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ABSTRACT
A local analysis technique is presented for the analysis of MSE data to deduce the safety factor q in tokamak discharges. The technique preserves the individuality of the measure by a simple rule of translation of magnetic field pitch angle measurements into q-values. Based on a geometric approach, and the observation that the flux surfaces shapes are strongly constrained by that of the last closed flux surface (LCFS), by the position of the magnetic axis and by a few more global parameters, it provides a robust, non-subjective, accurate technique that is useful for the experimental study of q-profiles and for the evaluation of its uncertainties. It also provides a useful tool for plasma control experiments as it does not submit data to a preliminary search of minima in a multi-parametric domain, a procedure that may lead to jumps in the time behaviour of the produced results.

1. INTRODUCTION
A crucial logical step is required when inferring, from local measurements obtained by Motional Stark Effect (MSE) polarimetry [1-2], the local values of the safety factor q as this parameter pertains to the entire flux surface, while the measurement determines magnetic the field pitch angle ζ only on a given point. Its evolution over the surface remains to be determined.

Usually to link the two quantities a search is performed, in a given functional class, for equilibria that best-fit the measured pitch angles together with other constraints. This is a powerful technique supplying solutions that are more or less closely adapted to the MSE data and that help to cross-validate them with other measured and theoretical information. It implies, however lack of a direct, point-to-point link between measurements and deduced q values. Furthermore the weights attributed to different channels (and to MSE relative to other diagnostics) may have to vary with their evolving condition or temporary malfunction so that the mutual influence among channels varies too, leading to difficulties in following the plasma evolution and comparing different cases. Also, the procedure may fail to describe properly certain cases or induce a bias in certain regions of the plasma.

Here we present a complementary technique that preserves the one-to-one relation mentioned above. It is based on the observation that global plasma parameters and especially the boundary shape of the discharge, as they can be determined by the magnetic measurements, strongly constrain the flux surfaces geometry in the bulk of the discharge. The resulting uncertainties are, therefore, moderate and allow a safe local deduction of q from every single data-point.

A similarly local technique applied to the measurement of the toroidal current density j_t was published recently [3]. Based on the assumption of constant elongation and negligible Shafranov-shift Δ, it uses a Taylor series expansion of the poloidal magnetic flux to derive j_t in terms of both the local value and the radial derivative of the poloidal field measurements. Application of our method, which assumes that the elongation k, as well as D and the triangularity d, varies from one flux surface to another, would only require the derivative. Here however we shall limit our analysis to the safety factor alone.

Our geometrical approach leads to a straightforward breakdown of the relation between measurement and deduced quantity into a sum of different terms. This makes explicit the role of the different geometric parameters and of their derivatives in determining q. It also provides, therefore, by combination of several contributions, an easy evaluation of its uncertainties, a task that might be more cumbersome with best-fitting procedures. Based on this technique, a unique recipe is adopted that always produces results with no convergence problems and without prior evaluation (and consecutive exclusion) of bad channels as any incorrect local measurement does not affect results obtained from the other. The analysis is robust and can therefore be safely performed automatically, used in real-time for plasma control purposes and readily deliver results immediately after the experiment.

Throughout our paper, we shall assume that the MSE measure points are in the plane of the magnetic axis, where the poloidal magnetic field is purely “vertical" or in the Z direction (in cylindrical coordinates R, ϕ, Z).

2. THE LINK BETWEEN BT/BP AND Q

The evolution of the poloidal magnetic field intensity over a flux surface is described by

$B_\rho(\rho,0) = B_{\rho 0}(\rho)\ (R_0+\Delta(\rho))\ R\rho,0)^{-1}\ (d\chi/d\rho)^{-1}$,

where dx is the distance between two neighbouring surfaces of minor radius ρ and ρ +dρ, and R_0 is the mid-major radius of the last closed flux surface.

