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Figure 1(a) shows the power spectrum of a family of toroidal Alfvén eigenmodes (TAEs) in JET Pulse No: 40332, previously considered in Ref. [1]. Each TAE begins life as a single mode which later develops sidebands. The TAEs are excited via a combination of NBI and ICRH heating. The different TAEs have toroidal mode numbers in the range 5–12 and frequencies that are Doppler-shifted due to plasma rotation. In the model analysis below, we concentrate on a single excited mode. To analyse the experimentally observed structures, the slow frequency drift of the modes (due to a slow change in the properties of the background plasma) is compensated for by a nonlinear mapping, the result of which is shown in Fig.1(b). This enables the power spectrum to be compared at different times on the same frequency scale. Figure 2 shows two cuts through the transformed power spectrum Fig.1(b) at  $t = 12:40\text{s}$  and  $t = 12:45\text{s}$ . We note that each of the dominant peaks in panel 2(a) has developed sidebands in panel 2(b).

The Berk-Breizman augmentation of the Vlasov-Maxwell system (“the VM(BB) system”) self-consistently models the resonant nonlinear coupling between energetic particles and the wave modes they excite. It is based on the one-dimensional electrostatic bump-on-tail model, with particle distribution relaxation and background electric field damping. We cast the model as follows [3], in terms of the particle distribution  $f(x, v, t)$  and the electric field  $E(x, t)$ :

$$\frac{\partial \tilde{f}}{\partial t} + v \frac{\partial \tilde{f}}{\partial x} + E \frac{\partial \tilde{f}}{\partial v} = -v_a(\tilde{f} - F_0) \quad (1)$$

$$\frac{\partial E}{\partial t} + \int v(\tilde{f} - f_0)dv = -\gamma_d E \quad (2)$$

Here  $F_0$  denotes the combined particle source and loss function,  $v_a$  the particle relaxation rate,  $\gamma_d$  the combined effect of all background damping mechanisms that act on the electric field, and  $f_0$  the spatial mean of  $f$ . Spatial lengths are normalised to the Debye length  $\lambda_D$ ; velocities to the thermal speed  $v_{\text{th}}$ ; time to the inverse plasma frequency  $\omega_p^{-1} \equiv \lambda_D / v_{\text{th}}$ ; and  $E$  to  $m_e v_{\text{th}}^2 = e \lambda_D$ . The equations are solved using a code [4] which allows direct numerical solutions of the fully nonlinear self-consistent VM(BB) system across the entirety of  $(\gamma_d, v_a)$  parameter space for any  $F_0(v)$ . For both the simulations described in this paper, we initiate using a bump-on-tail distribution  $F_0(v) = F_{\text{bulk}} + F_{\text{beam}}$  where  $(2\pi)^{1/2} F_{\text{bulk}} = (\eta/v_c) \exp(-v^2/2v_c^2)$  and  $(2\pi)^{1/2} F_{\text{beam}} = [(1-\eta)/v_t] \exp[-(v-v_b)^2/2v_t^2]$ , and we choose  $\eta = 0.9$ ,  $v_c = 1.0$ ,  $v_b = 4.5$ ,  $v_t = 0.5$ , and spatial period  $L = 2\pi/k_0$  where  $k_0 = 0.3$ . Initial conditions are  $f(x, v, t = 0) = F_0(v)(1+10^{-3} \cos(k_0 x))$  and  $E(x, t = 0) = (10^{-3}/k_0) \sin(k_0 x)$ .

Let us study how the system proceeds from steady state (i.e. the field is a single mode with constant amplitude) to chaos. This is achieved by choosing a cut in  $(\gamma_d, v_a)$  parameter space, where one end corresponds to steady state and one to chaotic behavior. The cut chosen is the straight line segment  $\gamma_d = 1.0$ ,  $0.022 \leq v_a \leq 0.05$ : with  $\gamma_d$  unchanged as we move along the cut,  $v_a = 0.05$  and  $v_a = 0.022$  correspond to steady state and chaotic system behavior, respectively. We have

performed two numerical simulations: (a)  $v_\alpha = 0.06 - 10^{-7}t$  and (b)  $v_\alpha = 0.0265 - 10^{-8}t$ . Spectra  $|\tilde{E}_1(\omega)|$  of the electric field component  $E_1(t) = \int E(x, t) \exp(-ik_0 x) dx$  are shown in Fig.3. To examine the spectra in detail we take cross-sections of Fig.3 at different fixed values of  $v_\alpha$ ; a selection is shown in Fig.4.

