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ABSTRACT

In toroidal plasmas, multiple Alfvén Eigenmodes (AE’s) with several toroidal mode numbers can be excited by fast ion populations, and in some circumstances may lead’to rapid loss of fast particle confinement. Although independent of each other, multiple AE’s often oscillate with the same or nearly the same frequency. For example, for the case of externally excited AE’s, a broad spectrum of modes necessarily is driven at one single frequency. Mode number identification must be accomplished by interpreting signals from a finite number of external Mirnov coils, which typically are unevenly spaced in the toroidal coordinate. However, sensor signals are then the result of a superposition of multiple modes, so the task of spatial mode decomposition becomes a difficult problem. Previous efforts to resolve toroidal mode numbers involve Singular Value Decomposition (SVD) and/or Lomb periodogram techniques. We describe a new approach, based on the SparSpec method, and show that it may be superior in several respects.

1. BACKGROUND

Alfvén Eigenmodes in tokamak fusion reactors could lead to rapid loss of fusion products and pose problems for burning plasma scenarios [1, 2]. AE fast-ion drive and damping is dependent upon the toroidal mode number (n), and thus their stability is dependent on n (as well as other plasma parameters [2]). The verification of these predictions is of paramount importance and is the fundamental goal of the active MHD projects on JET [3] and Alcator C-Mod [4]. Generally, AE’s can be detected with diagnostics which observe internal plasma fluctuations (μ-wave reflectometry, ECE radiometry, laser interferometry, phase contrast imaging, …). These are essential to be able to characterize the radial mode structure and to compare the measurements with the results of numerical models such as MISHKA and NOVA-K. However, n-number determination to date is exclusively performed using the data from a finite number of magnetic pickup coils, typically unevenly spaced, which are located outside of the plasma.

In the presence of only one toroidal mode at a given frequency, n-number identification is straight forward: The data from each sensor is Fourier transformed, and the phases of the relevant frequency can be fitted to a straight line, the slope being equal to the n-number. When the fit is very bad or jumps from one n to another as a function of time, it may be an indication that there are several modes being detected simultaneously (see figure 1). This latter case can completely prevent n-number identification by traditional methods, such as straight line phase fitting. Multiple AEs with similar frequencies can occur for example when there is little or no plasma rotation (due to a lack of Doppler separation). Another way this situation can occur is when multiple stable AEs are excited by an external source at one single frequency [5, 6], as illustrated in this article.

2. THE SPARSPEC METHOD

Traditionally, the tools employed to estimate sinusoidal contributions to sets of unevenly sampled data have involved the so-called Lomb periodogram [7], and this method was recently applied to

Alfvén Eigenmode identification in the Wendelstein 7-AS stellarator [8]. This is essentially a least squares fitting to sinusoids, which works well if there is only one dominant mode, but has limitations when multiple modes are present in the data. Thankfully, the problem of finding periodic waveforms (sines and cosines) in unevenly sampled data is ubiquitous in the field of astronomy, where much work has been done. It is easily seen that temporal frequencies in astronomical data can correspond to spatial toroidal mode numbers in tokamaks, and that unevenly sampled data in time is the analog of data from unevenly distributed Mirnov sensors in the toroidal coordinate. Recently, a new method for fitting sinusoids to irregularly sampled data was proposed [9], which is implemented in the SparSpec computer code (freely available at: http://www.ast.obs-mip.fr/Softwares).

Least Square (LS) fitting of sinusoids from time series is a difficult problem because of the non-linearity of the model with respect to the frequencies parameters, so the LS criterion may have many local minima. For a single frequency, it leads to the maximization of the Lomb periodogram, but the problem is generally more difficult for several frequencies, even if the number of frequencies is known (which is rarely the case).

An alternative model is used in SparSpec that is linear in the frequency parameters: the data is modeled as a large number (possibly larger than the data size) of pure frequencies, discretized on a fixed, arbitrarily thin, grid. Among the many representations fitting the data, we seek the one with the fewest non-zero amplitude, i.e. a sparse spectrum. The solution is computed as the minimizer of a penalized LS criterion:

$$J(x) = \frac{1}{2}\left\|\mathbf{y} - W\mathbf{x}\right\|^2 + \lambda \sum_{k=-K}^{K} \left|x_k\right|$$

where $\mathbf{y} = \{y_1, y_2 \dots y_N\}^T$ is the vector of data taken at time t_n, W is a $N \times 2K+1$ matrix with elements $\boldsymbol{W}_{n,k} = \exp(i2\pi t_n f_k)$, and $\mathbf{x} = \{x_{-k}, \dots x_k\}^T$ the vector of complex amplitudes associated with frequencies f_k, k = -K…K (The term ‘frequencies’ here applies to time-sampled data, for tokamaks k is in fact the spatial n number). This criterion is convex, with no local minima, and a computationally efficient and convergent optimization strategy has been proposed in [9], based on a Block Coordinate Descent algorithm. The hyper-parameter l is unknown and must be fixed to obtain a satisfactory sparse solution: it can be interpreted as the maximum peak amplitude allowed in the periodogram of the residual. Because SparSpec discretizes the allowable frequencies, it is ideally suited for toroidal mode number analysis, because only modes with integer wave number are sought, $f_k = k$, $k = -K \dots K$.

