criteria have to been applied. The multi-input, multi-output techniques adopted in this paper are fully general and valid also for nonlinear systems. Their more general applicability derives by the use of the information in the outputs of the models.

2. THE MODEL OF THE CALIBRATION OF JET POLARIMETER

At JET an on-line calibration, to link the output voltage of the detectors to the effective Faraday rotation angle, is carried out, for each chord, before each shot [1].

Currently, the procedure of calibration is the following: the half-wave plate, located at the entrance of the vacuum vessel (see figure 1), is rotated (via a step-motor) of a well-known angle (α) and the phase shift (φ) is recorded at each angle, while the Faraday rotation is equal to twice the half-wave plate angle.

A new calibration code [2] has been developed to solve the problems of the polarimetry measurements mainly for the shots acquired during the campaigns 2008-2009, when the diagnostic has been set for covering all JET plasma regimes, particularly the high current ones. In the new code all optical components, crossed by the laser beam for the vertical chords, have been modeled using the Mueller Matrix formalism.

As it is well known, any fully polarized, partially polarized, or unpolarized state of light can be represented by a Stokes vector ($\vec{S}$). Any optical element can be represented by a Mueller matrix ($\boldsymbol{M}$) [3].

Referring to the polarimeter scheme reported in figure 1, the optical components which have to be included in the model to obtain a good fit of the results are: the half-wave plate and the wire grid located in front of the detectors. In additional to reproduce the "instrumental ellipticity" which afflicts the polarimetry measurements, a cascade of two retarders is used, because this polarizing element changes the phase of the optical beam. In the following figure 2, the results obtained with the new calibration code in comparison with the current calibration are reported. To validate the new calibration code the autocorrelation test has been carried out. More details and the results are shown in the next section.

3. THE MODEL VALIDATION APPROACH BASED ON RESIDUAL CORRELATIONS

The approach to model validation adopted in this paper can be formulated as a statistical hypothesis testing problem. Its structure can be typically reduced to two main steps. The first one consists of formulating a parameter free statistics in such a way that the statistical distribution of the statistic variable is known if the hypothesis to be tested is valid. The second step defines the domain, in which the variable has to fall in order for the null hypothesis to be rejected. In our case, the null hypothesis is to test is that the model is not adequate and therefore that the residuals are not random. It is therefore reasonable to assume the typical 95% confidence interval for normally distributed variables as the condition to reject the null hypothesis. The delicate problem is of course the definition of the parameter free statistics. In case of linear, SISO systems, it can be easily demonstrated that the autocorrelation function of the residuals and the cross correlations between the inputs and the residuals provide valid tests. Unfortunately, these tests are not adequate for nonlinear systems. The calibration curves of JET polarimetry, particularly the phase shift, clearly show that the system in strongly nonlinear. Also the measurements of the Faraday rotation and the cotton-Mouton are nonlinear as can be seen from relations (1) and (2). Moreover the diagnostic is a MIMO system with two outputs and several inputs.

A complete and adequate set of tests for a nonlinear, MIMO system is provided by the correlation between the residual and input vectors given by the following two relations:

These tests are based on higher order correlation functions, can be defined as [4, 5]:

$$\phi_{\xi\eta}(\tau) = E\left[\xi(\tau)\,\eta(t+\tau)\right] \tag{3}$$

$$\phi_{\xi\eta}(\tau) = E\left[\vartheta(\tau)\,\eta(t+\tau)\right] \tag{4}$$

where $\xi(\tau)$ is the residuals vector, $\eta(\tau)$ the vector of the outputs time their residuals and $\vartheta(\tau)$ the inputs vector. So these vectors can be calculated as:

$$\xi(t) = \varepsilon_1^2(t) + \ldots + \varepsilon_q^2(t) \tag{5}$$

$$\eta(t) = Y_1(t)\,\varepsilon_1(t) + \ldots + Y_q(t)\,\varepsilon_q(t) \tag{6}$$

$$\vartheta(t) = u_1^2(t) + \ldots + u_r^2(t) \tag{7}$$

If the cross correlations in (3) and (4) remain in the 95% confidence interval, then it is reasonable to consider the model adequate, at least for the level of noise of the measurements. On the other hand, if the cross-correlations exceed this value, the model is to be considered somehow defective and a series of tests can be performed to identify the main reason for the weakness.

4. THE VALIDATION OF THE CALIBRATION MODEL

Since the developed model for the polarimeter calibration is non linear it is necessary to perform the non linear correlation tests to check the validity of the model.