The results presented in this paper suggest that key aspects of the frequency splitting observed in JET Pulse No: 40332 are captured by the VM(BB) model in its fully nonlinear self-consistent form (Eqs.1 and 2) as implemented in the code of Refs. [3, 4]. Our results also provide strong support for the conjecture that period doubling bifurcation underlies the observed plasma phenomenology. These results open the way to relating observations of frequency splitting to the two key model parameters ( $\gamma_d, v_\alpha$ ) of the VM(BB) system. This work was supported by Euratom and the UK EPSRC.

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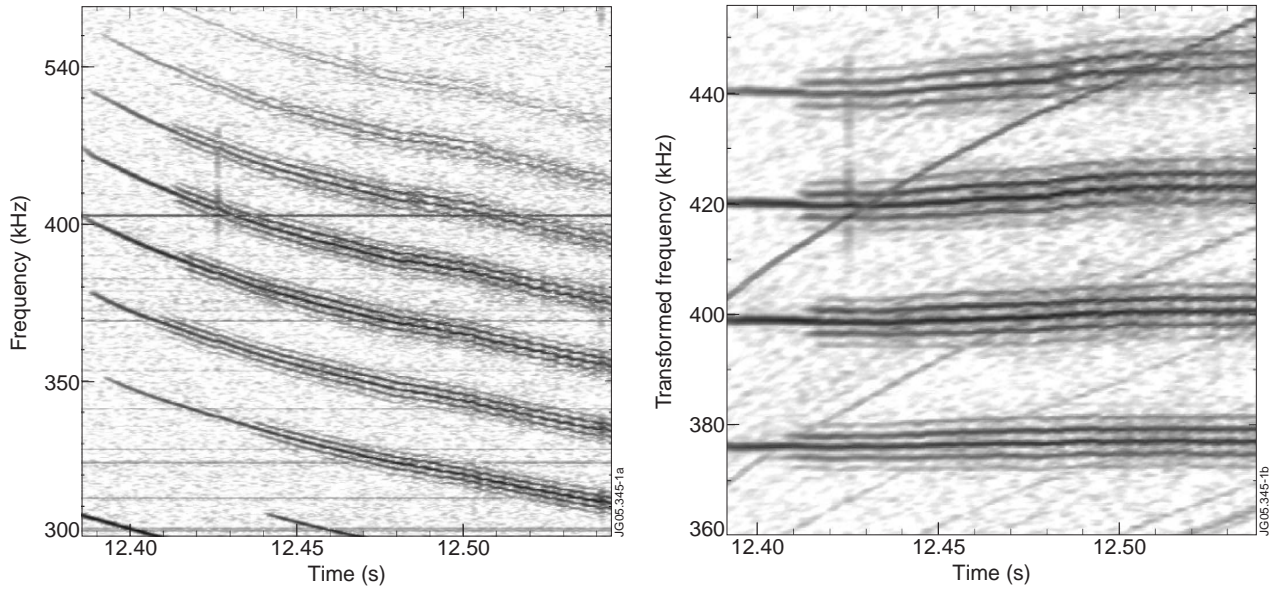


Figure 1: (a) Experimental observation of TAEs in JET Pulse No: 40332 undergoing frequency splitting. The slow frequency drift is due to slow variation of macroscopic plasma parameters. (b) Transformed power spectrum of JET Pulse No: 40332. Frequency has been nonlinearly stretched to eliminate the slow frequency drift. This transformation allows cross sections corresponding to different times (as in Fig.2) to be compared on the same frequency axis.

