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ABSTRACT.
The main confinement parameters, like the internal inductance l_i and the diamagnetic poloidal beta β_{DIA}, are of particular relevance for a reliable real-time control system of next step tokamaks. These quantities have been obtained at JET (Joint European Torus), with a precision more than satisfactory for real-time applications, through a method, known as BETALI, that uses the Shafranov integrals S_1, S_2 and S_3.

Since S_1, S_2 and S_3 are defined on the plasma boundary, an upgrade of the fast boundary code XLOC, that meets stringent real-time and precision constraints, has been expressively developed to determine the Last Closed Flux Surface.

BETALI has been verified on several experimental plasma configurations, giving very encouraging results both in the limiter and x-point phase of the discharges. The compatibility with the time restrictions has also been tested successfully.

This application has therefore been implemented and it has already been used during last JET campaign.

1. INTRODUCTION

In the last years the new perspectives opened by the advanced tokamak scenarios have emphasised the requests for feedback control of several plasma parameters. This necessity is of particular relevance in the case of plasma scenarios aimed at the study of Internal Transport Barriers (ITBs), which are very promising in terms of performances and could benefit from the use of feedback control tools [1].

With all these long-term programmes in view, not only local quantities, like the current density and the q profiles [2], but also plasma confinement parameters like the internal inductance
l_i and the diamagnetic poloidal beta
β_{DIADIA} are of great interest. In JET it has therefore been decided to develop a fast algorithm, BETALI, for the real-time determination of these parameters.

As far as the plasma shape is concerned, a new technique [3] has been devised to determine the Last Closed Flux Surface (LCFS), on the basis of the poloidal magnetic flux Y[1] reconstruction obtained by the external magnetic measurements (section 2). Once the boundary has been identified, crucial surface-dependent quantities can be calculated. In particular, the Shafranov integrals (S_1, S_2, S_3) and moments (Y_i, i = 1..n) can be evaluated (section 2). The former are a prerequisite to the determination of the diamagnetic poloidal beta β_{DIA} and of the plasma internal inductance l_i, a quantity that also enters into the calculation of some of the main plasma parameters, like the stored MHD energy W_{MHD} and the confinement time τ_E (section 3).

The results of the described application have been extensively compared with the estimates of more complex off-line programs, like the equilibrium code EFIT [4], which solves the complete Grad-Shafranov equation, and FAST [5], and the agreement has been more than satisfactory, as described in section 4. In that section the performance of BETALI will be reported as well, showing the feasibility of using this code in real-time.

2. IDENTIFICATION OF THE PLASMA BOUNDARY AND OF THE RELATED INTEGRAL QUANTITIES

The starting point in the determination of the LCFS is the mapping of the Ψ function on a cross section of the machine. The boundary code XLOC allows the fast determination of Ψ using the vacuum region approximation and matching the real-time requirements by expressing it with a sixth order Taylor expansion [6].

As soon as the poloidal flux at the boundary Ψ_{LCFS} is available, its value has to be tracked down in the vessel cross section in order to determine the points belonging to the LCFS. This has been achieved by using 100 segments, called Gaps, whose layout can be seen in fig.1, together with an example of the calculated LCFS. Since Ψ is a function of the radial and vertical coordinates R and Z and has a monotonic trend from inside to the outside the plasma, the previously determined Ψ_{LCFS} is looked for along the Gaps by means of the fast bisection method. Indeed the monotonicity guarantees the existence of a unique solution of $\Psi(R, Z) = \Psi_{LCFS}$ along each Gap, and the bisection allows to find it with an efficiency of *O(log2 N))* cycles, where N is the number of points of each Gap. Accordingly, the point of each Gap whose flux value is the closest to Ψ_{LCFS} can be considered as belonging to the LCFS. At this point providing geometric quantities like the plasma volume V, the minor and major radii a and R_0 and the elongation k is immediate.

