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** See annex of J. Pamela et al, "Overview of JET Results ", (Proc. 20th IAEA Fusion Energy Conference, Vilamoura, Portugal (2004).*

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1. INTRODUCTION

The JET polarimeter system[1] measures the Faraday rotation and the Cotton-Mouton phase shift angle linked(proportional) to the line integral of poloidal magnetic field times the electron density and line integrated plasma density respectively. It has been shown [2] that this ideal link is verified in the experimental data only for line integrated densities (in units of 1020m-2.) neL20<1. At high density this linear link disappears and the two effects mutually interfere[2,3,4]. In this condition it is useful to have a tool to predict the polarimetry measurements using data produced by other diagnostics, to compare the theory and polarimetric measurements. This paper presents a study aimed in particular to validate the ability of a model to predict the Cotton-Mouton effect at high plasma density and temperature, i.e. at ITER relevant plasma parameters. The Faraday rotation and the Cotton-Mouton phase shift angle can be calculated by means of the rigorous solution of Stokes equations[4], which define the spatial evolution of the polarisation of the laser beam inside the plasma. A simplified analytical solution(AS) could be found using an ordering between the components of the vector appearing in the Stokes equations which is typical of Tokamak plasma. The AS could be used (i)for understanding the mutual effect between the Faraday and Cotton-Mouton at high density and current, (ii)to assess the range of plasma parameters were there is a linear dependence between Cotton-Mouton phase shift angle and line integral of plasma density and (iii)to calculate the line integral of plasma density once the Cotton-Mouton phase shift and Faraday rotation are known. The paper is organised as follows: in sec II the method of rigorous solution to Stokes equations is introduced and the comparison with measurements is carried out; in sec III the AS is obtained and the comparison with the rigorous solution and other analytical approximations present in the literature is discussed ;in sec IV conclusions are given.

2. SOLUTIONS OF STOKES EQUATIONS.

The geometry considered includes the propagation of a laser beam along a (vertical) chord (taken as z-axis) in a poloidal plane of a Tokamak. The toroidal magnetic field ($\vec{B}t$) is perpendicular to this plane and the angle of the electric field vector of the input wave with $\vec{B}t$ is 45^{o}. The polarisation of a beam can be described using Stokes vector $\vec{S}$, whose components are expressed in terms of the ellipticity (χ) (which is linked to the Cotton-Mouton phase shift angle ϕ) and Faraday angle (ψ).The spatial evolution along the z-axis of the polarisation of a beam is given by the Stokes equation [4]:

$$\frac{d\vec{S}}{dZ} = \vec{\Omega} \times \vec{2} \tag{1}$$

where Ω = (T_1, T_2, T_3), where T_1 = C1* n* Bt^2; T_2 = 2* C1* nBr* Bt;T_3 = C3n* Bp (Bt is the toroidal magnetic field (T), Bp the poloidal magnetic field (T), Br the radial magnetic field (T), n the electron density (m^{-3}), C1 = 1.794 10^{-22} and C3 = 2 10^{-20}) [4] , and Z = z/ ka , where k is the elongation and a the minor radius. The relation between the measured Faraday rotation(y) and the Cotton-Mouton phase shift (f) angles and the components of Stokes vector is:

$$\frac{S2}{S1} = \tan 2\psi \tag{2}$$

$$\frac{S3}{S2} = \tan \phi \tag{3}$$

A simple approximate solution [4] is found to (1) if the quantities $W3 = \int = dZ\, T_3\, C3 * \int n\, Bp\, dZ$ and $W1 = \int = dZ\, T_1\, C1 * \int ndZ\, Bt^2$ satisfy to the conditions $(Wi)^2 << 1$). In this condition the Stokes vector and the Faraday and Cotton-Mouton phase shift angle are given by

$$S1 = -W3 = -C3^* \tan \int n\, Bp\, dZ = 1/ \tan 2\psi \tag{4}$$

$$S2 = 1 - (W1^2 + W3^2) / 2 \approx 1 \tag{5}$$

$$S3 = W1 = C1^* \int ndZ\, Bt^2.\ \tan \phi \tag{6}$$

The fig.1a shows a comparison between the measurements of Cotton-Mouton phase shift angle (blue simbols) and numerical solutions of Stokes equations (black stars) for the Pulse No: 60870 (B_t=2.7T, IP=3.5MA, neL20=2.5, W1=0.36, W3=1.84), polarimeter channel 2. The red and green dashed traces are calculated using the upper and lower limit given to the measurement of plasma density made by the LIDAR Thomson Scattering. The magnetic field components which enter the evaluation of W1 and W2 and vector $\vec{\Omega} = (T_1, T_2, T_3)$ are taken from the equilibrium. The fig.1b shows a comparison of the measured Cotton-Mouton phase shift angle (blue symbols) and the approximate solution(black stars) (4, 5, 6). It is clear that the approximate solution is not adequate, and this is because $(W3)^2 > 1$, for this shot.

3. APPROXIMATE ANALITICAL SOLUTION

The conditions $(Wi)^2 << 1$ are restrictive and more general approximate solutions to the equations (1) can be found, noting that the following inequalities between the components of the vector $\vec{\Omega}$ hold for tokamak plasma:

$$T_3 > T_1 >> T_2 \tag{7}$$

Figure 2 shows the values of T_i for the Pulse No: 60870 : the condition (7) is verified.

