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ABSTRACT.
This paper deals with the design of a new plasma current and shape controller for the JET tokamak. This new controller is aimed at improving the performance of the present controller so as to allow the control of extremely shaped plasmas with higher values of elongation and triangularity. The design approach makes use of a linearized mathematical model linking the voltages applied to the active coils with a number of geometrical descriptors characterizing the plasma shape. The control problem under investigation is characterized by the fact that the number of parameters to be controlled is larger than the number of control inputs. To overcome this problem a singular value analysis is carried out to identify the principal directions of the algebraic mapping between coil currents and geometrical descriptors. These principal directions are then assumed as controller outputs so as the original multivariable rectangular control problem is transformed into a square problem. Moreover the singular value decomposition approach makes it possible to solve this modified problem by means of a number of SISO problems.

1. INTRODUCTION

Many physical experimental activities on JET will require plasmas with high triangularity and elongation. This work has been carried out in the framework of a project aimed at investigating the possibility of obtaining extremely shaped plasmas with the existing active circuits and control hardware [1]. The aim is to reach ITER-like shaping with high value of triangularity. One of the steps needed to achieve this objective is the redesign of the JET shape controller, since the present controller does not consider the possibility of operating with high elongation.

This paper presents the features of a new controller and some preliminary results obtained in simulation. To control the plasma shape in JET, there are 8 knobs, namely the currents flowing in 8 PF coils, while the P1E circuit is devoted to control the plasma current. In the present JET shape controller (SC), a number of coil currents are feedback controlled at given preprogrammed time trajectories, while the remaining coil currents are used to control a limited number of geometrical parameters, usually no more than 5. Extensive simulations have shown that, in the presence of highly shaped plasmas, this approach, even guaranteeing small errors on the controlled geometrical variables, is no longer able to ensure a satisfactory invariance of the overall shape. In this paper we propose a new approach to the shape control in which all the available coil currents are used to control, at the best, the overall plasma shape remaining within the current limits imposed by the power supply. In particular, the plasma shape is defined in terms of 48 geometrical descriptors: 40 gaps chosen all around the vacuum vessel and 8 additional geometrical descriptors for the *X*-point, the strike points and the plasma current centroid.

The performance of the new shape controller (XSC) have been tested in simulation in two different situations: the rejection of the disturbances (namely β_p, l_i and I_p variations) acting on the plasma during flat top phases; and the transition from a non-extreme diverted equilibrium to an extreme equilibrium, with high triangularity.

2. THE PLANT MODEL USED FOR THE XSC DESIGN

The first step for the design of the XSC consists in describing the dynamics, on a slow time scale, of the plasma-circuits system by means of a mathematical model in the state space form. The inputs to the system are divided in *Control Inputs* V_C, namely the PF coil voltages, and *Disturbances* w. The basic relationship describing the overall plasma-circuits dynamics can be written as [2]:

$$\begin{cases} \dfrac{d}{dt}\Psi(I_T, w) + RI_T = V_T \\ G = \eta\left(\dfrac{I_T}{I_{Pl}}, w\right) \end{cases} \tag{1}$$

where $I_T = (I_{P1E}\ I_C^{\ T}\ I_{Pl})^T$, $V_T = (V_{P1E}\ V_C^{\ T}\ 0)^T$, Ψ is the poloidal flux, R the resistance matrix, I_{P1E} is the current flowing in the P1E circuits, I_C is the vector of the currents flowing in the remaining active circuits, I_{Pl} is the plasma current, and G is the vector of the geometrical vector to be controlled. Linearizing eqn. (1) around a nominal equilibrium specified in terms of (V_{T0}, I_{T0}, w_0), the model describing the dynamics of the variations has the form:

$$\begin{cases} L^* \dfrac{d(\delta I_T)}{dt} + R\delta I_T = \delta V_T - E^* \dfrac{d(\delta w)}{dt} \\ I_{Pl0}\delta G = C\delta I_T + F(I_{Pl0}\delta w) \end{cases} \tag{2}$$

where the Jacobian matrices L^*, E^*, C, F can be evaluated with the aid of the CREATE-L code [2].

