

D. Dodt, R. Fischer, A. Korotkov, D.C. McDonald  
and JET EFDA contributors

# Electron Density Profiles from the Probabilistic Analysis of the Lithium Beam at JET

“This document is intended for publication in the open literature. It is made available on the understanding that it may not be further circulated and extracts or references may not be published prior to publication of the original when applicable, or without the consent of the Publications Officer, EFDA, Culham Science Centre, Abingdon, Oxon, OX14 3DB, UK.”

“Enquiries about Copyright and reproduction should be addressed to the Publications Officer, EFDA, Culham Science Centre, Abingdon, Oxon, OX14 3DB, UK.”

# Electron Density Profiles from the Probabilistic Analysis of the Lithium Beam at JET

D. Dodt<sup>1</sup>, R. Fischer<sup>1</sup>, A. Korotkov<sup>2</sup>, D.C. McDonald<sup>2</sup>  
and JET EFDA contributors\*

*JET-EFDA, Culham Science Centre, OX14 3DB, Abingdon, UK*

<sup>1</sup>*Max-Planck-Institut für Plasmaphysik, EURATOM Association, D-85748 Garching, Germany*  
<sup>2</sup>*EURATOM-UKAEA Fusion Association, Culham Science Centre, OX14 3DB, Abingdon, OXON, UK*

*\* See annex of F. Romanelli et al, "Overview of JET Results",  
(Proc. 22<sup>nd</sup> IAEA Fusion Energy Conference, Geneva, Switzerland (2008)).*

Preprint of Paper to be submitted for publication in Proceedings of the  
36th EPS Conference on Plasma Physics, Sofia, Bulgaria.  
(29th June 2009 - 3rd July 2009)



## INTRODUCTION

The lithium beam diagnostic provides a powerful edge plasma diagnostic. It enables the determination of electron density profiles in the scrape-of-layer and pedestal region with high spatial and temporal resolution ( $\Delta t \approx 50\text{ms}$ ,  $\Delta x \approx 1\text{cm}$ ). A fast neutral lithium beam (beam energy at JET: 55keV) is injected into the plasma and the line radiation emitted by the collisionally excited lithium is observed at different spatial positions using a periscope optic. The deconvolution of the emission profiles is based on a collisional-radiative model [1, 2]: The interaction of the Li atoms with the plasma is described by a system of coupled linear differential equations for the population densities  $N_i$  of the excited states  $i$  as functions of the beam coordinate  $x$ :

$$\frac{dN_i(x)}{dx} = \sum_{j=1}^{n_{Li}} \{n_e(x) a_{ij} + b_{ij}\} N_j(x), \quad \begin{aligned} N_1(x=0) &= 1 - N_{2,0} \\ N_2(x=0) &= N_{2,0} \\ N_j(x=0) &= 0 \quad \forall j > 2,0 \end{aligned} \quad (1)$$

(number of considered excited states:  $n_{Li}$ , rate coefficients for electron induced processes  $a_{ij}$ , rate coefficient for ion induced processes and spontaneous emission:  $b_{ij}$ , the lithium beam enters the plasma at  $x = 0$ , the population of the  $i = 2$  state at this position is called  $N_{2,0}$ . For more details see [3])

A detailed description of the diagnostic setup at JET consisting of the beam source, the observation optic and the spectrometer, can be found in [4]. As in other experiments, the setup at JET allows for the determination of the spatial variation of the beam emission, but not the absolute value of its intensity. For vanishing initial beam excitation  $N_{2,0}$ , the initial value problem (1) for  $n_e(x)$  is invariant under a change of the absolute scale of the emission. Accordingly, the absolute calibration of the optical system is not needed to reconstruct the electron density.

Different numerical methods can be used for the reconstruction of the electron density from the measured lithium line emission. Equation (1) can be algebraically rearranged to obtain an explicit equation for  $n_e(x)$ . Reconstruction methods based on this explicit equation have been used e.g. at JET ([4]) and ASDEX Upgrade ([1]) these will be referred to as *standard* method in the following. Recently a new method for the reconstruction of electron density profiles based on a probabilistic data analysis has been implemented [3] (and references therein).