Also relevant to our application is the possibility of calculating, on the LCFS, the integral quantities used in the determination of the confinement properties of the plasma. In particular, the so-called Shafranov integrals S_1, S_2 and S_3 [7,8], are necessary to determine l_i and some of the main plasma confinement quantities (see next section). S_1, S_2 and S_3 are integrals of the various components of the poloidal field on the LCFS and are defined as [8]:

$$S_1 = \frac{1}{B_{pa}^2 V} \int_S \mathrm{B}_p^2 (R\bar{e}_R + Z\bar{e}_Z - R_c\,\bar{e}_R) \cdot \bar{n} dS \tag{1}$$

$$S_2 = \frac{1}{B_{pa}^2 V} \int_S \mathrm{B}_p^2\, R_c\,\bar{e}_R \cdot \bar{n} dS \tag{2}$$

$$S_3 = \frac{1}{B_{pa}^2 V} \int_S \mathrm{B}_p^2\, Z\bar{e}_Z \cdot \bar{n} dS \tag{3}$$

where S is the plasma surface, $\boldsymbol{e}_R$ and $\boldsymbol{e}_Z$ the vectors of R and Z, R_c a constant major radius (taken equal to 2.96 m, the radial coordinate of the vessel centre), B_{pa} the poloidal field for normalisation[2] and $\boldsymbol{n}$ the vector normal to the plasma surface.

To assess the quality of the results, our evaluation of the Shafranov integrals has been compared with the same quantities provided by EFIT, used generally in JET to obtain information on the magnetic configuration. In fig. 2 a typical example of the S_1, S_2 and S_3 evolution during a discharge is reported, showing the good agreement between our calculation and the EFIT outputs.

Other essential quantities that can be determined from the knowledge of the magnetic field at the LCFS, are the Shafranov moments [9,10], that are defined as:

$$Y_m = \frac{1}{\mu_0 I_p} \oint_\Gamma F_m B_p \, dl \tag{4}$$

where Γ is the contour of the plasma cross-section and F_m is a weighting function satisfying $\nabla \times \nabla \times F_m \hat{e}_\varphi = 0$, where $\hat{e}_\varphi$ is the toroidal coordinate vector.

In our application a particular role is assumed by the first two moments Y_1 and Y_2, since the former allows the determination of the coordinates of the current centroid [10], while the latter can be directly correlated to l_i (see below).

3. INTERNAL INDUCTANCE AND CONFINEMENT PARAMETERS

If the plasma is sufficiently elongated, the Shafranov integrals lead to the calculation of the plasma internal inductance l_i [8] according to the relation:

$$l_i = \frac{1}{\alpha - 1} \left[S_1 + S_2 \left(2 - \frac{R_t}{R_c} \right) - 2S_3 \right] \tag{5}$$

where R_t is the radial current centroid and the parameter a is defined as

$$\alpha = 2 \int_V (\overline{B_p} \cdot \overline{e_z})^2 \, dV / B_p^2 \, dV \tag{6}$$

In the determination of l_i the most delicate point is the evaluation of the volume integral a. Since in real-time the magnetic field values inside the plasma are not available, relation (6) becomes unusable. This quantity can be approximated in terms of k [8] by:

$$\alpha \cong \frac{2k^2}{k^2 + 1} \tag{7}$$

but this expression is not completely satisfactory.

To reduce the errors on l_i below 10%, an ideal parameter a_{ID} has been introduced, derived from (5) using all BETALI quantities except l_i, which has been taken from EFIT since this is the value that has to be reproduced. It has been found that α_{ID} shows a strong and clear dependence on k, Y_2 and on the quantity α_S, which can be thought of as the analogous of (6) but computed on the surface S instead than in the volume V. The α_{ID} parameter has been therefore considered as a linear combination of these three main quantities and their mixed products. Its values, determined from a database of 21 pulses representative of the most typical plasma configurations, has been used to derive the linear combination coefficients by inversion of the following super-determined matrix equation:

$$[\alpha_{ID}] = [X] \cdot [C] \tag{8}$$

In (8), [X] is the six-column matrix containing the parameters on which a_{ID} depends and [C] is the vector containing the 6 unknown coefficients. The number of patterns considered was 3000. The coefficients determined in this way have then been used to perform the calculation of a in real-time. A comparison of a time evolution of a BETALI estimate of

l_i and the corresponding signal from EFIT is reported in fig. 3, showing the good agreement between the two values during the entire shot. The time variations of l_i during the discharge are well reproduced and this is a fundamental key for the real-time control.