For the design of the controller we neglect the plasma resistance, and consider only the control inputs ignoring the disturbances. This leads to the fact that one of the currents can be expressed as a linear combination of the others, and hence eliminated from the state variables. In particular for the output equation in (2) we obtain

$$I_{Pl0}\delta G = \tilde{C}_C\delta I_C + \tilde{C}_{Pl}\delta I_{Pl} \tag{3}$$

3. THE XSC DESIGN

The XSC essentially consists of 2 parts: an inner Current Controller, and an outer Shape Controller.

Concerning the current controller, in the design procedure we firstly consider a positive feedback loop for the compensation of the resistive drops on the PF coils. In particular it is assumed (see [3]):

$$V_T = V_T' + RI_T \tag{4}$$

Then a decoupling matrix is calculated in accordance with the procedure described in [3], in such a way that each PF coil current can be imposed independently from the others.

This current decoupling controller already provides the control of the plasma current through the feedback of I_{Pl} and its comparison with I'_{Pl} that plays the role of plasma current reference signal. Nevertheless in the XSC an additional plasma current feedback loop has been introduced mainly aimed at attenuating the effects on the plasma current of low frequency oscillations in the external disturbances and in the PF coil currents which are not exactly decoupled with the plasma current. In particular an additional Proportional + Integral (*PI*) feedback loop was closed on the $I'_{Pl} - I_{Pl}$ channel. This channel, considering the current decoupling controller, is described by a transfer function of the form:

$$W_{I'_{Pl}-I_{Pl}}(s) = \frac{1}{1+s\tau_{I_p}} \tag{5}$$

where τ_{I_p} is the time constant used in the JET design procedure (see [3]). Its value, in that procedure, is fixed to 0.67s.

Basing on the transfer function (5), and considering a PI controller described by a transfer function

$$W_{PI}(s) = K_P\left(1+\frac{1}{s\tau_I}\right) = \frac{K_P}{\tau_I}\frac{(1+s\tau_I)}{s} \tag{6}$$

in order to guarantee a first order closed loop behavior with a specified time constant τ_p it has been assumed:

$$\tau_I = \tau_{I_p}, \qquad K_P = \frac{\tau_I}{\tau_P} \tag{7}$$

As for the value of the time constant τ_p, it has been chosen: $\tau_p = 0.67s$.

One of the assumptions of the XSC design procedure is that all the input-output channels between each PF reference current and the corresponding PF actual current have the same dynamic behavior. The current decoupling controller has been already designed so as to guarantee, on each of these channels, a first order dynamics with unit gain and a time constant equal to that of the corresponding power supply. In order to guarantee the same first order dynamics for all the input-output $I'_{C,i} - I_{C,i}$ channels (see Figure 1), an additional feedback control loop with a PI controller was introduced on each channel. The PI controllers were designed using the same procedure described previously for the plasma current control. Their parameters were tuned so as to guarantee the same time constant $\tau = 100ms$ for all the 8 PF coil currents. The scheme of the plasma-circuit system with the decoupling controller and the additional PI loops is shown in Figure 1.

The outer shape controller calculates the coil current set-points I_{C_ref} for the inner current control loop (see Figure 1). These set-points consist of the sum of a nominal part and of a feedback term as shown in Figure 2. The design of the shape controller is performed using:

1. the input-output model between the reference currents I_{Pl_ref}, I_{C_ref} and the actual currents I_{Pl}, I_C of the inner current controlled system (see Figure 1). Neglecting the unavoidable coupling terms which can be considered as disturbances on each single input – single output (SISO) channel, each SISO model for the shape controller design can be written in the simplified form:

$$I_{C,i}(s) = \frac{1}{1+s\tau} I_{C\_ref,i}(s), \qquad i = 1,...,8, \tau = 0.1 \tag{8}$$