## 2. PROBABILISTIC LITHIUM BEAM DATA ANALYSIS

The new probabilistic approach [3] combines the well established atomic model for the beam composition and attenuation with a description of the experimental noise of the emission measurement (forward model). The set of electron density profiles which are compatible with a given measurement are inferred according to Bayes' theorem. In this approach, the relevant uncertainties, of which the most important are the uncertainty of the background radiation and the detection noise, are taken into account in a consistent way. The determination of 'confidence intervals' for the reconstructed density profiles is an inherent feature of the approach.

The probabilistic method is able to cope with ambiguous data (partly measured emission profiles,

missing absolute calibration), revealing the whole set of valid solutions. Furthermore, physical boundary conditions, like constraints on the monotonicity of the electron density, or density information from independent diagnostics are straight-forward to incorporate [3].

The correct quantification of the uncertainty of the obtained electron density profiles requires a comprehensive assessment of the uncertainties of the employed emission data. The spectral fitting of the detector raw data has been reimplemented with an emphasis on the correct description of this uncertainty.

### 3. UNCERTAINTY OF THE LI(2P) EMISSION MEASUREMENT

As described in [4], the light emission of each spatial channel of the observation system is measured spectrally resolved in a region from 670nm to 674nm by a CCD chip. The spectral information is used to infer the intensity of the lithium beam and background radiation, as well as the exact angle of the periscope mirror that moves during the plasma pulse due to mechanical stress. Meanwhile, the beam-source allows for a fast chopping of the beam, providing the possibility for a direct measurement of the background radiation. In Figure 1, an example for the spectral fitting procedure is shown. The uncertainty of the spectral measurement  $\sigma_I$  was determined by considering the slowly varying signal in the neutral gas calibration shots and found to be

$$\sigma_I^2 \approx 20.3\text{adu}^2 + I \cdot 0.15\text{adu} \quad (2)$$

(Intensity  $I$  of considered Pixel of the CCD measured in analogue-to-digital-units: adu). This is somewhat larger than the specification of the manufacturer of the CCD:  $\sigma_I^2 \approx 12.1\text{adu}^2 + I \cdot 0.15\text{adu}$ . In order to account for the variation of the background over time, the variance  $\sigma_{\text{bg,fluct}}$  of the fluctuation of the background signal of each pixel between adjacent background measurements is determined for a given JET pulse. The uncertainty of the background subtracted signal is obtained from the sum of the squares of both uncertainties:  $\sigma_{I,\text{bg}}^2 = \sigma_{\text{bg,fluct}}^2 + \sigma_I^2$ . The uncertainty of the background-subtracted signal (right part of Figure 1) is given by the sum of the squares of the uncertainty of the spectral measurement and the uncertainty of the background. The signal is fitted by a Gaussian function for the Lithium line. The uncertainty of the fitted parameters is obtained by the employed software package and was checked to coincide with the standard deviation of an equivalent Monte-Carlo sampling of the likelihood of the fit. The emission of each spatial channel is corrected for the sensitivity of the channel, which is determined from a beam emission measurement performed in neutral hydrogen gas. The uncertainty of the used calibration factors is estimated using the different time frames of the calibration pulse. The overall uncertainty of the emission is obtained from the sum of the squares of the calibration uncertainty and the uncertainty of the fit.

### 4. FIRST RESULTS OF THE PROBABILISTIC RECONSTRUCTION

The profiles reconstructed with the probabilistic method are shown in comparison to the results of

the standard analysis procedure. The plasma scenario chosen for the comparison is the ohmic phase of JET Pulse No: 74930 ( $I_p = 1.7\text{MA}$ ,  $B_t = 2.9\text{T}$ ).

## 5. OBTAINED EMISSION PROFILE

In Figure 2, the emission profile, which is the result of the spectral fitting is shown together with the emission profile of the standard lithium beam analysis and the result of the forward calculation.

The sum of the squared residuals  $\chi^2 = \sum_{i=1}^N \frac{(D_i - D_{sim,i})^2}{\sigma_i^2}$  between model ( $D_{sim,i}$ ) and measurement ( $D_i$ ) is  $\chi^2 \approx 18.7$ , which corresponds roughly to the number of data points (24) and therefore indicates consistency of the statistical data model. The emission profile used in the standard analysis is in good agreement with the new fitting procedure, only the data points at high  $Z$  values show a slightly larger emission intensity.