A batch processing code was developed for tokamak data in the following ways: First, the SparSpec code was adapted to take complex data as input. Then, because of the penalization term in J(x), the minimizing amplitudes are initially underestimated: a LS fitting is then performed to re-estimate amplitudes of the detected frequencies. The code was benchmarked with Monte Carlo simulations using simulated, noisy data. It was found to be very robust, and very fast (6 msec for one time point with 11 complex valued signals on 2.2GHz processor). An example calculation is shown in figure 2, where data was constructed from four random modes (random n, amplitudes, phases), and with 5% random Gaussian noise added to each sensor. The SparSpec frequency grid was restricted to $-40 < n < 40$. SparSpec correctly identifies the dominant four modes; the small amplitude solutions are due to the added noise.

3. RESOLVING STABLE ALFVÉN EIGENMODES OF JET

For the MHD spectroscopy diagnostic at JET, the data from eleven unevenly spaced Mirnov coils (all at identical poloidal angles) is obtained via synchronous detection hardware, i.e. a very sharp bandpass filter (+/- 500 Hz) produces the in phase and quadrature signal components only at the antenna frequency, and these signals are then used to facilitate real time resonance detection and tracking [8]. The damping rate of the detected resonant mode is determined post-shot by fitting the data to a forced, damped harmonic oscillator model.

While frequently AE resonances involves a single, dominant toroidal mode number, it is sometimes the case that several modes are excited by the system, which then compete at nearly the same frequency. Using SparSpec, individual modes can be resolved and damping rates determined for each mode separately. A typical example is shown in figures 3 and 4 for JET Pulse No: 69586, in which three toroidal AE's with mode numbers $n = -1, 0, 2$ exist simultaneously near $t = 72$sec.

When a very small time interval is analyzed, the changes in amplitude and phase of an individual mode as a function of incremental changes in antenna frequency can be used to calculate the damping rate, g, of that particular mode in that time interval [3]. One such calculation is shown in figure 5, for $n = -1$ at $t = 71.9$sec. When plotting the in phase and quadrature components in the complex plane, a circle results. It is interesting to note that the resonant frequencies of the three AE's differ slightly from each other, based on the fact that for each mode the peak amplitude occurs at different times. Such information would not be possible without applying the SparSpec method to the Mirnov data first.

CONCLUSION

A new method for identifying toroidal mode numbers in Mirnov data from toroidal plasmas has been found and benchmarked. Embodied in the SparSpec code, and originally developed in the field of astronomy for analysis of unevenly time-sampled astronomical data, this new method fits signals which are unevenly sampled in the toroidal coordinate to a sum of an arbitrarily large number of toroidal modes with integer mode numbers. By assigning a penalty to solutions that invoke larger numbers of modes, SparSpec determines the best fit with the sparsest spectrum. The method has proven to be very robust, and is especially useful for resolving the amplitudes and phases of multiple Alfvén eigenmodes which are ringing with the same or nearly the same frequency. The method also is superior in determining n-numbers when there is only one dominant mode present, as compared with traditional straight line phase fitting techniques. Examples involving stable Alfvén eigenmodes in JET, excited by an array of external antennas, were used to illustrate the efficacy of the method.

ACKNOWLEDGEMENT

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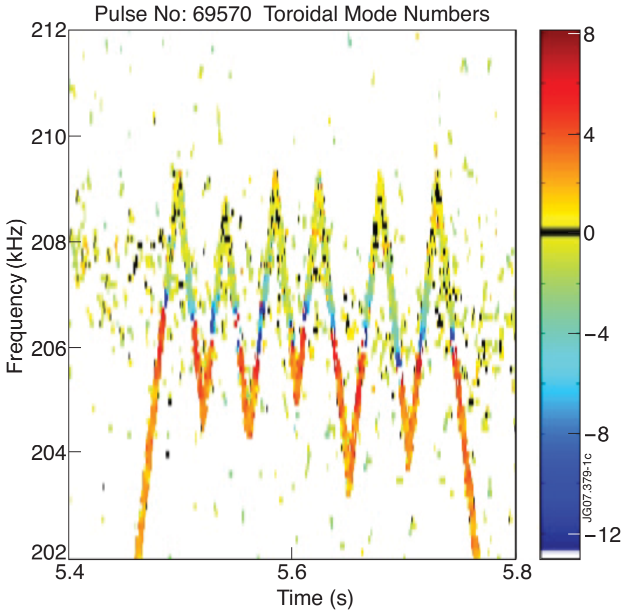