2. the CREATE_L output algebraic mapping given by the second equation in (2). As already said before, for the design of the controller we neglect the plasma resistance, and ignore the disturbances w. Considering the plasma current variations as a disturbance to be rejected, the model to be used for the design of the shape controller can be written as:

$$\delta G(s) = \frac{\tilde{C}_C}{1+s\tau}\frac{\delta I_{C\_ref}(s)}{I_{Pl0}} \tag{9}$$

Equation (9) implies that:

a) the geometrical descriptors variations can be controlled acting on the PF current reference signals scaled with the nominal plasma current I_{Pl0};
b) all the geometrical descriptor variations have the same dynamics of the coil current variations, i.e. a first order dynamics with a time constant τ. Therefore the design of the shape control can be based on the $\tilde{C}_C$ matrix.

In the following, we assume that $\tilde{C}_C$ is $n \times 8$, with n (number of controlled geometrical descriptors) greater than 8 (number of active control currents).

As far as the shape control problem is concerned, in principle it can be seen as the problem of determining, at each time instant t, the current values able to put the geometrical descriptor errors at zero. Since, using the 8 currents, only 8 linear combinations of the geometrical descriptors can be regulated to zero, the problem can be reformulated as that of determining the 8 linear combinations which minimize the quadratic norm of the overall geometrical descriptor error vector. On the other hand, once these 8 linear combinations have been selected, the 8 values of the PF coil currents are univocally determined. Hence this approach could lead to high values of the PF currents. To have additional degrees of freedom, one can reduce the number of linear combinations of geometrical descriptors error to be regulated to zero, and determine the minimum norm current vector which solves the problem. A straightforward solution to both these optimization problems is given by the Singular Value Decomposition approach.

Let us consider the Singular Value Decomposition of $\tilde{C}_C$:

$$\tilde{C}_C = U * S * V^T \tag{10}$$

where S is a square diagonal matrix containing the singular values in decreasing order. It can be proved that:

i. the matrix $S^{-1} * U^T$, multiplied by the geometrical descriptor error vector, gives the 8 linear combinations which solves the first optimization problem;
ii. to exploit at the best the PF coil currents, it is convenient to eliminate the linear combinations associated with the smallest singular values; hereafter this elimination is obtained multiplying by zero these combinations;

iii. the matrix V, when multiplied by the 8 linear combinations modified as in the previous step, gives the 8 PF coil currents which solve the second optimization problem.

To extend this approach to a dynamical case in which the geometrical descriptors need to be controlled on a time interval and not for a given time instant, we can use the control scheme shown in Figure 3. In that Figure, m indicates the number of linear combinations of geometrical descriptor errors that are forced to zero. For the JET case, $m = 5$.

The above procedure has been extended to the case in which the geometrical descriptor errors are weighted with a matrix $Q = \tilde{Q}^T\tilde{Q}$ and the PF coil currents are weighted with a matrix $R = \tilde{R}^T\tilde{R}$. In this case, it is sufficient to calculate the SVD of the following matrix:

$$\hat{C}_C = \tilde{Q}\tilde{C}_C\tilde{R}^{-1} = USV^T \tag{11}$$

The PIDs have been designed so as to guarantee the desired dynamics of the closed loop system, and hence of the geometrical descriptors. To this aim, for the design of the PIDs, we can consider the scheme shown in Figure 4.

It is simple to prove that the open loop transfer matrix between $\delta\overline{G}_{ref}$ and $\delta\overline{G}$ is given by:

$$F(s) = K_{PID}(s)\frac{I_8}{1+s\tau} \tag{12}$$

where $K_{PID}(s)$ is the diagonal transfer matrix of the PIDs, and each diagonal term has the form:

$$K_{PID,i}(s) = K_P\left(1+\frac{1}{s\tau_I}+\frac{s\tau_D}{1+s\tau_D/N}\right) \tag{13}$$

The diagonal structure of the transfer matrix $F(s)$ allows us to design the various PID using a SISO approach. Since we want to have the same dynamics for all the geometrical descriptors, it is sufficient to design just one PID.