As consequence of condition (7), the terms with component T2 can be neglected in the equations (1). The expressions for the Faraday angle and Cotton-Mouton phase shift can now be obtained from the simplified Stokes equations:

$$\frac{S_1}{S_2}(z) = \tan 2\psi = -\frac{1_1}{\tan(W_3)} \quad ; \quad \frac{S_3}{S_2}(z) = \tan\phi = \left[\frac{\int dy T_1(y) \cos(W_3(y))}{\cos(W_3(z))} \right] \tag{8}$$

The fig.3 shows a comparison between the rigorous solution(blue symbols) , the approximated solution obtained using the inequality Wi^2<<1(red symbols), and the AS (black symbols) for Pulse No: 60870,channel2.A good agreement is found between the AS and the numerical solution. The fig.4 shows the ratio of tanf/W1 versus W3 as it is calculated from (8) for a discharge with edge safety factor $q_{95} = 3$: it is deduced that the phase shift is close to the line integral of plasma density only for W3<<0.5. The fig.4 can be used to extract the value of the W1, so the value of the line integral of plasma density, once the Cotton-Mouton (S3/S2) and Faraday angles are known.

CONCLUSIONS

The Stokes equation is a model useful to analize the polarimeter data: good agreement is found between data and calculations . A new analytic approximate solution is obtained from Tokamak ordering, which is in agreement with the numerical solution and experimental data also in conditions where $(Wi)^2$>1.This analytic solution can be used to evaluate W1 (i.e. the line integral of plasma density) once the W3 (i.e. the Faraday rotation) and Cotton-Mouton phase shift are known.

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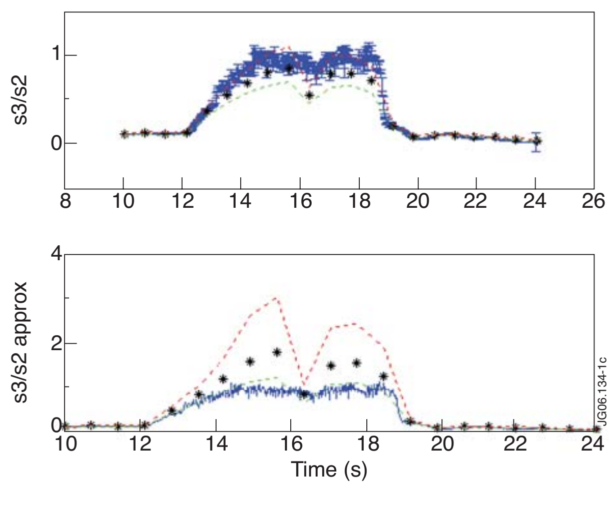

Figure 1: Tan ϕ = S3/S2 versus time.

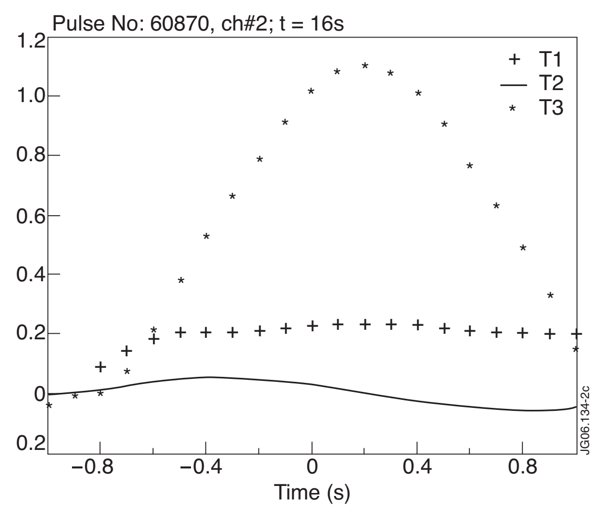

Figure 2: T_i versus normalized Z for Pulse No: 60870, channel2.

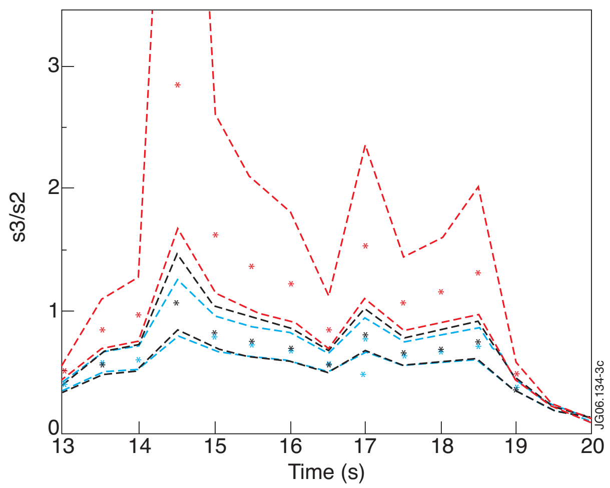

Figure 3: Tan ϕ = S3/S2 versus time for Pulse No: 60870.

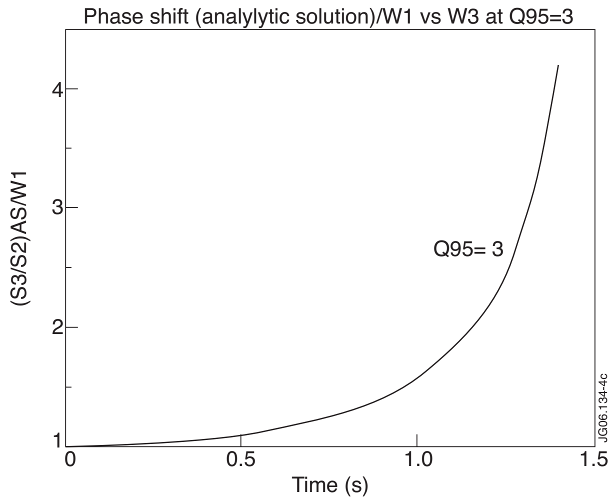

Figure 4: Model calculation of Tanϕ/ W1=(s3/s2)/W1 versus W3.