Figure 1: Stable TAE's excited by external antennas in JET Pulse No: 69570. Active antennas sweep in frequency to repeatedly scan over a detected resonance. Determination of n-number using simple straight line phase-fitting of three closely spaced Mirnov coils shows drastic jumps in n-number for incremental frequency changes and indicates the presence of multiple modes.

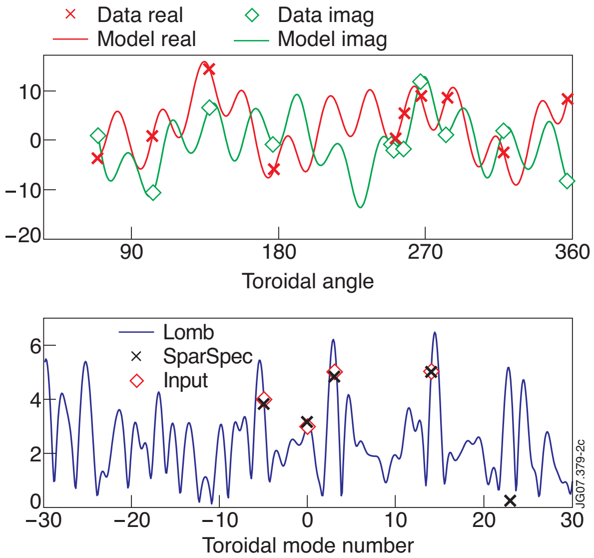

Figure 2: SparSpec calculation using simulated data: Top: data (points) and model fit (lines). Bottom: Input mode amplitudes versus estimation from SparSpec.

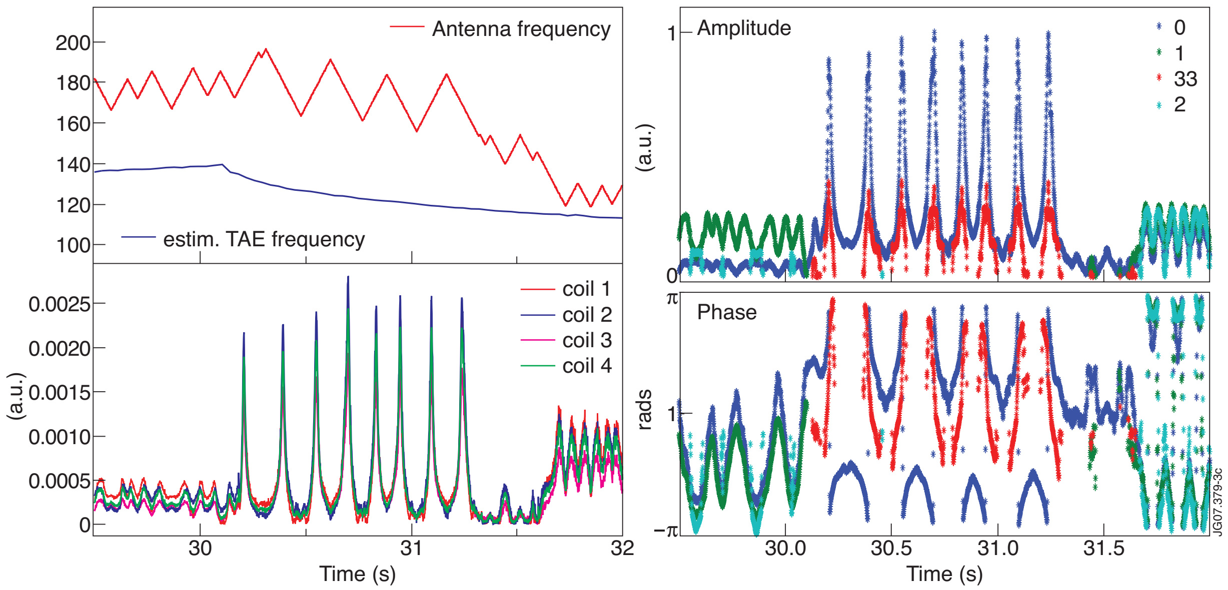

Figure 3: Left: Active antenna TAE resonance tracking for JET Pulse No: 69586: Right: SparSpec calculation for 69.5 < t < 72 sec. n = −1 until t = 70.2s, then n = 0, then n = −1,0, and 2.