The design has been carried out assuming $N = 5$, taking into account the phase-lag introduced by the various power supplies, and in such a way to guarantee a second order closed loop transfer function on each SISO channel characterized by a time constant of 60ms and a damping factor equal to 0.7.

4. SIMULATION RESULTS

To test the effectiveness of the proposed approach we designed a controller aimed to control the highly shaped plasma shown in Figure 5. This plasma is characterized by a 95% elongation of 1.734, a 95% triangularity of 0.353, β_p = 1.18, l_i = 0.83 and I_p=–1.6MA. We have assessed in simulation the capability of the XSC of maintaining the plasma shape at the nominal equilibrium despite the presence of large β_p and l_i variations. The considered disturbance variations are shown in Figure 6.

During a time interval of 6*s* the worst performance in terms of displacement of the plasma shape with respect to the nominal one has been observed after 2*s*. The comparison between the nominal plasma equilibrium and this worst case is shown in Figure 7. This figure shows that the XSC is able to control the whole shape of the plasma during the flat top for the considered highly shaped plasma.

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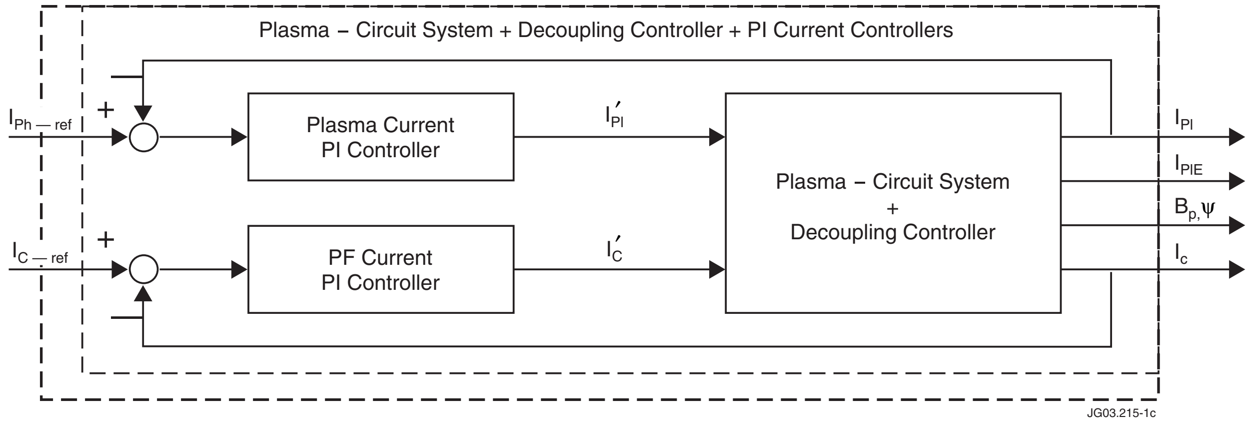

Figure 1: The plasma-circuit system with the decoupling controller and the PI current controllers.

