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**See Annex of J. Pamela et al., "Overview of Recent JET Results and Future Perspectives", Fusion Energy 2000 (Proc. 18th Int. Conf. Sorrento, 2000), IAEA, Vienna (2001).*

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MOTIVATION

The goal of a real-time equilibrium code is to identify:

- the plasma boundary
- the flux surface geometry outside and inside of the plasma
- the current density profile
- derive safety factor profile and other important parameters from obtained equilibrium

In order to meet the real-time requirements, an entirely new code EQUINOX has been designed and implemented in C++ using the latest software engineering techniques.

1. MATHEMATICAL LAYOUT AND ALGORITHM

In order to find the plasma equilibrium, we solve the following stationary, nonlinear, bidimensional differential Grad-Shafranov equation:

$$
\begin{gathered}
-\Delta^* \psi(r, z) = J_P\,(r, \bar{\psi}(r, z)) \\
\Delta^* = \frac{\partial}{\partial r}\left[\frac{1}{\mu_0 r}\,\frac{\partial}{\partial r}\right] + \frac{\partial}{\partial z}\left[\frac{1}{\mu_0 r}\,\frac{\partial}{\partial z}\right]
\end{gathered}
\tag{1}
$$

where $\bar{\psi}$ is the normalized poloidal flux

The right-hand side of the equation is composed of the two functions to be identified:

Pressure function P and diamagnetic function F.

$$
J_P = (r, \bar{\psi}) = rP'(\bar{\psi}) + \frac{1}{\mu_0 r}\,F\,(\bar{\psi})\,F'(\bar{\psi})
\tag{2}
$$

The use of Picard type (fixed-point) algorithm allows the calculation of the inverse of finite element stiffness matrix, while the limitation to the first wall reduces mesh size and removes the nonlinearity of the magnetic permeability of iron at JET. Global optimisation of the code during its creation allows further acceleration, finally obtaining a code that is 2-3 orders of magnitude faster than offline equilibrium codes.

The following formulation (3) of the equation (2) forms the basis of finite element method:

$$
\begin{gathered}
\text{Find } \psi \in H_0^1\,(\Omega) \text{ such that} \\
\int_\Omega \frac{1}{\mu_0 r}\,\nabla\psi\,\nabla\nu\;\,d\Omega = \lambda \int_{\Omega_P} j_P\,(r, \bar{\psi})v\;\,d\Omega\;\;\forall v \in H_0^1(\Omega)
\end{gathered}
\tag{3}
$$

It can be written as

$$
K\underline{\psi^{n+1}} = B(\psi^n, {\Omega_P}^n)\underline{u^n} - \underline{h}
\tag{4}
$$

where: K – stifness matrix, B – source term matrix, u – vector of fluxes at nodes of the mesh, h – a vector modifying source term because of Dirichlet boundary conditions (the values of poloidal fluxes), n – iteration index

The fitting problem can be written as:

$$C\underline{\psi^{n+1}} = C(K^{-1}B(\psi^n, \Omega_P{}^n)\underline{u}^n - K^{-1}\underline{h}) \cong \underline{g} \tag{5}$$

wher: g- Neumann boundary condititions (poloidal magnetic field, plus ev. faraday rotation angles or MSE measurements), C – magnetic (+ev. polarimetry) measurement matrix

Please note that if the fluxes are known from previous iteration, the left-hand sidematrix can be precomputed:

$$CK^{-1}\, B(\psi^n, \Omega_P{}^n)\underline{u}^n \cong \underline{g} + CK^{-1} \cong \underline{h} \tag{6}$$

This leads to very efficient algorithm. In the real-time versions of the code a perturbation algorithm is used. The perturbation algorithm uses direct solver to calculate the residuals for a few cases close to the previous solution. The minimisation of the cost function is performed using binomial approximation of the minimum bracketed by neighbouring solutions. This prevents the code from choosing globally optimal but transient and potentially divergent solutions.

The code relies on the boundary code reconstruction providing total plasma current, toroidal magnetic field, magnetic flux values and poloidal magnetic field on the first wall of vacuum vessel. This improves the portability of the code, since we are not asking for the boundary itself.

From the practical point of view, the boundary codes are traditionally required to give an accurate plasma boundary even if this boundary sometimes is discontinuous or we can observe local oscillations due to high-degree polynomial extrapolation used in those codes. Hopefully, while using the boundary conditions on the first wall we use the values where they are the most exact, while hiding tokamak magnetic measurement-specific issues in boundary codes. Fig.1. illustrates this approach for JET tokamak.

There are two major versions of the code:

1. EQUINOX-M, using magnetic measurements only, gives accurate plasma geometry and optionally electronic density (with interferometry) and other profiles (Ti, Te...) .