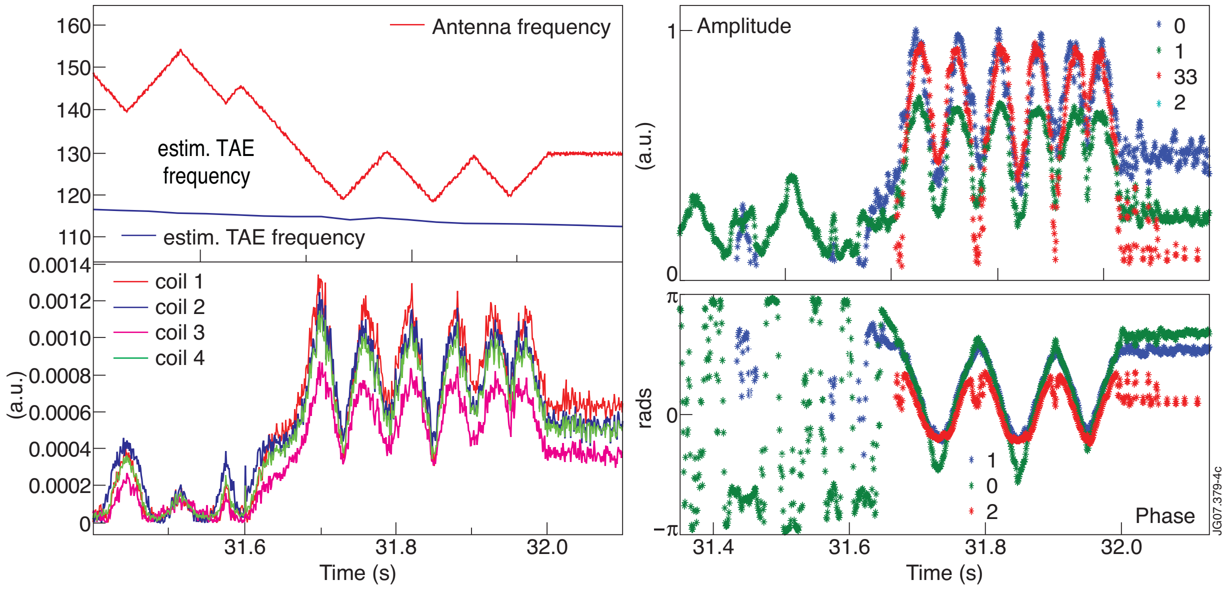

Figure 4: Left: Detail of JET Pulse No:9586 from 71.4 < t < 72.1. Three modes with n = −1, 0, 2 compete with nearly equal amplitudes. Right: SparSpec resolves amplitude and phase of each mode.

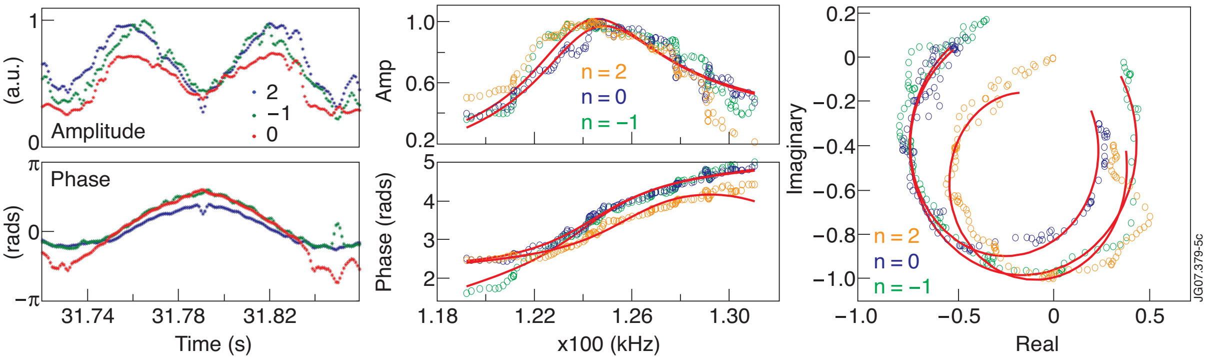

Figure 5: Top: Detail of SparSpec calculation near t = 71.9s. Three modes can be seen to have slightly different resonant frequencies. Middle: Normalized amplitude and phase of n = −1 versus antenna frequency near t = 71.9s. Bottom: Damping calculation for n = −1: $freq_{res}$ *= 124 kHz,* γ/ω *= 2.3%.*