Adopting for the flux surfaces equations the:

$R=R_0+\Delta(\rho)+\rho\ \cos(0+\delta(\rho)\ \sin 0)$, $\quad Z = k(\rho)\ \rho\ \sin 0$,

(which describe well, for our purposes, virtually all of JET [4] or DIII-D [5] equilibria, up to ρ/α = 0.8-0.95, α being the value of ρ on the LCFS) we obtain

$$d\chi/d\rho = [k/H(k,\delta,0)]\ [1+F_1+F_2\, d\Delta/d\rho + F_3\, d\ln k/d\ln\rho + F_4\, d\delta/d\ln\rho] \equiv k\ G/H.$$

Here H = (ds/dθ)/ρ and s is the arc length on the poloidal cross-section of the flux surface. About the functions $F_i(\delta,\theta)$ we note the relations:

$$F_i(\delta,0) = F_i(\delta,\pi) = 0,\quad i + 1,3,4 \qquad F_2=\cos 0,$$

implying that, on the magnetic axis plane,

$$B_p(\rho,0=0,\ \pi) = B_{p0}(\rho)(1\pm d\Delta/d\rho)^{-1}, \qquad (1)$$

where $\varepsilon = \rho/(R_0 + \Delta(\rho))$. A similar relation holds for the toroidal field B_t: $B_t(\theta=0,\pi) = B_{t0}\ (1 \pm \varepsilon)^{-1}$.

The link between B_p/B_t and q is then given by

$$q = \frac{1}{2\pi}\int \frac{1}{R}\frac{B_f}{B_p}\, ds_p = \varepsilon\, \frac{B_{t0}}{B_{p0}}\ 2\int_0^{\pi} \frac{G}{1 + \varepsilon\cos(0 + \delta\sin 0)}\, d0$$

$$= \varepsilon\, \frac{B_{t0}}{B_{p0}}\, \frac{k}{\sqrt{1-\varepsilon^2}} \left[1 + f_1 + f_2\frac{d\Delta}{d\rho} + f_3\frac{d\ln k}{d\ln\rho} + f_4\frac{d\delta}{d\ln\rho}\right] \qquad (2)$$

The four functions fi(e,d), calculated as powers series of d, are to the second order in that variable

$$f_1 \approx \frac{1}{2}\left[2t_2 - \varepsilon^2 t_1\right]\frac{\delta}{\varepsilon^3} + \frac{1}{2}\left[6t_2 - 3\varepsilon^2 - \varepsilon^4\right]\frac{\delta}{\varepsilon^4};$$

$$f_2 \approx -t_2\frac{1}{2} - \left(2t_2 - \varepsilon^2\right)\frac{\delta}{\varepsilon^2} - \frac{1}{4}\left(18t_2 + \left(5t_1 - 14\right)\varepsilon^2\right)\frac{\delta^2}{\varepsilon^3};$$

$$f_3 \approx \left(-t_2 + \varepsilon^2\right)\frac{1}{\varepsilon^2} + \left[-t_2 + (1+t_2)\frac{\varepsilon^2}{2}\right]\frac{\delta}{\varepsilon^3} - \frac{1}{4}\left[6t_2 + (5t_1 - 8)\varepsilon^2 + 2\varepsilon^4\right]\frac{\delta^2}{\varepsilon^3};$$

$$f_4 \approx \left[-t_2 + \left(1 - \frac{1}{2}t_1\right)\varepsilon^2 \quad \frac{1}{\varepsilon^3} + \left[-3t_2 + \left(4 - \frac{5}{2}t_1\right)\varepsilon^2 - \varepsilon^4\right]\frac{\delta}{\varepsilon^4} + \left[-8t_4 + (13 - 9t_1)\varepsilon^2 + \left(\frac{3}{2}t_1 - 5\right)\varepsilon^4\right]\frac{\delta^2}{\varepsilon^5}\right.$$

where $t_1 = \sqrt{1 - \varepsilon^2}$ and $t_2 = 1 - t_1$.