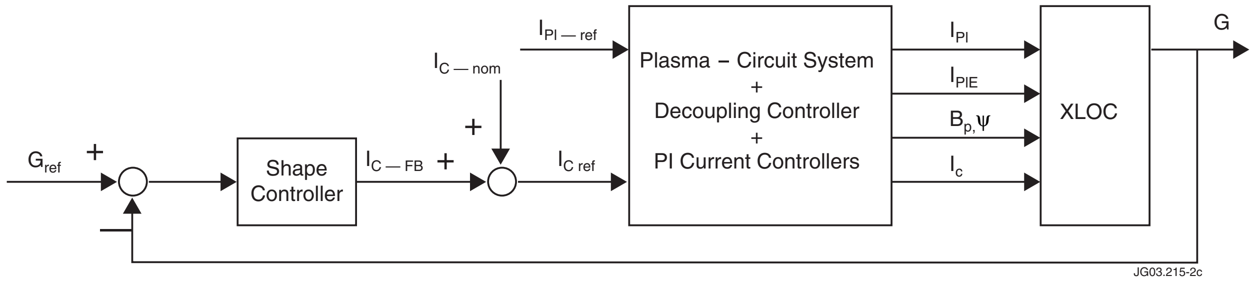

Figure 2: The overall control system.

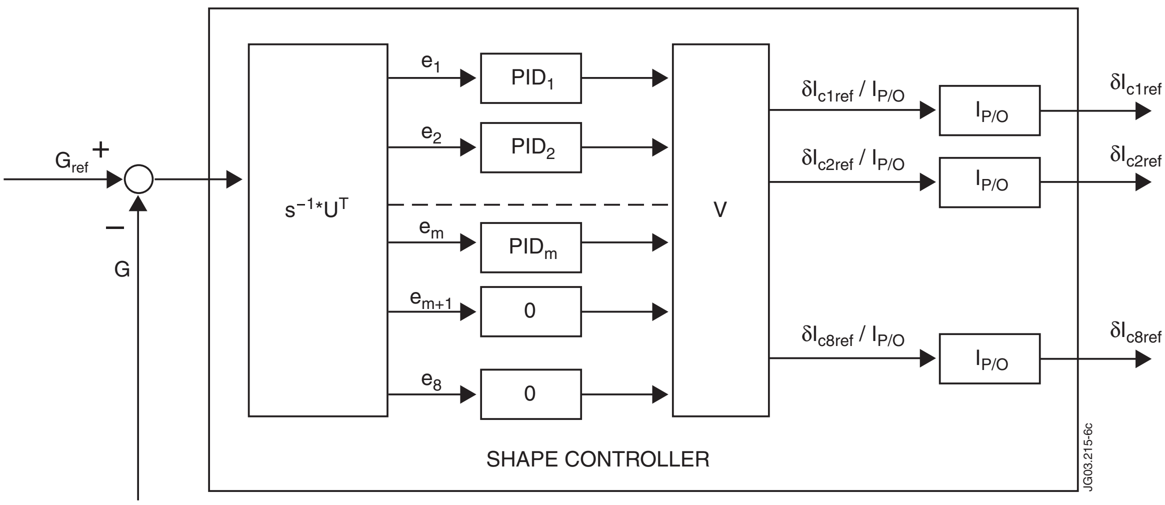

Figure 3: The shape controller structure.

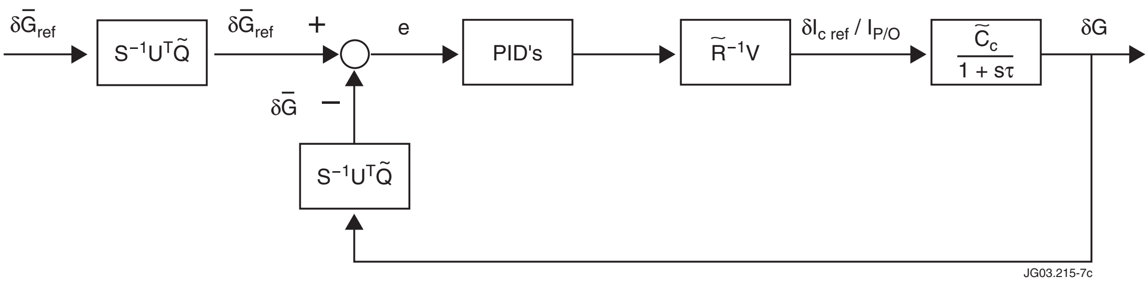

Figure 4: The closed loop scheme for the design of the PIDs.