Referring to the equations 5, 6 and 7, for the calibration model, in which the input is the half-wave plate signal and the outputs are the Faraday angle and the Cotton-Mouton effect, they can be written as:

$$\xi\ (t) = \varepsilon^2{}_{FAR}\ (t) + \varepsilon^2{}_{cm}\ (t) \tag{8}$$

$$\eta\ (t) = Y_{FAR}\ (t) \cdot \varepsilon_{FAR}\ (t) + Y_{cm}\ (t) \cdot \varepsilon_{cm}\ (t) \tag{9}$$

$$\vartheta\ (t) = u^2{}_{HWP}\ (t) \tag{10}$$

In the following figures the results of the non linear global tests are shown. In particular, the figure 3 (Top) represents the correlations between the squared residuals and the residual and output product (see equations 4 and 5), while the figure 3 (Bottom) the correlation between the squared residuals and the squared input. In both case the result is completely inside the confidence limit, validating the developed calibration model.

CONCLUSIONS

The new code to simulate the calibration of JET polarimeter is based on an equivalent optical model. The quality of the results has been assessed with a correlation test valid for nonlinear systems. In the future it is planned to apply the same model validation approach also to the full propagation code already developed to simulate the measurements of the Faraday rotation and Cotton-Mouton during plasma.

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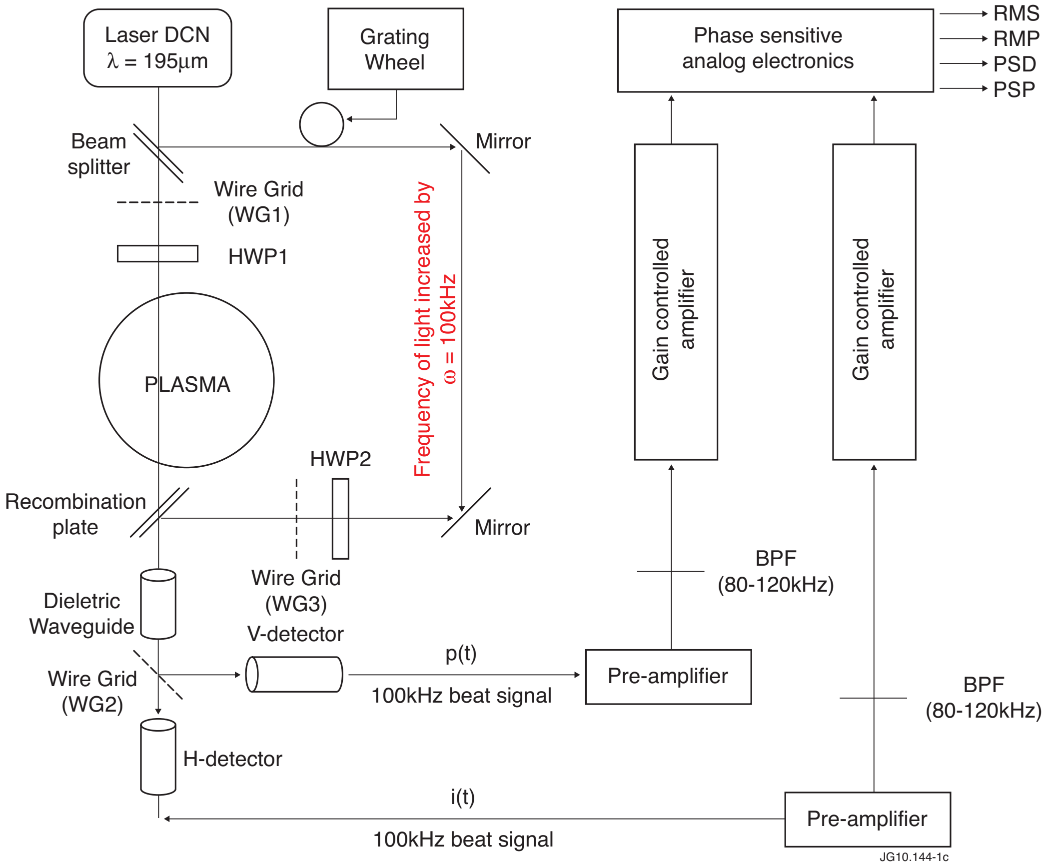