## 6. RECONSTRUCTED ELECTRON DENSITY PROFILES

In Figure 3, the reconstructed electron density is shown and compared to the result of the standard analysis procedure and the density profile obtained by the core LIDAR system. In the region of weak beam attenuation, both reconstruction methods show a very similar result. For the observation channels at large  $Z$ , the standard analysis obtains a slightly higher density, which is consistent with the larger beam emission found by the procedure. In the region of high beam attenuation, a strong increase of the uncertainty of the probabilistic reconstruction can be observed. This is consistent with the findings at ASDEX ([3]). The probabilistic reconstruction recognizes the diminishing information content of the data concerning this part of the profile. It should be noted, that the standard lithium beam analysis uses a density constraint at the inward side of the density profile, which was not included in the probabilistic reconstruction. The necessity of additional density constraints, especially for cases of non-vanishing initial beam excitation are subject to ongoing studies.

## SUMMARY AND OUTLOOK

A probabilistic analysis method for the lithium beam diagnostic, was adapted to the requirements of the diagnostic at JET. The detailed study of the physics model of the diagnostic, identifying and quantifying all important uncertainties, was performed. The electron density profiles obtained with the new procedure are in agreement with the existing lithium beam analysis. In addition, the uncertainty of the reconstruction is obtained. The probabilistic nature of the analysis allows to encode the information which can be inferred from the measured emission data in a formalized way: the full joint distribution of the parameters of interest accounting e.g. of correlations in the reconstructed electron density profile. This allows the integration of the obtained edge density profiles with additional information from other sources in a consistent way. As a next step, the improvement, which could be achieved at ASDEX Upgrade, for the reconstruction of profiles with

a low signal to noise ratio, and allowed an improved time resolution [3], will be assessed. The combined analysis of the lithium beam with data from the LIDAR and the interferometry diagnostics at JET is planned and will be based on their probabilistic analysis [5], and the incorporation of uncertainties introduced by the magnetic mapping [6].

## ACKNOWLEDGEMENTS

This work has been conducted under the European Fusion Development Agreement. The authors wish to thank Jakob Svensson, Mathias Brix and Andrew Meigs for helpful discussions and suggestions. This work is dedicated to our dear colleague and committed scientist Anatoli Korotkov. We are in deep sorrow, because he passed away unexpectedly during the preparations of this publication.

## REFERENCES

- [1]. J. Schweinzer et al., *Plasma Physics Controlled Fusion* 34 1173 (1992)
- [2]. R. Brandenburg, et al., *Plasma Physics Controlled Fusion* **41** 471 (1999)
- [3]. R. Fischer et al., *Plasma Physics Controlled Fusion*, **50** 1-26 (2008)
- [4]. M. Brix, A. Korotkov, M. Lehnen, et al. Proc. 28th Conf. on Controlled Fusion and Plasma Phys. (Madeira) *Europhys. Conf. Abstracts* 25A, 389, (2001)
- [5]. O.P. Ford et al., *Bayesian Combined Analysis of JET LIDAR, Edge LIDAR and Interferometry Diagnostics*, this volume, (2009)
- [6]. J. Svensson, A. Werner et al., *Plasma Physics Controlled Fusion* **50** 085002 (2008)

