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**ABSTRACT.**
The continuous wavelet transform scalogram, and recently the Choi-Williams distribution, have both been used to improve upon the short-time Fourier transform spectrogram in the analysis of some nonstationary phenomena in fusion plasmas. Here, a comparison is made with real fusion plasma signals that shows the advantages of the Choi-Williams distribution over wavelets as a complementary tool to the spectrogram.

## 1. INTRODUCTION

The spectrogram may not always be the best tool to analyze some nonstationary fusion plasma signals [1-3].Wavelets are well-known in fusion research,[4] particularly the Morlet wavelet, [5] the scalogram constituting an alternative to the spectrogram. Actually, the scalogram is a natural extension of the spectrogram. In simple terms, going from the spec-trogram to the scalogram is a matter of using smaller windows with higher frequencies and vice versa [6]. So, while constituting an enhancement, the scalogram is not fundamentally different from the spectrogram, both tools being limited in terms of time-frequency resolution [7]. Recently, the Choi-Williams distribution has been effectively used to analyze nonstationary phenomena in fusion plasmas for which the spectrogram did not produce the best possible results [2, 3]. In principle, the Choi-Williams distribution is superior to wavelets, as it can yield excellent time–frequency resolution [2, 3]. However, with very good resolution comes the existence of artifacts,[1-3, 8] of which the spectrogram and scalogram are practically free of. Still, the Choi-Williams distribution allows artifact reduction, at the cost of some time–frequency resolution[2, 3]. So, despite having better resolution than the scalogram, problems caused by artifacts can hinder the applicability of the Choi-Williams distribution. Since artifacts strongly depend on the analyzed signals [8, 9] a comparison between the Choi-Williams distribution and the scalogram using real fusion plasma signals is in order. Here, such a comparison is made and the advantages of using the Choi-Williams distribution over wavelets are shown. The comparison is for phenomena in the JET tokamak, such as precur-sors of Edge Localized Modes (ELM) and washboard (WB) modes,10 sawtooth (ST) crashes and Neoclassical Tearing Modes (NTM) in discharges with Ion Cyclotron Resonant Heating (ICRH),11 and Alfvén cascades.12 In the remainder of this article, the comparison between the Choi-Williams distribution and the scalogram is reported in Sec. II, and results are discussed in Sec. III.

## 2. RESULTS

The discrete-time spectrogram of a signal s (n) sampled at frequency $f_s$ is [1].

$$P(n, \theta) = \left| \frac{1}{\sqrt{2\pi}} \sum_{m=-\infty}^{+\infty} s(m)\, h(n-m)\, \exp(-im\theta) \right|^2$$

where the sample number $n = tf_s$ and the normalized frequency $\theta = 2\pi\ f/f_s$ are functions of time $t$ and frequency $f$. The window $h(n)$ has length $l$, being zero except for $-(l-1)/2 \leq n \leq +(l-1)/2$, whereby the time resolution of the spectrogram is $\delta t_P = (l - 1)/(2f_s)$ [2, 3]. The complex Morlet mother wavelet is basically a sinusoid with a gaussian envelope,

$$\Psi(\tau) = \frac{1}{\sqrt{\pi F_b}} \exp(i2\pi F_c \tau) \exp\left(-\frac{\pi^2}{F_b}\right),$$

being often preferred for its harmonic character and good time–frequency localization [7]. $F_c$ is the center frequency. $\Delta = \sqrt{\ln(10^3)}\, F_b$ is the effective half-length of $\Psi(\tau)$, obtained from the bandwidth parameter $F_b$ by requiring that $\exp(-\Delta^2/F_b) = 10^{-3}$. The translated and scaled wavelet is $\Psi_{a,b}(\tau) = \Psi[(\tau - b)/a]/\sqrt{a}$, where b is the translation and $a$ is the scale. The scalogram S $(a, b)$ is the square of the continuous wavelet transform,

$$S(a, b) = \left| \int_{-\infty}^{+\infty} s(\tau) = \Psi^*_{a,b}(\tau) = d\tau \right|^2,$$

where time and frequency are given by $t = b$ and $f = F_c/a$, respectively, according to the transformation undergone by $\Psi_{a,b}(\tau)$. Since the half-length of $\Psi_{a,b}(\tau)$ is $a\Delta$,[14] the time resolution of the scalogram is $\delta t_S(f) = F_c\Delta/f$, which, contrary to $\delta t_P$, depends on frequency. The discrete form of the Choi-Williams distribution is [2, 3, 13].