Once the plasma internal inductance has been properly estimated, it can be used to determine the MHD beta, b_{MHD}, with [7]

$$\beta_{MHD} = 0.5S_1\ [1 - 0.5 \cdot (1 - \frac{R_t}{R_c})]\ S_2 - 0.5l_i, \tag{9}$$

and to provide the MHD energy, W_{MHD}, with [5]

$$W_{MHD} = 4.7le^{-7} \cdot R_0 I_P^2 \beta_{MHD}. \tag{10}$$

b_{DIA} can then be expressed in terms of S_1, S_2 and of the plasma diamagnetic parameter m, which is supplied directly by XLOC, as follows [8]:

$$\beta_{DIA} = \mu + S_1 + \left(\frac{R_t}{R_c}\right) S_2, \tag{11}$$

whose related diamagnetic energy, W_{DIA}, is given by [5]:

$$W_{DIA} = 4.7le^{-7} \cdot R_0 I_P^2 \beta_{DIA}. \tag{12}$$

From W_{DIA} several important confinement parameters can be derived, like the diamagnetic toroidal beta β_{TOR} and the diamagnetic normalised beta β_{NORM} [11]:

$$\beta_{TOR} = \frac{19.123e^{-6} \cdot W_{DIA} R_0^2}{V \cdot B_{vac}^2} \tag{13}$$

$$\beta_{TOR} = \frac{56.605 \cdot W_{DIA} R_0\, a}{I_p \cdot V \cdot B_{vac}} \tag{14}$$

where B_{vac} is the toroidal field in the vacuum computed at R_c.

On the base of l_i and β_{DIA} also τ_E can be evaluated [5]:

$$\tau_E = \frac{W_{DIA}}{P_T - \frac{dW_{DIA}}{dt}} \tag{15}$$

where P_T is the total input power, given by the sum of the ohmic input power and the powers coming from the additional heating systems, namely the neutral beam injection, the radio-frequency heating, and the lower hybrid heating. The ohmic input power itself can be evaluated by means of l_i [5].

4. OVERVIEW OF THE RESULTS AND PERFORMANCES

4.1 OVERVIEW OF THE RESULTS

A list of the most important calculated quantities is given in table I. The application has been benchmarked successfully on a large variety of plasma discharges and can be considered of general validity. The most restrictive condition for the applicability of the routines is on the elongation: it must be sufficiently high ($k > 1.3$ for JET) in order to have a reliable value of S_3 and therefore of l_i. Moreover for small values of k the parameter α is very close to 1 and so the expression in (5) tends to diverge.

As an indication of the accuracy of our routines, in fig. 4 a statistical graphical comparison between the BETALI and EFIT estimates of l_i is shown, based on a database of 54 pulses. The closer the points are to the bisectrix, the more the real-time estimates are similar to the off-line ones.

The mean relative error (MRE[3]) and the mean squared error (MSE[4]), with respect to EFIT, of some of the evaluated plasma parameters, are summarised in table II. MREs of about 10% must be considered more than satisfactory for real-time control applications. The database is the same as the previous one.

Similarly table III shows the errors with respect to FAST for two other confinement parameters. In this case the values are worse, but it must be pointed out that the determination of these parameters is more difficult since it involves the real-time calculation of derivatives, and FAST itself gives, for these signals, a level of accuracy of 20-25%.

4.2 PERFORMANCE

BETALI has been written in C++ and the machine used to test it is a PC with a 400MHz Pentium II as processor running Windows NT.

The calculation of the LCFS is very fast with the described code, taking on average 600ms per cycle. The accuracy in the determination of the boundary spans from ≈0.01m near the equatorial plane at Z = 0m to ≈0.04m in the upper and lower regions of the plasma. This is mainly due to two reasons: the approximation of Ψ with a Taylor expansion and the uncertainty in the measurements of the magnetic probes. On the other hand, all the parameters (altogether more than 30) can be evaluated in about 900ms per cycle. It turns out that the overall updating time is about 1.5ms. This value is well below the sampling period of 10ms at which BETALI is requested to supply data to the JET control applications.

CONCLUSIONS AND FURTHER DEVELOPMENTS

The presented application allows to provide the main geometrical and confinement (if the plasma elongation is greater than 1.3) quantities with accuracy and performance that are perfectly appropriate for the real-time control applications in JET. The method is also very robust and suitable to both limiter and x-point configurations.

An optimised version of BETALI has been implemented into the JET real-time control system and its outputs have been used in the last experimental campaigns. In particular, the internal inductance is a relevant parameter for the current profile and can be given reliably since the very beginning of the pulse.