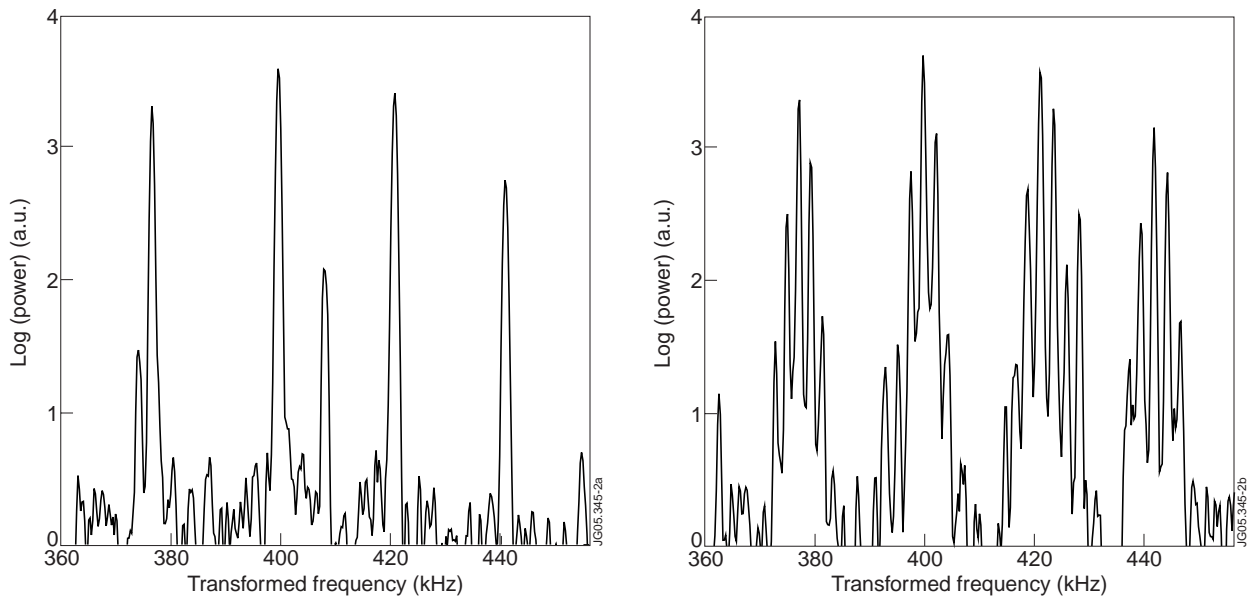


Figure 2: Cuts through the transformed power spectrum shown in Fig.1(b) at times (a)  $t = 12.40s$  and (b)  $t = 12.45s$ . Panel (a) displays four distinct modes. Panel (b) shows that sidebands have formed beside the dominant modes.

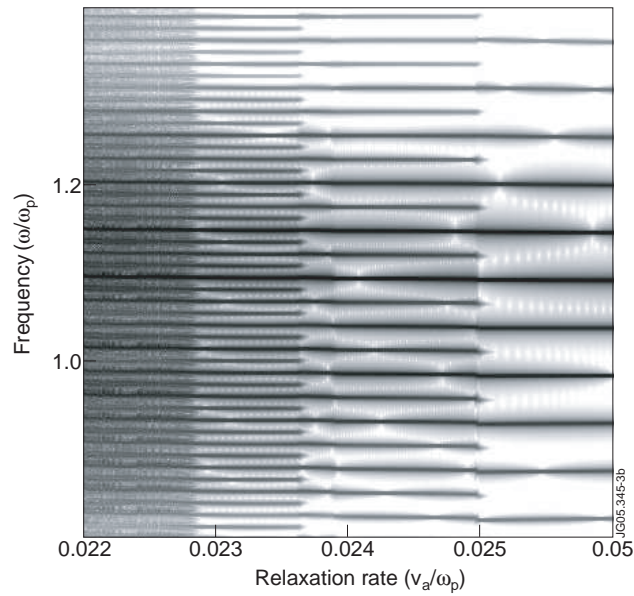
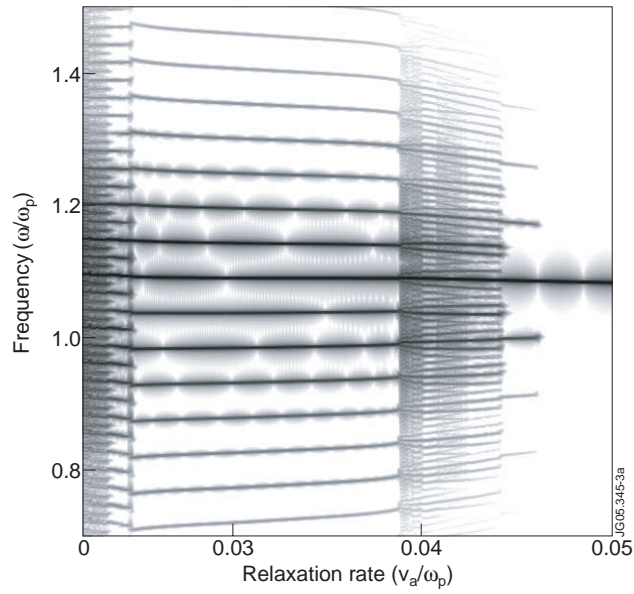


Figure 3: Field spectra  $|\tilde{E}_1(w)|$  as a function of time for the two simulations (a) and (b) described in the text. Plot (a) shows, as  $\nu_a$  decreases, the transition from a single mode, through a region of complicated behavior, to a significant interval with a number of modes with well-defined spacing, to a region where separate modes are indistinguishable. Plot (b) shows details that cannot be observed in plot (a). It shows two period doublings, at  $\nu_a = 0.0250$  and  $\nu_a = 0.0236$ . At  $\nu_a \approx 0.02284$  either a further period doubling or transition to chaos occurs.