These functions have, of course, finite limits for $\varepsilon \to 0$, their asymptotic forms (truncated to second order in ε) being in that limit

$$f_1 \approx \frac{3}{8}\varepsilon\delta - \left(\frac{1}{8} - \frac{3}{16}\varepsilon^2\right)\delta^2;$$

$$f_2 \approx -\frac{1}{2}\varepsilon - \frac{1}{4}\varepsilon^2\delta + \frac{1}{16}\varepsilon\delta^2;$$

$$f_3 \approx \frac{1}{2} - \frac{1}{8}\varepsilon^2 + \frac{1}{8}\varepsilon\delta + \frac{1}{16}(1-\varepsilon^2)\delta^2;$$

$$f_4 \approx \frac{1}{8}\varepsilon - \frac{1}{8}(1-\varepsilon^2)\delta - \frac{1}{8}\varepsilon\delta^2;.$$

We note that $f_1(\varepsilon,0) = 0$ and that, therefore, for a purely elliptic cross section and $d\Delta/d\rho = 0$ the exact formula is

$$q = \varepsilon\,(B_{t0}/B_{p0})\,k\,(1-\varepsilon^2)^{-1/2}.$$

The expression of q of eq. (2) provides a useful breakdown of the different effects contributing to the determination of the safety factor. Its cylindrical approximation, $q_{cyl} = \varepsilon\,(B_{T0}/B_{P0})$, is just modestly increased, for small ε, by the toroidal effect (the factor $(1-\varepsilon^2)^{-1/2}$ in eq. (2)) but is strongly affected by the linear dependence upon the elongation k when this is considerably larger than one. Other effects (involving the functions f_i) usually become increasingly significant for ρ increasing above $a/2$ and they may account for up to 20-25% of the q value at $\rho/a \approx 0.9$. The highest among these contributions is usually due to dk/dr followed by the one due to dD/dr that may become more relevant, even for $\rho < a/2$, for higher beta poloidal.

3. GEOMETRY OF FLUX SURFACES

The geometry of flux surfaces is predictable with its uncertainties if the plasma outer shape, magnetic axis position and a few other global parameters are known.

Indeed both on JET and on DIII-D we found the elongation at the magnetic axis k_0 to be strongly linked to the edge elongation $k(a)$, the internal inductance l_i, the poloidal beta β_p, Δ_0/a, where $\Delta_0 = R_m \; R_0$ is the Shafranov-shift on the magnetic axis (whose major radius coordinate is R_m), the upper and lower triangularity. A more thorough analysis was conducted on DIII-D (where a wider variety of plasma shapes is more frequently used) based on equilibria from the MSE-constrained EFIT procedure [2]. From a set of 400 random-chosen pulses, we used all the time-slices where the procedure converged. For this set (~ 45000 time-slices) regressions can be found of k_0 versus the above parameters leading to predictors k_{p0} of k_0 such that $\sigma \approx 0.05$, where σ is the standard deviation of k_0/k_p. Also, for more than 95% of the data $0.9 < k_0 / k_{p0} < 1.1$ and, for more than 99.9%, $0.83\, k_{p0} < k_0 < 1.21\, k_{p0}$. The set densely explores (i.e. with the densest 95% of data) the ranges $1.5 \le k(a) \le 2$, $0.7 \le l_i \le 1.6$, $0.12 \le \beta_p \le 1.8$, and more sparsely (the densest 99.9%) the wider ranges $1.3 \le k(a) \le 2.1$, $0.55 \le l_i \le 2.5$, $0.01 \le \beta_p \le 3.2$, with k_0 covering densely the interval $1.2 \le k_0 \le 1.7$ and more sparsely $1.2 \le k_0 \le 1.7$. It is certainly representative of, if not exhaustively embodying, the wide variety of plasma configurations run on DIII-D.