$$CW(n, \theta; \sigma) = 2 \sum_{\tau,\mu=-\infty}^{+\infty} s(n + \mu + \tau)\, s^*(n + \mu + \tau)\, h_\tau(\tau)\, h_\mu(\mu) \exp(-i2\tau\theta)\, I(\mu, \tau; \sigma),$$

where the function $I(\mu, \tau; \sigma)$ is approximately given by

$$I(\mu, \tau; \theta) \approx \exp\left(-\frac{\mu^2}{4\tau^2}\right)\sigma \Big/ 2\sqrt{\frac{\mu^2}{4\tau^2}}\,|\tau|,$$

and $h_\tau(n)$ and $h_\mu(n)$ are windows of length $l_\tau$ and $l_\mu$, respectively [15]. For large $\sigma$ values, $CW(n, \theta; \sigma)$ has very good time–frequency resolution but little artifact reduction. Reduction of artifacts, along with some loss of resolution, is achieved by decreasing $\sigma$. Intermediate values of $\sigma$ yield a low level of artifacts and good time–frequency resolution. To avoid aliasing, $s(n)$ is replaced with the corresponding analytic signal [1–3,13]. The effective time resolution of $CW(n, \theta; \sigma)$ is $\delta tCW = (l_\mu - 1)/(2f_s)$ [2, 3].

The spectrogram, scalogram, and Choi-Williams distribution will now be applied to some strongly nonstationary fusion plasma signals. A logarithmic scale is used in all plots [2, 3]. Windows $h(n)$ and $h_\tau(n)$ are of the Hanning type, whereas $h_\mu(n)$ are rectangular windows [1].

The signals represented in Figs. 1 and 2 have been studied previously using the spectrogram and the Choi-Williams distribution [2, 3]. Figure 1 shows a type-I ELM precursor at 15kHz interrupting WB modes at 25kHz – 50kHz. Signal components below 13kHz have been removed by filtering to avoid artifacts caused by low-frequency components. While the scalogram has better frequency resolution than the spectrogram, showing a clearer separation of the WB modes, its time resolution, $\delta t_S$ (30kHz) = 1.2ms, is slightly worse than $\delta t_P$ = 0.5ms, particularly at lower frequencies. On the other hand, the Choi-Williams distribution achieves better resolution than the scalogram in time, with $\delta t_{CW}$ = 0.5ms, and also in frequency, WB modes being clearly resolved. Artifacts pose no problems at all. In Fig.2, a ST crash appears as a broadband event at 20.6095s (vertical line), along with several modes including the (m = 3, n = 2) NTM at 5kHz. In such a discharge with low N and ICRH, NTM may start with, or after the ST crash. So, good time resolution is required. The time resolution of the spectrogram, $\delta t_P$ = 2ms, is not quite satisfactory, the region around the ST crash appearing blurred. The dependency of the time resolution of the scalogram on frequency is evident, $\delta t_S$ (5kHz) = 7.4ms being a large value. In this case, for which good time resolution is important at all frequencies, the scalogram actually gives a worse representation than the spectrogram. Notice that decreasing $F_c$ to improve the time resolution would lead to worse frequency resolution, particularly at higher frequencies. Again, the Choi-Williams distribution, for which $\delta t_{CW}$ = 1ms, represents the modes and the ST crash with better time resolution than the spectrogram and the scalogram, although higher frequency modes appear somewhat masked by artifacts. For the signal represented in Fig.3, the spectrogram has a time resolution $\delta t_P$ = 2ms. In this case, using the scalogram instead of the spectrogram presents no advantages at all. In fact, since frequencies are high and the frequency range is relatively narrow, the time resolution of the scalogram, $\delta t_S$ (125kHz) = 3ms, does not change significantly with frequency. Only the Choi-Williams distribution, here calculated with a high value of $\sigma$, achieves a sharper time–frequency representation, with a time resolution $\delta t_{CW}$ = 0.5ms.

## 3. DISCUSSION

A comparison between the Choi-Williams distribution and the scalogram based on the continuous wavelet transform has been done for real examples of nonstationary fusion plasma signals. For completeness, the spectrogram based on the short-time Fourier transform, which remains the most widely used tool to analyze the aforementioned signals, has also been calculated. Differences between the scalogram and spectrogram can be significant, as in Fig.1, or relatively small, as in Fig.3. Moreover, such differences may not always be in favor of the scalogram, as in Fig. 2 where the frequency dependency of the time resolution of the scalogram is actually detrimental to the quality of the time–frequency representation. On the other hand, the Choi-Williams distribution has always yielded better time–frequency resolution than the spectrogram and the scalogram, although artifacts sometimes masked signal components, such as the higher frequency modes in Fig.2. So, the overall conclusion is that using the Choi-Williams distribution is advantageous if the spectrogram fails to produce acceptable results, as long as the signal structure is not too complex, that is, with too many modes too close together in the time–frequency plane, so that artifacts can be adequately reduced. Wavelets, on the other hand, although sometimes improving upon the spectrogram, are unable to render sharp time–frequency representations as those produced by the Choi-Williams distribution.

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[14]. $S\,(a, b)$ has been evaluated by interpolating $s\,(n)$ and using the IMSL routine DQDAG to calculate $S\,(a, b) = \left| \sqrt{a} \int_{-\Delta}^{+\Delta} s\,(a, x + b)\, \Psi^{*}\, s\,(x)\, dx \right|^{2}$ for $a$ and $b$ corresponding to the points of a regular time–frequency grid.