P'(psi_bar) is a parabola with null boundary condition and fixed peakedness (1 degree of freedom), FF'(psi_bar) is a parabola with null boundary condition (2 degrees of freedom)

2. EQUINOX-J using magnetics and polarimetry in order to identify and control hollow plasma profiles.

P'(psi_bar) is a parabola with null boundary condition and fixed peakedness (1 degree of freedom), FF'(psi_bar) is piecewise linear with null boundary condition (3 degrees of freedom).

The number of degrees of freedom for P’ function was limited because the solution of the Grad-Shafranov equation (1) depends more strongly on FF’ than on P’, thus using more degrees of freedom for P’ would reduce the remaining residuals in the fitting functions because of the stiffness of FF’. On the other hand we must leave some minimal stiffness on FF’ in order to increase robustness. The use of regularisation would not help in this case: the regularization coefficients for P’ has to be strong enough, so that the function remains exactly linear. Because we have observed that the peakedness of P’ vs normalized flux was almost constant in EFIT-intershot results at JET, we used this peakedness for P’ and fixed it. The use of electronic density, electron and ion temperature to constrain the P’ is under investigation.

So far, several basis function has been tested (various splines, polynomials, sinusoidal), however the use of piecewise linear spline allowed the best decoupling of the edge and the plasma core, allowing the robust identification of hollow profiles without the use of 6 or more degrees of freedom like in the case of polynomials.

2. VALIDATION OF THE RESULTS

In general, equilibrium reconstruction using magnetic measurements only is in good agreement between EQUINOX and EFIT. The same applies to the results of equilibrium reconstruction using polarimetry (Faraday rotation angles). However, during additional heating phases the equilibria using polarimetry detects reversed profiles, while magnetic codes have no traces of them. As verified using simulated measurements, this is not due to the stiffness of fitting functions used with magnetic equilibria but to the lack of flux measurement inside the plasma.

Finally, EQUINOX-J using polarimetry agrees well with EFIT+polarimetry results (Fig.3) but is more robust since it does not requires any specific regularisation since the robustness of the code has been checked on a large variety of plasma conditions.

It is important to note the large sensitivity of QAX to the position of magnetic axis. In fact, even a minor jump of 3cm in RAXIS can correspond to the change of QAX close to 0.6 and also an error of 6cm gives such a large error in QAX that the decision of whether we have a hollow profile or not starts to be difficult because of error bars on QAX.

The position of magnetic axis can be verified using Thompson Scattering (LIDAR) in the sense that we can reject wrong equilibriums. However, because of the noise of LIDAR it is almost impossible to write a code performing this task automatically. On the other hand, it seems to be at least difficult to obtain good plasma geometry using polarimetry, EQUINOX-M can be used to obtain the plasma geometry in agreement with both boundary codes and EFIT-intershot (Fig. 4), while for analysis of the hollow profiles an equilibrium code incorporating both magnetic measurements and polarimetry (EQUINOX-J) should be used.

The last observation concerns the lack of sense of the equilibria obtained with negative plasma current on the center: even if most of the equilibrium codes can solve the Grad-Shafranov equation (1) using such plasma currents, even the simplest sensitivity study show an enormous sensitivity of magnetic surfaces and the magnetic axis to the measurements with such profile of plasma current. Every time Equinox code converged to such solutions using real data (after manipulations with basis functions used for fitting), the axis was moving even by 10cm every 20ms what is completely instable compared with Thomson Scattering and other available measurements.

CONCLUSION

A careful implementation together with a new algorithm allowed a giant leap in execution time, towards a 6-12ms on an Anthlon 1700+ PC machine per time frame.

EQUINOX code was tested on 5 operating systems, 6 different compilers, >1000 different shots and installed for two tokamaks with different geometry demonstrating its portability.

The code is mature and very robust, with particularly good worst-case results and completely unmanned operation.

ACKNOWLEDGEMENTS

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Q95 and RAXIS in good agreement between magnetic equilibria