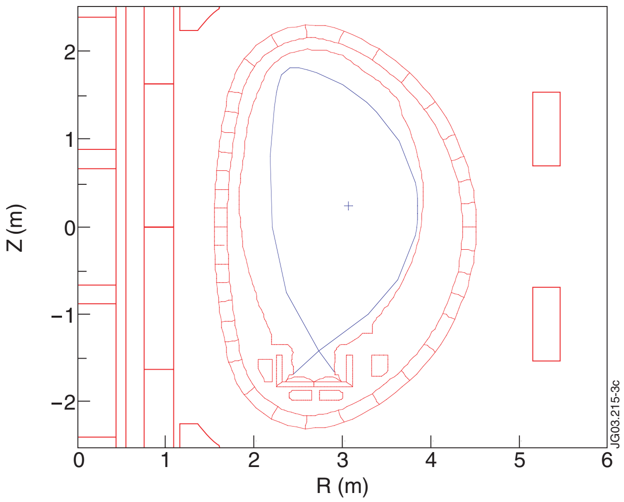

Figure 5: The Plasma equilibrium used to test the XSC.

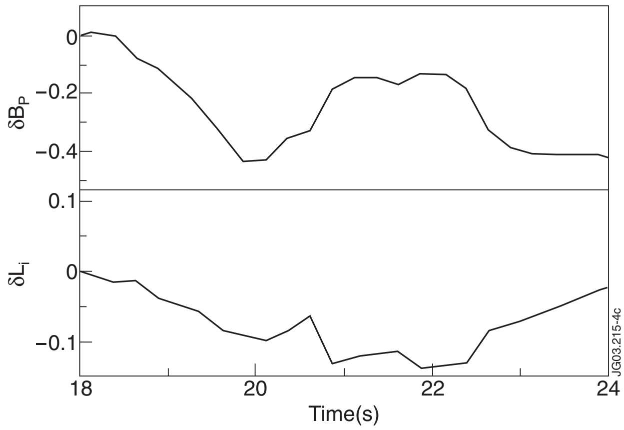

Figure 6: β_p and l_i variations.

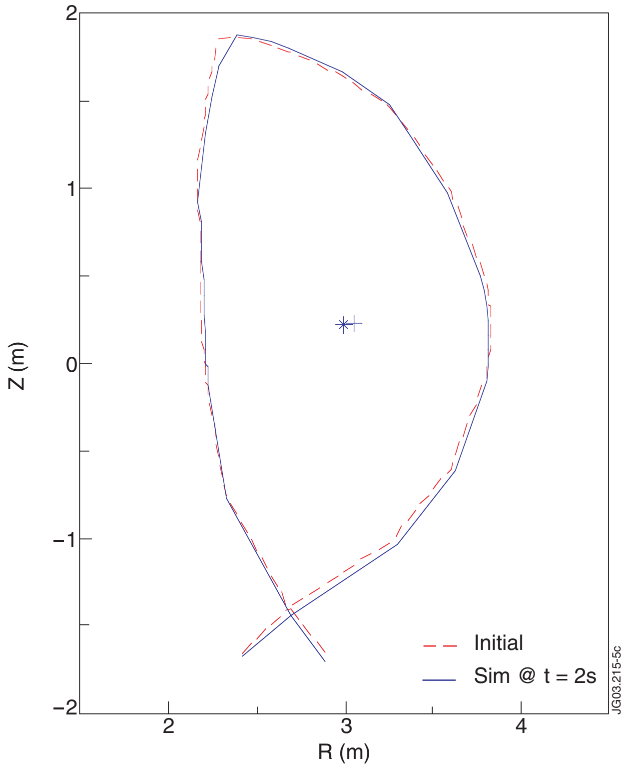

Figure 7: Comparison between the nominal plasma shape, and the plasma shape after 2s (worst case).