[15]. Notice that subscripts $\tau$ and $\mu$ are labels, not variables.

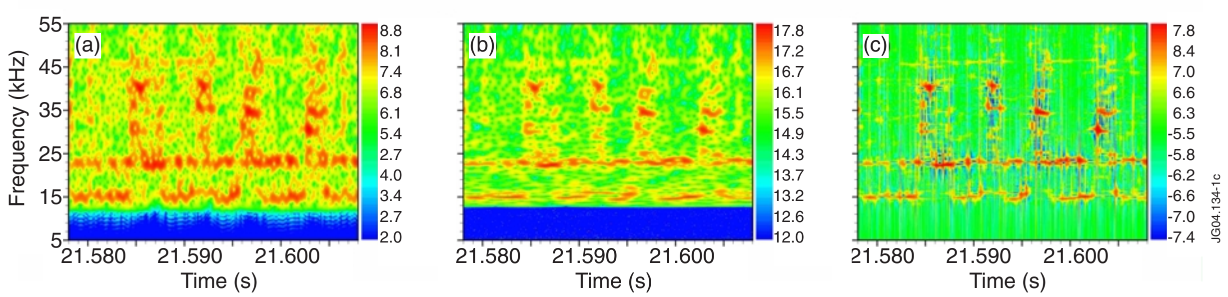


*Figure 1: Analysis of a magnetic pick-up coil signal (JET Pulse No: 55976), using (a) the spectrogram with l = 255, (b) the scalogram with $F_c$ = 1Hz and $F_b$ = 200$s^2$, and (c) the Choi-Williams distribution with $l_\tau$ = 2047, $l_\mu$ = 255, and $\sigma$ = 0.5.*

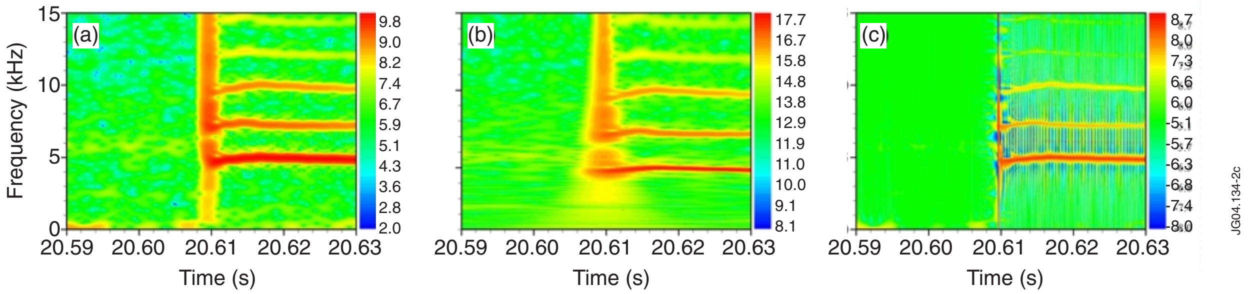


*Figure 2: Analysis of a magnetic pick-up coil signal (JET Pulse No: 50668), using (a) the spectrogram with l = 1023, (b) the scalogram with $F_c$ = 1Hz and $F_b$ = 200$s^2$, and (c) the Choi-Williams distribution with $l_\tau$ = 2047, $l_\mu$ = 511, and $\sigma$ = 1.*

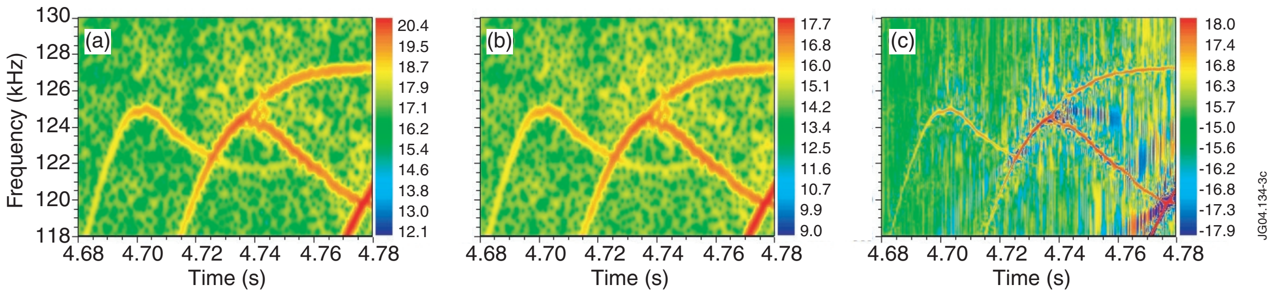


*Figure 3: Analysis of a magnetic pick-up coil signal (JET Pulse No: 63092), using (a) the spectrogram with l = 4095, (b) the scalogram with $F_c$ = 1Hz and $F_b$ = 20000$s^2$, and (c) the Choi-Williams distribution with $l_\tau$ = 8191, $l_\mu$ = 1023, and $\sigma$ = 100.*