Using one of these empirical *scaling laws*, the k profile is given by $k(\rho) = k_{p0} + c(k(a)\; k(0))\; (\rho/a)^{a_k}$, where $c_k \approx 0.3\; 0.5$ and $a_k \approx 4\; 6$ can be optimised depending on configuration details (e.g. X-point or limiter). We have $c_k < 1$ to account for the elongation drop in the outer layers of the confined plasma (say where $\rho/a > 0.9\; 0.95$). The relative uncertainty (two sigma) on $k(\rho)$ then varies from ±10% in the inner core to ±5% at $\rho/a = 0.9$. The uncertainty on $dk/d\rho$ is estimated as $\pm[(0.4 \bullet dk/d\rho)^2 + (k(a)-k_{p0})^2/a^2]^{1/2}$.

We assume $\delta(\rho) = \delta(a)[\, c_{\delta 1}\, (\rho/a) + c_{\delta 2}\, (\rho/a)^{a_\delta}]$ for the triangularity with $c_{\delta 1} \approx 0.1\; 0.2$, $c_{\delta 2} \approx 0.25\; 0.4$ and $a_\delta \approx 10\text{-}15$, with uncertainty estimated as ±20% for d(r), and as $\pm[(0.5 \bullet d\delta(\rho)/d\rho)^2 + \delta(a)^2/a^2]^{1/2}$ for $d\delta/d\rho$.

For the Shafranov shift's profile we use

$$\Delta(\rho) = \Delta_0\{(1-(\rho/a)^2)^{m+2} + [1-(1-(\rho/a)^2)^2]\,(1-(\rho/a)^n)\}. \quad (3)$$

It permits to specify independently the edge derivative of Δ with respect to ρ, $\Delta'(a)$, and its second derivative on axis, $\Delta''(0)$, with two exponents, $m = a^2\, \Delta''(0)/(2\Delta_0)$ and $n = \; a\, \Delta'(a)/\Delta_0$. For high core pressure gradients, $\Delta''(0)$ can be significant and affect the deduced q-values. In these cases, for higher $\Delta''(0)$, $\Delta(\rho)$ may have an inflection point ($\Delta'' = 0$, see fig. 1b). Also, when $\Delta_0 > a\, |\Delta'(a)|$ the profile must have an inflection (fig. 1c), as $\Delta'(0)$ has to be zero.

We get R_m, needed to determine Δ_0, from the MSE data by zero-search in the pitch angles trend. This leads to values of R_m that are typically stable within a few millimetres, its total systematic error being 1-2 cm. Alternatively other estimators of R_m may be used, such as supplied at JET by the magnetic reconstruction program XLOC [6], but the uncertainty on R_m would be probably higher. $\Delta'(a)$ is obtained as $\Delta'(a) = [B_p(\rho,0)\,(1+\varepsilon(a))]/[\,B_p(\rho,p)\,(1-\varepsilon(a))]$, from eq. (1) where $B_p(\rho,0)$ and $B_p(\rho,\pi)$ are deduced from plasma boundary reconstruction procedures.

To determine $\Delta''(0)$ we use the relation

$$\Delta''(0) = -\frac{1}{3}\frac{d^2(B_p/B_t)}{dR^2}/\frac{d(B_p/B_t)}{dR}, \qquad (4)$$

deduced from the condition that q is function of ρ alone, as eq. (1) reads $q(\rho) = |B_t/B_p|\,(1+sign(R\ R_m)\Delta')^{\ 1}\ g(\rho)$, where g is also a function of ρ alone. So, requiring that , about the magnetic axis, $|B_p/B_t|\,(1+sign(R\text{-}R_m)\Delta')$ be itself function of r alone yields eq. (4).

MSE measurements usually allow systematic, stable determinations of the r.h.s. of eq. (4) from polynomial fits of these measured values: on DIII-D the fluctuation of the parameter m is typically $\sigma_m \approx 0.1$ and the systematic error $\delta m \approx 0.2\ 0.5$. It is seen to range between zero (and somewhat less i.e. *apparently* involving a slightly *positive* value for $\Delta''(0)$) and values above 3.