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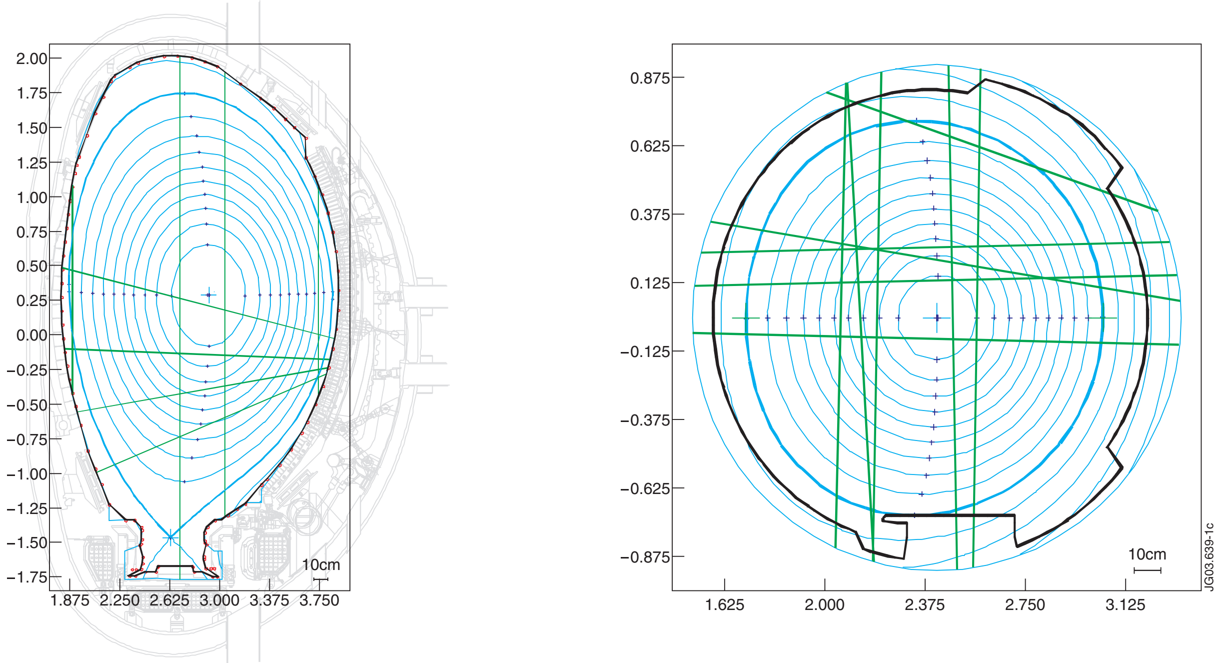

Figure 1:

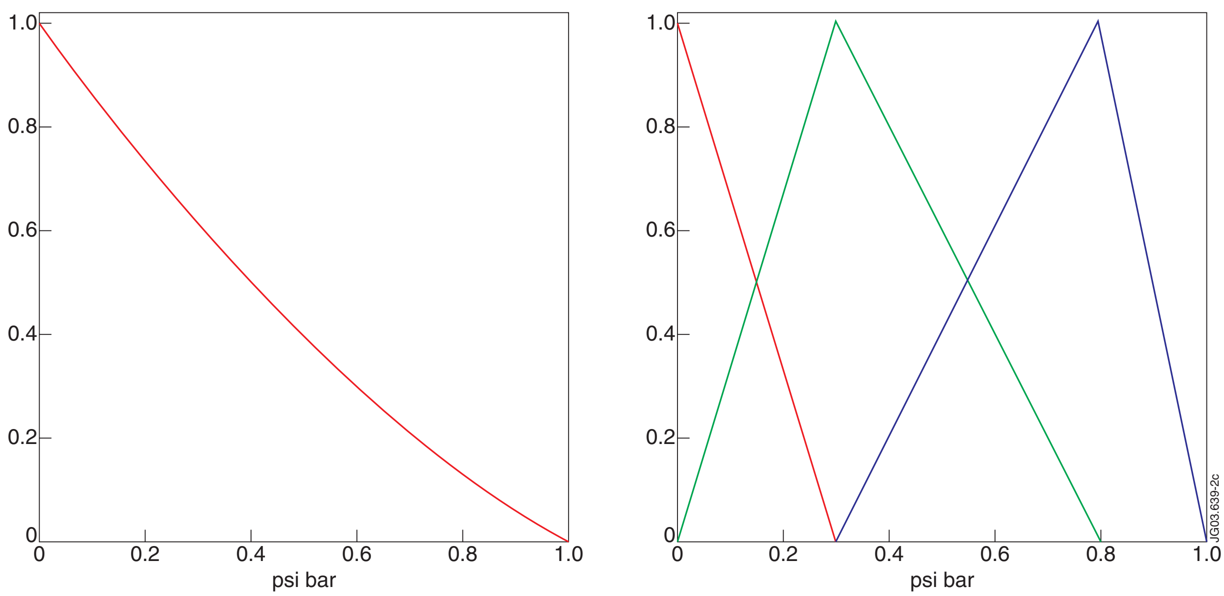

Figure 2: Basis functions used in EQUINOX-J

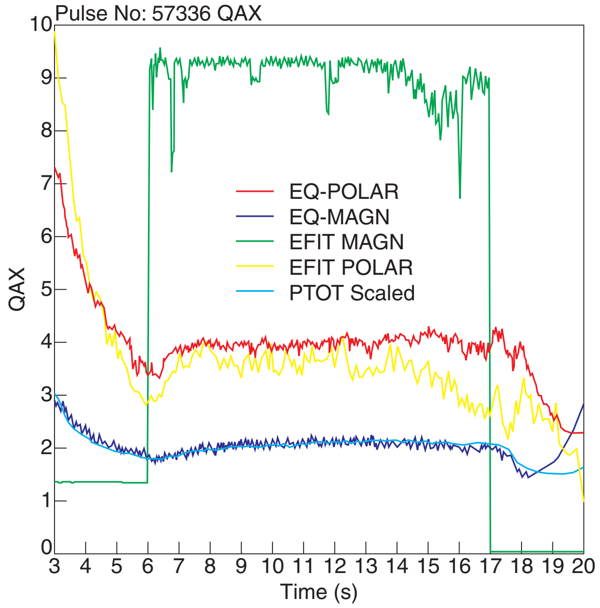

Figure 3(a): QAX using polarimetry or magnetics

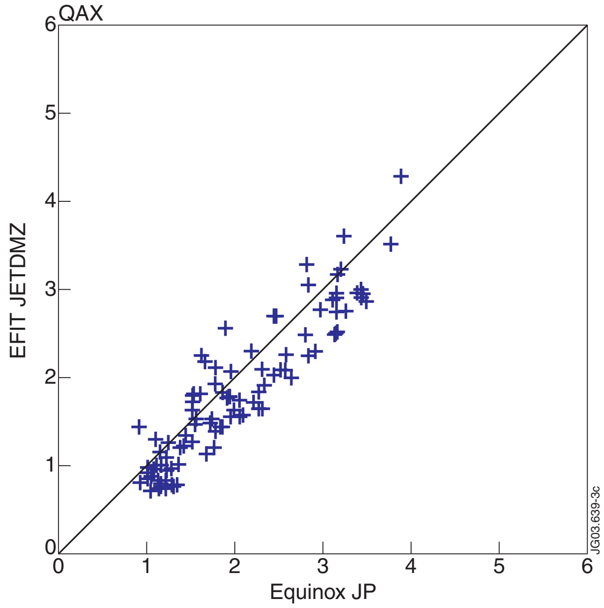

Figure 3(b): QAX using polarimetry

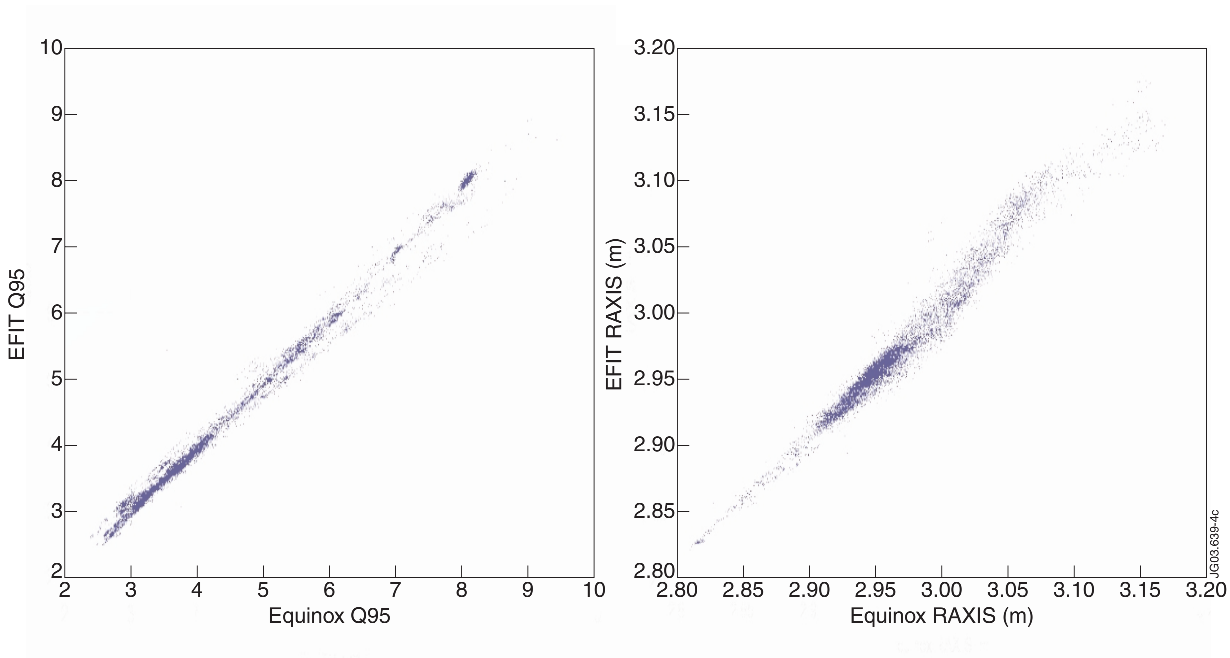

Figure 4: Q_{95} and RAXIS in good agreement between magnetic equilibria