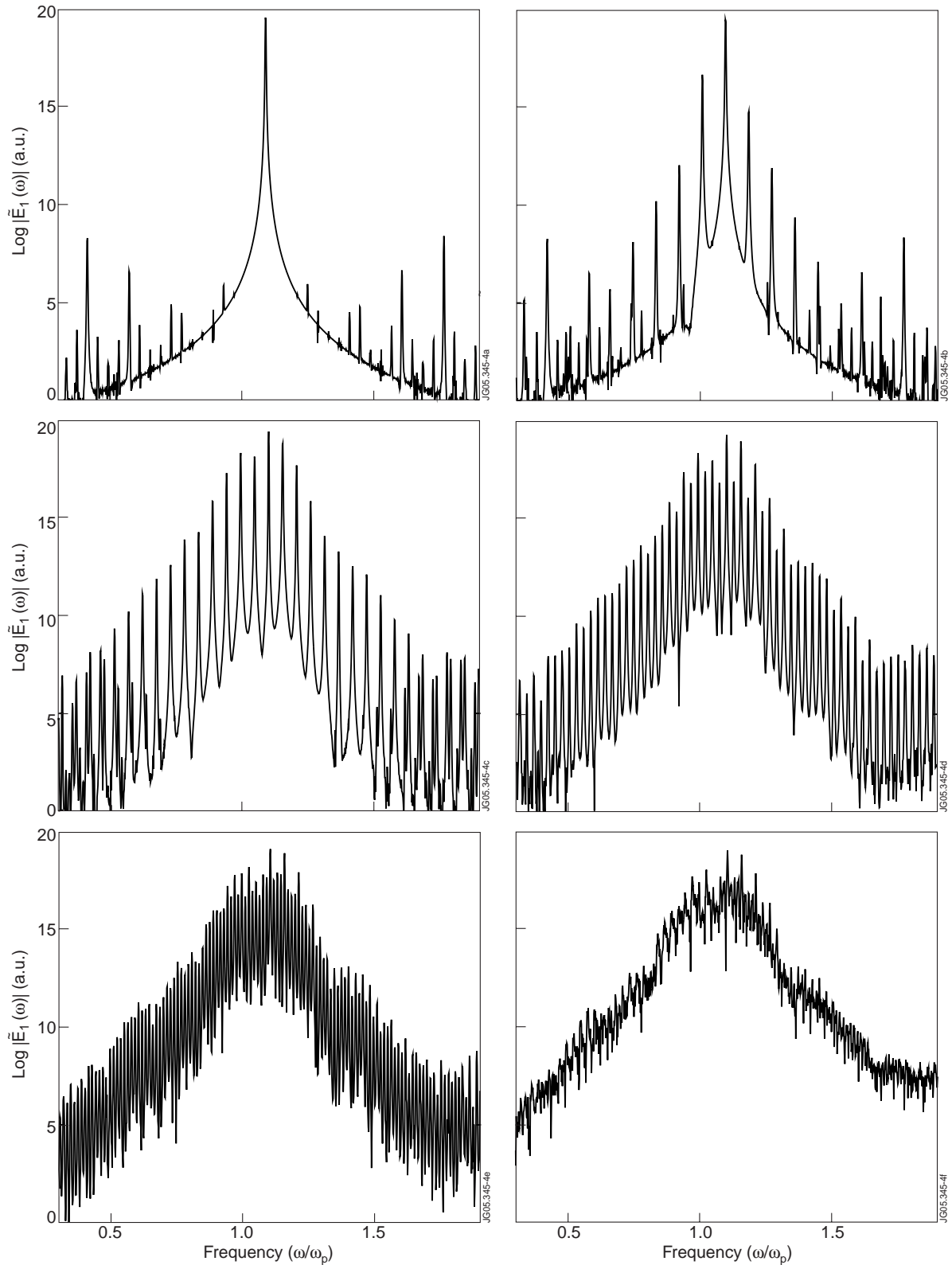


Figure 4: Cross sections through Fig.3 showing power spectra for various values of the model parameter  $v_a$ : (a) ( $v_a = 0.05$ ) system is dominated by a single mode; (b) ( $v_a = 0.045$ ) distinct sidebands have appeared; spectrum still dominated by the central mode; (c) ( $v_a = 0.03$ ) plurality of significant sidebands; (d,e) ( $v_a = 0.0245; 0.023$ ) period-doubling bifurcations: distance between sidebands is halved; (f) ( $v_a = 0.022$ ) the chaotic regime.