Figure 1: Schematic of the Polarimeter vertical chords

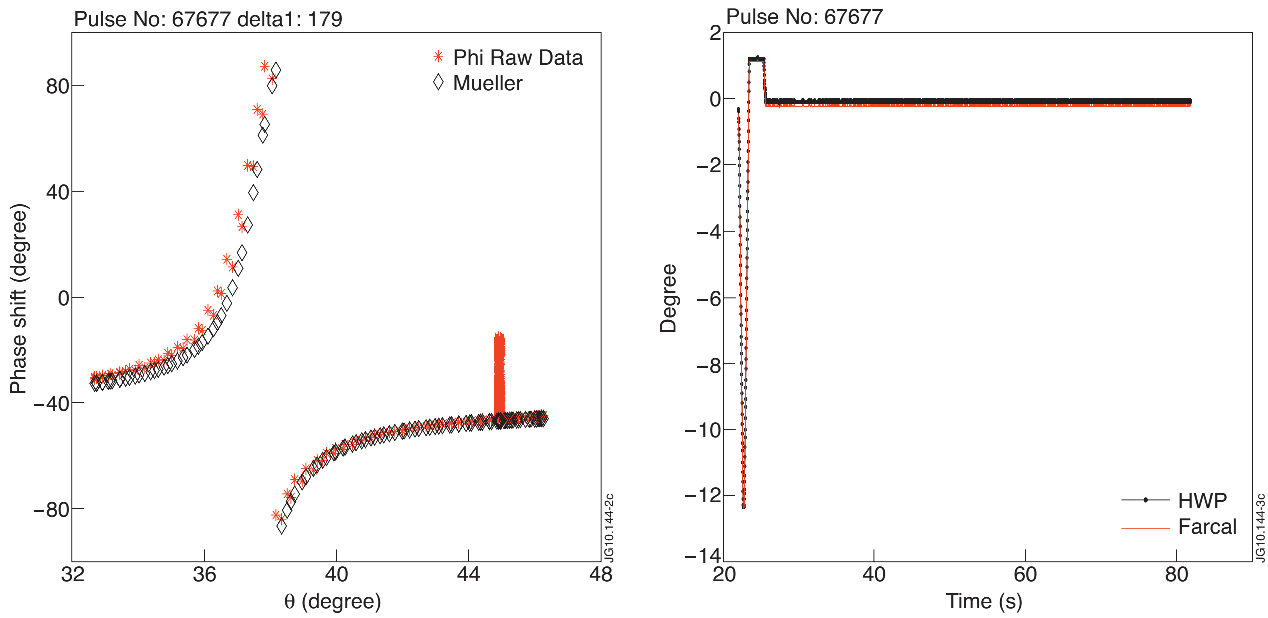

Figure 2: (Left) Comparison between the experimental calibration curve (red asterisk) and the estimate of the new calibration model (black diamond). Pulse No: 67677 chord #3 (Right): Calibration curve of Faraday angle (Pulse No: 67677 chord #3). Comparison between the experimental calibration curve (black dot-line) and the estimate of the new calibration model (red line).

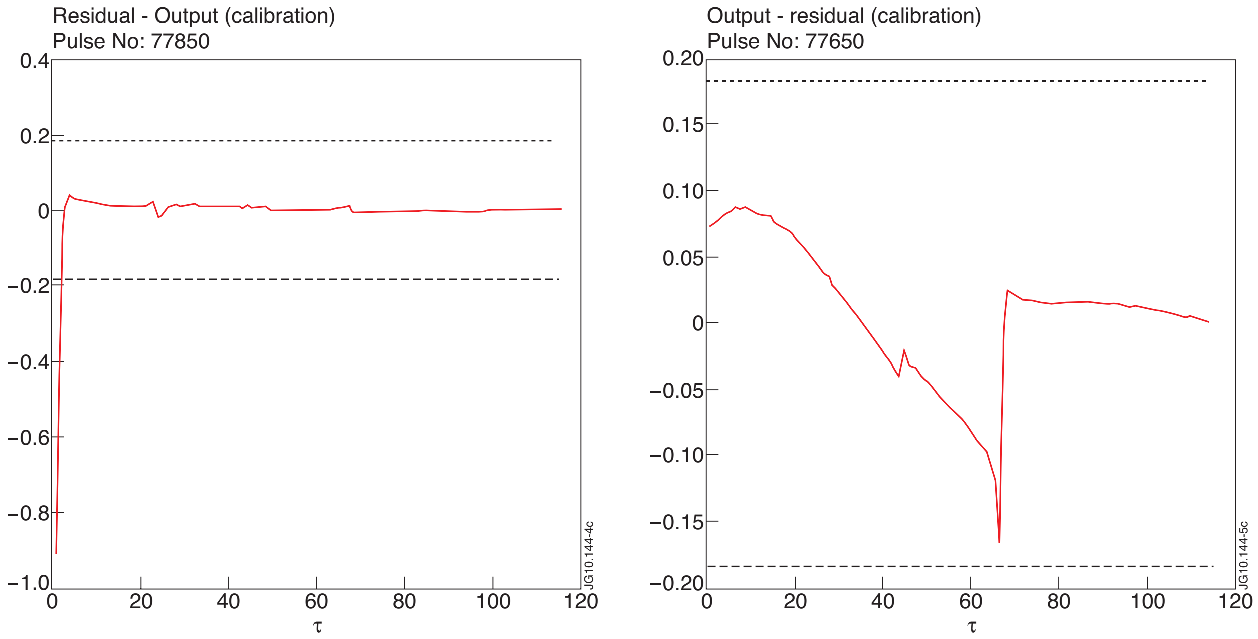

Figure 3: (Left): non linear correlation between the model outputs and the sum of the squared residuals. (Right): non linear correlation between the model outputs and the sum of the squared input. Dashed lines indicate a 95% confidence interval which correspond to 1.96/√N, where N is the number of sampled points of the trajectory.