Omitting to constrain $\Delta''(0)$ by the trend of the measured B_p/B_t over the magnetic axis (and adopting for example a parametric expression for $\Delta''(r)$ simpler than eq. (3), and implying $\Delta''(0) = 0$) would lead to an average non-zero radial derivative in the trend of the deduced q values at $R = R_m$, inconsistent with it being a function of ρ alone. This effect becomes *apparent* from the data when $|m| > 1$.

The core beta poloidal, defined as $\beta_{p0} \equiv -\frac{(1+k_0^2)^2}{1+3k_0^2}\frac{8\pi^2}{\mu_0}\frac{p''(0)}{I''^2(0)}$ where $p''(0)$ and $I''(0)$ are the second derivatives of the pressure and of the toroidal current at $\rho = 0$, is linked to $\Delta''(0)$ by the relation

$$\beta_{p0} = -R_{mag}\Delta''(0) - \frac{k_0^2}{1+3k_0^2}\ \frac{2R_{mag}\delta'(0)}{1+3k_0^2} \quad [7].$$

So one might hope to deduce this parameter (a measure of the "peakedness" of the core pressure profile), from MSE data as the second term in the equation above is very slowly varying and the third is usually small. At the present state of the art this procedure looks marginal as the uncertainty on b_{p0} may be as high as 1.

We estimate the uncertainty on Δ, as $\pm\ \{[(1\ \rho/a)\ \Delta R_m]^2 + 0.04\ (\rho/a\ (1\ \rho/a)\ \Delta_0)^2\}^{1/2}$ and the relative uncertainty on $d\Delta/d\rho$ at ± 50%.

4. RADIAL ELECTRIC FIELD CORRECTION

Viewing the same position from two different angles allows correction of the effect of radial electric field E_r on the B_p/B_t measurement. We have

$$B_p = B_{p0} + (B'_{p0} - B_{p0})/(1\ \ \xi),$$

for the corrected value of the poloidal field, where B_{p0}, B'_{p0} are the two uncorrected values, $\xi = (A_1A'_5)/(A_5A'_1)$, and A_1, A_5 and A'_1, A'_5 are geometric parameters characterising the two views [8]. It appears that when $\xi < 0$ ($\xi > 0$) B_p is inside (outside) the interval $[B_{p0}, B'_{p0}]$ and when $\xi \approx 1$ it is far from both B_{p0} and B'_{p0}. In the first case, the most favourable, the E_r effect has opposite signs for the two views, so that the two uncorrected determinations of q will set an upper and a lower limit for it. Correspondingly, the correction will slightly reduce the uncertainty on B_p (and its effect on q) with respect to the uncorrected values, even when the uncertainty on E_r is high, the best situation being $\xi = 1$ where there is a reduction by $\sqrt{2.}$. For positive ξ, the uncertainty will increase, by a factor larger than $\sqrt{5}$ if $1/2 < x < 2$ (and diverging at $x = 1$), and equal to, or below, $\sqrt{5}$ outside that interval. Dual view for E_r-correction is used on DIII-D for a large fraction of the bulk plasma [9]. Most pairs of channel observing nearly coincident positions have $x < 0$ or $\xi < 0.2$, suggesting routine use of the procedure irrespective of uncertainties on E_r. For a small minority of "intersections", however, $\xi \approx 0.8$.and correction would lead to unreliable results in any case.

5. OVERALL UNCERTAINTIES AND DISCUSSION

Results from our technique applied to DIII-D data are displayed on fig. 2, where
q values, as determined with and without E_r-correction, are shown with their 2σ error bars.