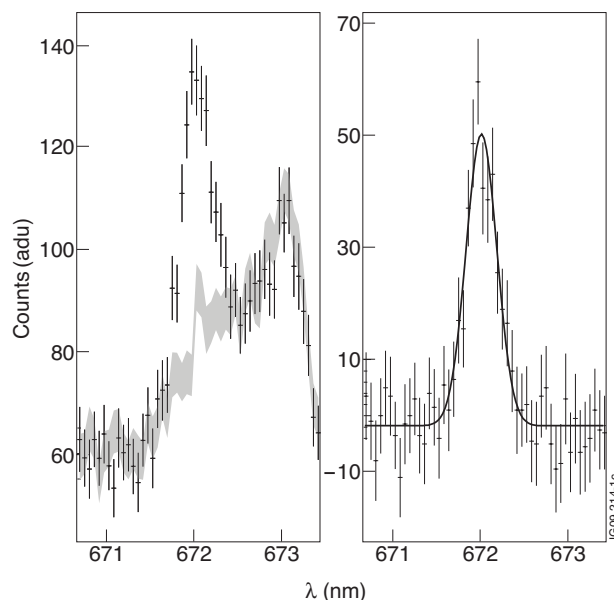


Figure 1: Measured beam emission spectrum of the 5th spatial channel of the observation system. The measured spectrum (left, black points with error bars) is shown together with the estimate for the background radiation obtained from the adjacent beam-off time frames (gray, shaded area shows uncertainty.). In the right part of the figure, the signal is shown after subtracting the background. The black line depicts a Gaussian fit to the signal.



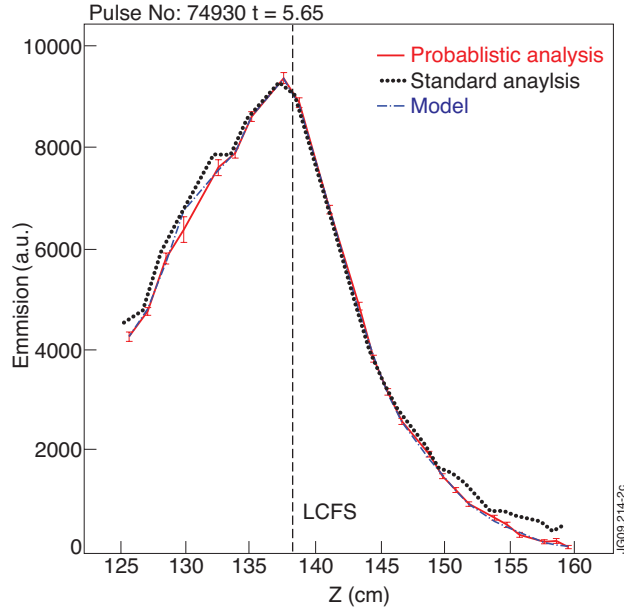


Figure 2: Emission profile as obtained by the fitting procedure. The red curve shows the measured emission with its uncertainty. The blue curve shows a simulated emission profile, which was obtained from the forward calculation, using parameters fitted to the shown data. The black curve shows the emission profile obtained from the standard lithium beam analysis. The dashed line shows the position of the Last Closed Flux Surface (LCFS), as obtained by the plasma equilibrium reconstruction EFIT.

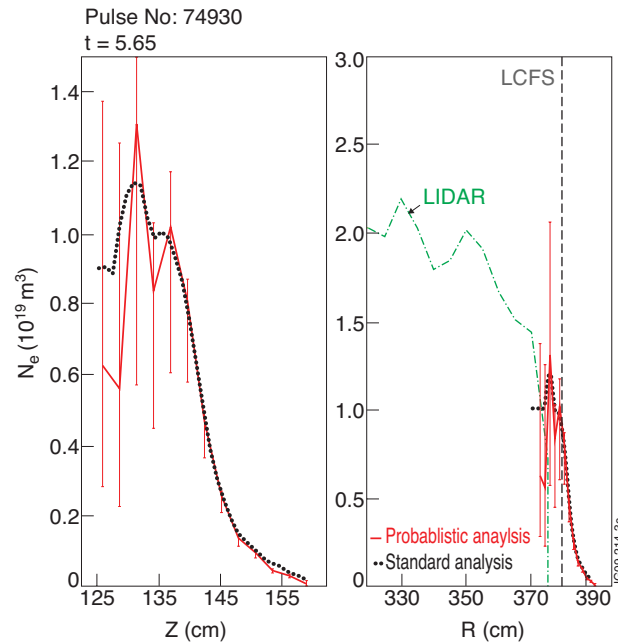


Figure 3: Reconstructed electron density profiles in the early phase of Pulse No: 74930 ( $t=5.65$ ). The result of the standard lithium beam analysis is shown in black, while the red curve with error bars shows the result of the probabilistic analysis. The right part of the plot shows the profiles mapped onto the midplane together with the result of the LIDAR measurement (green curve).