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Parameter class	**Parameter name**
Shafranov integrals and internal inductance parameters	first Shafranov integral S_1
	second Shafranov integral S_2
	third Shafranov integral S_3
	internal inductance l_i
Geometric parameters	minor radius a
	major radius R_0
	elongation k
	volume V
Confinement parameters	diamagnetic poloidal beta β_{DIA}
	diamagnetic energy W_{DIA}
	diamagnetic toroidal beta β_{TOR}
	diamagnetic normalised beta β_{NORM}
	MHD beta β_{MHD}
	MHD energy W_{MHD}
	total input power P_T
	confinement time τ_E

Table I Main real-time outputs of BETALI.

Plasma parameters	**MRE (%)**	**MSE**
l_i	2.59	0.0012
S_1	3.3	0.001
S_2	4	$1.87e^{-4}$
S_3	2.82	0.0001
β_{DIA}	10.79	0.0007 $[\%]^2$
β_{TOR}	11.14	0.0023 $[\%]^2$
β_{NORM}	10.96	0.0029 $[\%]^2[m]^2[T]^2/[MA]^2$
W_{DIA}	11.43	$1.96e^{10}$ $[J]^2$
k	1.26	0.0007

Table II: Mean relative errors (MRE) and mean squared errors (MSE), with respect to EFIT, of some of the plasma parameters evaluated by BETALI.

Plasma parameters	**MRE (%)**	**MSE**
P_T	7.42	$2.82e^{11}$ $[W]^2$
τ_E	17.16	0.022 $[s]^2$

Table III: Mean relative errors (MRE) and mean squared errors (MSE), with respect to FAST, of some of the plasma parameters evaluated by BETALI.

Footnote

[1] $\Psi(R, Z) = \frac{1}{2\pi}\int_{\Sigma} \bar{B}_P(R, Z) \cdot \bar{n} d\Sigma$, where R and Z are respectively the radial and vertical coordinates, $\mathbf{B}_P$ is the poloidal magnetic field, S is a circular surface centred on the axis of the machine and **n** the versor normal to it.

[2] $B_{pa} = \sqrt{\frac{m_0^2 R_c I_p^2}{2V}}$, where I_p is the plasma current and m_0 is the diamagnetic parameter in the vacuum.

[3] $MRE = \left(\sum_{i=1}^{N} \left\| \frac{S_i^R - S_i^B}{S_i^R} \right\| \right) / N$, where S^R is the reference off-line signal, S^B is the analogous real-time signal from BETALI, and N is the total numbers of samples.

[4] $MSE = \left(\sum_{i=1}^{N} (S_i^R - S_i^B)^2 \right) / N$.

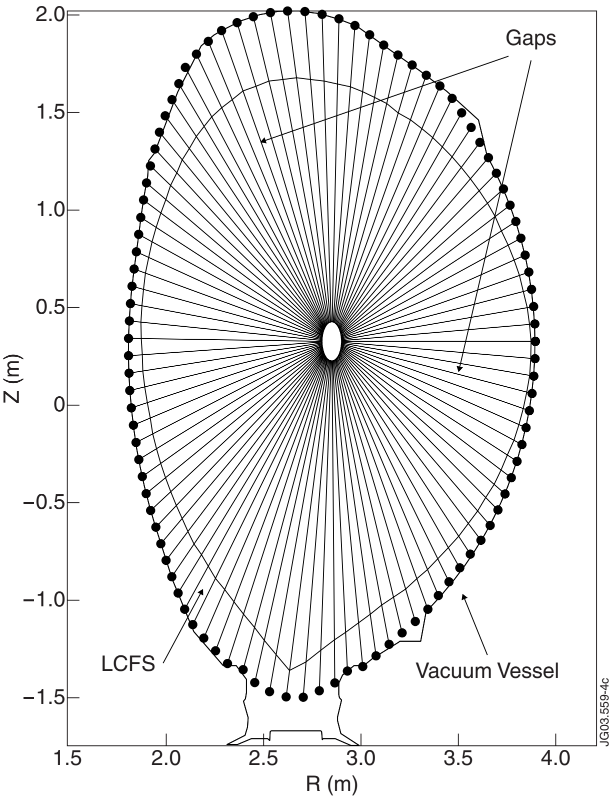

Figure 1: Cross section of JET vessel with the Gaps layout and an example of the calculated Last Closed Flux Surface.