These are determined by combining the uncertainties resulting from errors on pitch angles, on R_m (affecting $\Delta(\rho)$ and therefore $\rho(R)$, $\varepsilon(\rho)$ and the relation of B_{p0}/B_{t0} with B_{p0}/B_{t0}) on $k(\rho)$, $\delta(\rho)$, $\Delta'(\rho)$, $k'(\rho)$ and $\delta'(\rho)$. The uncertainty of these local values of q increases in the proximity of the magnetic axis, and indeed for the channels that are closest to it, the result has been conventionally been set to zero as the uncertainty becomes very large. (It should be observed that an *average* stable value of q on axis can easily be obtained from the derivative of the measured angles. We have not drawn this value on the plot of fig.2 where local values of the q experimental determinations appear). These uncertainties are not necessarily higher than those applicable to other techniques. Indeed integrating information from one diagnostic with other information, while producing plausible overall pictures compatible with most of the data, does not necessarily supply the best expected local values for measured parameters, and may induce bias on those values.

In our analysis we must of course integrate information from other diagnostics and from theory. We try however to point out dependencies and to analyse influences on uncertainties from external information. Our method amounts to a simple, non-subjective, accurate (as it qualifies its own accuracy) rule of point-to-point translation of the measurements into q-values. The data so produced are immediately available to evaluate the experiment without prior intervention of an equilibrium analyst. Their interpretation is straightforward and the quality of the original data transparent in the processed results. The method is robust for plasma control purposes, as it does not submit data to a preliminary search of minima in a multi-parametric domain. In such procedures, small perturbations of the input data may lead to appearance of previously non-existing, distant new minima and consequent jumps in the results from one time point to the next.

Preparations for plasma control experiments using this technique are in progress at JET.

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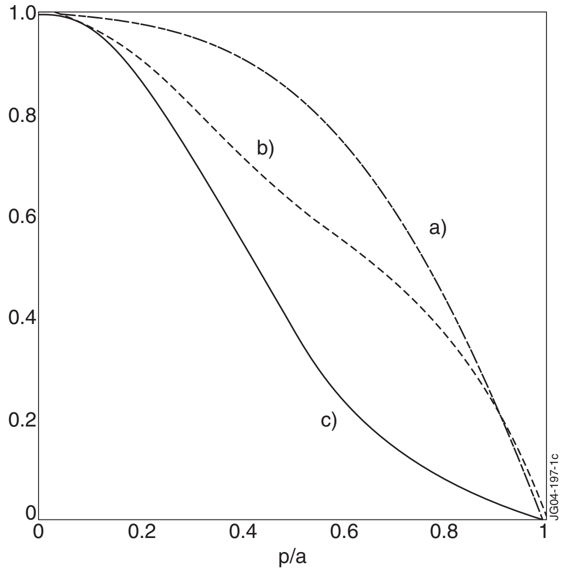

Figure 1: Profiles of Δ/Δ_0 according to eq. (3) with parameters [m, n] equal to [0.4, 2.4] (a), [2.5, 2.4] (b) and [2.5, 0.4] (c).

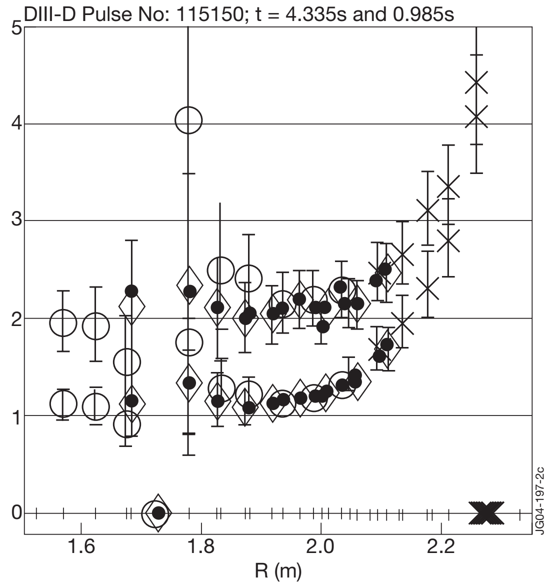

Figure 2: q-values as deduced with point-to-point analysis on a DIII-D pulse with (smaller, dark symbols) and without (larger symbols) E_r *-correction.*