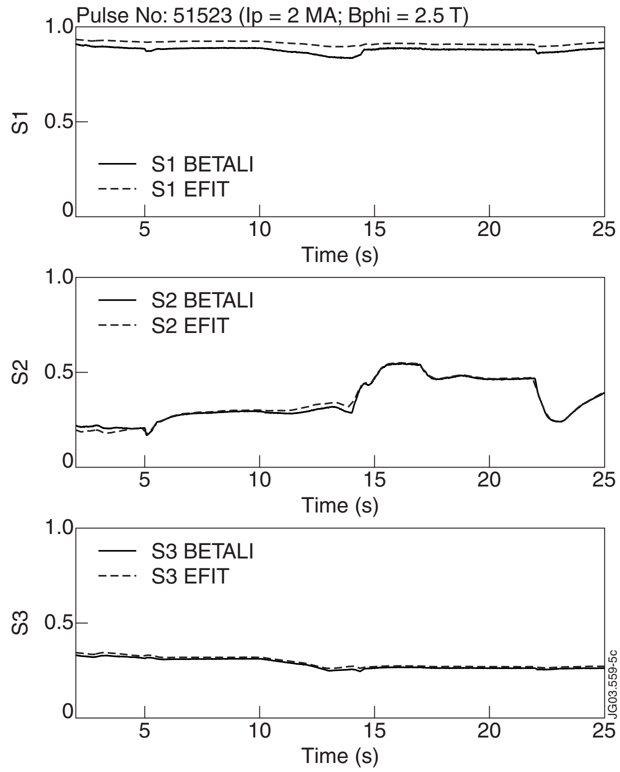

Figure 2: Comparison of the values of S_1, S_2 and S3 determined by BETALI (solid line) and EFIT (dotted line) for the Pulse No: 51523; Ip is the plasma current and Bϕ is the toroidal magnetic field.

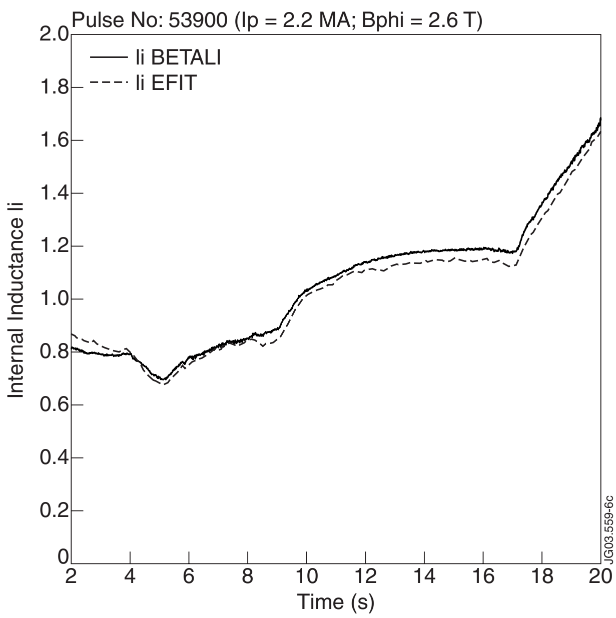

Figure 3: Time evolution of BETALI (solid line) internal inductance compared with the one from EFIT (dotted line) for Pulse No: 53900; Ip is the plasma current and Bϕ is the toroidal magnetic field.

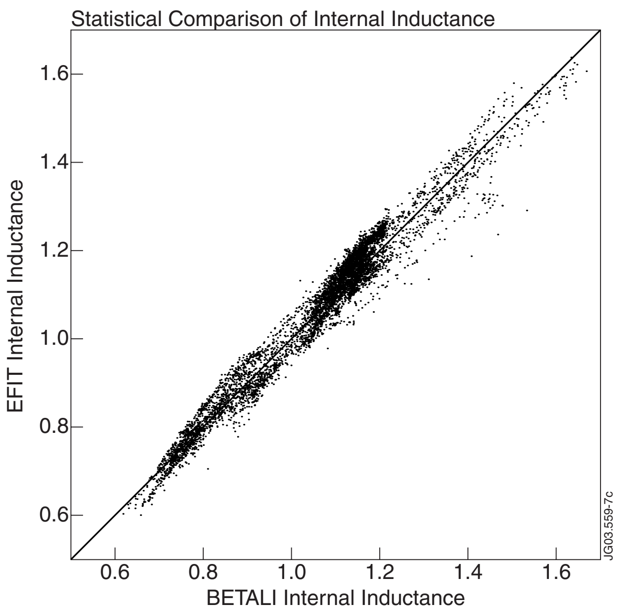

Figure 4: Statistical graphical comparison of the l_i *estimates computed by BETALI and by EFIT.*